Problems in the Analyses of Final-State Resonances

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I. INTRODUCTION

The clear evidence, accumulated in the last few years, of important deep symmetries of the strong interactions has lent a new sense of direction to the experimental studies of the resonance states of elementary particles. Our attitude towards the meaning of fundamental particles has changed and we now regard many of the resonance states as having a fundamental character in the same sense as the nucleon or pi meson. Furthermore, the symmetry descriptions which suggest the fundamental character of these states also result in quite specific predictions concerning some of the properties of the states and result in specific questions concerning other properties. Since many of these states can be prepared and observed only as final-state interactions, an understanding of the complications of final-state interactions is an essential part of our understanding of elementary particles and the strong interactions.

There is a parallel between our view of the strong interactions and the elementary particles today and a view of nuclear forces and light nuclei which Wigner[1] showed us thirty years ago. It is attractive, in this setting, to emphasize this parallel in a brief review of elementary-particle theory designed to establish a context for our discussions of final-state interactions.

In nuclear physics, the importance of the deuteron, the stable isosinglet triplet state of the nucleon–nucleon system, is hardly to be considered more important or more fundamental than the unbound isotriplet singlet states of the proton–proton, neutron–proton, or neutron–neutron system. These six states, counting spin multiplicities, belong to, and constitute, a single SU_4 supermultiplet or representation with a dimension of six. In the approximation that SU_4 symmetry is exact, that only Wigner and Majorana forces obtain, the six states would be degenerate and have the same space radial wave function. In the same way we can not now consider that the pi meson, stable with respect to the strong interactions, is more important, or at least more nearly fundamental than the eta meson which is a state in the continuum, unstable against decay through the strong interactions. The eta and pi appear to be members of the same representation or multiplet of a SU_3 group, and in the approximation that SU_3 symmetry is exact, the eta and pi are degenerate.

[1] E. Wigner, Phys. Rev. **51**, 106 (1937).

It is possible that the striking SU_3 symmetry[2] can be usefully considered as part of a larger symmetry. Even as the isosinglet and isotriplet states of two nucleons constitute two different representations of SU_2, the group of transformations invariant under charge independance, where SU_2 is a subgroup of SU_4, it is possible that SU_3 is a subgroup of a larger group. In this case different representations of SU_3 comprise a single representation of this larger group. One of the interesting possibilities, presented here as an example of current thought, is that a larger group including spin can be constructed[3] which transforms as SU_6. In the same way that the separate isosinglet and isotriplet nucleon–nucleon representations of SU_2 comprise the six elements of a single SU_4 representation, the scalar octet, vector octet, and vector singlet meson representations of SU_3, might constitute a single representation of SU_6 with a dimension of 35. In a similar vein, the spin-$\frac{1}{2}$ baryon octet and spin-$\frac{3}{2}$ decuplet, discrete representations of SU_3, might constitute a single SU_6 multiplet with a dimension of 56.

The majority of these states are in the continuum, and a majority are only accessible experimentally through the analyses of final-state interactions. Table I lists these states together with important quantum numbers, masses, and some other relevant parameters. Besides these states listed in Table I, which might be considered to comprise the ground-state multiplets, the existence of some other states has been established experimentally, and still others are suggested with various degrees of certainty. These might represent members of higher representations or excited states of the basic meson and baryon particles. Almost all of these are accessible only through studies of final-state interactions and an understanding of their character should also provide important information concerning both the group character of the interactions and the detailed dynamics.

While it is a tautology that all possible information concerning these states must be relevant to a complete understanding of the strong interactions and elementary particles, specific characteristics bear on specific questions in a manner which allows an ordering of importance or at least immediacy to the determination of these characteristics.

[2] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[3] F. Gursey and L. Radicatti, Phys. Rev. Letters, **13**, 299 (1964); B. Sakita, preprint, University of Wisconsin.

TABLE I. Mass and symmetry parameters of elements of a proposed baryon SU_6 multiplet of dimension 56.

State	Mass	Y	T	T_a	V	V_3	SU_3
η^0	938.3	1	$\frac{1}{2}$	$-\frac{1}{2}$	1	-1	8
p^+	939.6	1	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	8
Σ^-	1197.0	0	1	-1	$\frac{1}{2}$	$-\frac{1}{2}$	8
Σ^0	1192.3	0	1	0	1, 0	0	8
Σ^+	1189.4	0	1	$+1$	$\frac{1}{2}$	$+\frac{1}{2}$	8
Λ^0	1115.4	0	0	0	1, 0	0	8
Ξ^0	1314.3	-1	$\frac{1}{2}$	$-\frac{1}{2}$	1	$+1$	8
Ξ^-	1320.8	-1	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	8
$N_{\frac{3}{2}}^-$	1238	1	$\frac{3}{2}$	$-\frac{3}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	10
$N_{\frac{3}{2}}^0$	1238	1	$\frac{3}{2}$	$-\frac{1}{2}$	1	-1	10
$N_{\frac{3}{2}}^+$	1238	1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	10
$N_{\frac{3}{2}}^{++}$	1238	1	$\frac{3}{2}$	$\frac{3}{2}$	0	0	10
Y_1^-	1385	0	1	-1	$\frac{3}{2}$	$-\frac{1}{2}$	10
Y_1^0	1385	0	1	0	1	0	10
Y_1^+	1385	0	1	1	$\frac{1}{2}$	$\frac{1}{2}$	10
$\Xi_{\frac{1}{2}}^-$	1530	-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	10
$\Xi_{\frac{1}{2}}^0$	1530	-1	$\frac{1}{2}$	$\frac{1}{2}$	1	1	10
Ω_0^-	1675	-2	0	0	$\frac{3}{2}$	$\frac{3}{2}$	10

With this scale of values, the group properties of a state are most important. The values of the discrete quantum numbers, the spin, parity (to the extent it is operationally defined), hypercharge or strangeness, and electric charge, are necessary to establish the place of the state in any hypothetical large symmetry such as SU_3. Even as states belonging to the same representation of SU_2 under charge independence, such as the three singlet states of the two-nucleon system, must have the same spin and parity, the elements of each representation of SU_3 must have the same spin and parity. Even as the elements of the larger group SU_4, in nuclear physics, have different, though specified, spins, and the same parity, elements of the octet and decuplet which may make up a single elementary-particle representation of SU_6 must have the same parity as well as specific predicted values of the spin.

Some SU_3 representations, the octet, for example, have certain positions of the T_3, Y plane occupied by more than one element: T_3 and Y do not uniquely specify an element. When the degeneracy of all states of a representation is removed by the symmetry breaking forces the resultant eigenstates, which have the same T_3 and Y assignment, are found to have different values of isotopic spin: to a good approximation they belong to different representations of that SU_2 symmetry. In this way the Λ^0 and Σ^0 are labeled as $T=0$, and $T=1$ states; the η and π^0 are also $T=0$, and $T=1$ states. There are other SU_2 symmetries, subgroups of SU_3, such as U spin. Linear combinations of Σ and Λ or π and η comprise the eigenstates of U spin. The basic octet and decuplet baryon states plotted on a T_3, Y plane are shown in Fig. 1 together with an indiction of representations of the SU_2 subgroups, isospin, and U spin.

The mass of the states is an essential parameter qualitatively, as the multiplet character is, as a practical matter, only noticeable among states whose masses do not differ too widely. The precise value of the energy is important since the energy differences between elements of a representation allow a measure of the strength and character of the symmetry breaking part of the whole interaction. It appears that these splittings are consistent with the hypothesis that the symmetry-breaking interaction transforms as the hypercharge Y, a component or vector of the eight representation of SU_3, and that the splittings, which are of the order of 100 MeV/c^2, can be adequately calculated using first order perturbation theory.[4] The parallel in nuclear physics is clear: The isotopic-spin multiplets of light nuclei are split by the Coulomb interaction which transforms as the isovector T_3, an element of the three-dimensional representation of SU_2.

The electromagnetic splittings among the isotopic-spin multiplets of an SU_3 representation again appear to be rather precisely discribed by first-order perturbation theory.[5] Here, the electromagnetic interaction again transforms as a vector of the eight representation, but in a different direction than the strong interaction splitting: $Q=T_3+Y/2$.

The mass splittings are not precisely known for the unstable particles where the resonance widths are comparable to and larger than the suggested accuracy of the mass formulas. Rather precise measurements of the masses of the resonances are then desirable.

Even as the symmetry-breaking interaction generates a mass splitting among the elements of an SU_3 representation and the electromagnetic interaction generates splittings among the isotopic-spin submultiplets, these interactions will be expected to mix wave functions.

[4] M. Gell-Mann (Ref. 2); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 929 (1962).

[5] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).

The electromagnetic interaction can be expected to mix the Λ^0 and Σ^0 isotopic-spin states[6]; the real Λ^0 and real Σ^0 will not be eigenstates of isotopic spin, but will be mixtures of $T_0{}^0$ and $T_1{}^0$ states. The real π^0 will be a mixture of the states which would correspond to the π^0 and η^0 in the absence of electromagnetic forces. The mixing in these cases is not large but might be observed by noting the presence of the reaction: $K^-+p\rightarrow Y_0{}^*(1520)\rightarrow\Lambda_0+\pi^0$, forbidden by charge independence.

Larger effects may be expected from the symmetry-breaking interaction which transforms as Y. The energy splittings resulting from this symmetry breaking interaction are quite large and we might reasonably expect appreciable mixing of different SU_3 representations to result from this interaction. The $Y_1{}^*$ (1385), nominally belonging to the 10 representation may then have a mixture of other representations. In general, this should effect partial decay widths. In a pure SU_3 symmetric universe the decay rates of $Y_1{}^{*+}\rightarrow\Lambda+\pi^+$, $Y_1{}^{*+}\rightarrow\Sigma+\pi$, and $N^*_{\frac{3}{2}}(1238)\rightarrow p+\pi^+$, will be in the ratio of 0.5:0.33:1.0. These predictions are changed in two ways by the symmetry-breaking interactions: There are kinematic effects resulting from the differences in available phase space and centrifugal barriers resulting from the changes in mass levels, and dynamic effects resulting from the impurity of representation in the wave functions of the real $Y_1{}^*$ and N^*, as well as the decay products. The kinematic effects can be approximated by considering that the width for the P-wave decay of these states varies as p^3, where p is the center-of-mass momentum. The predictions are then modified to: 0.34:0.08:1.0. The accepted experimental ratios are about: 0.50:0.02:1.0. The rather large difference requires a considerable representation mixing. While the mixing expected from the symmetry-breaking part of the interaction is in a direction to improve agreement,[7] there is still no quantitative understanding of the small value of the $Y_1{}^*\rightarrow\Sigma+\pi$ partial width.

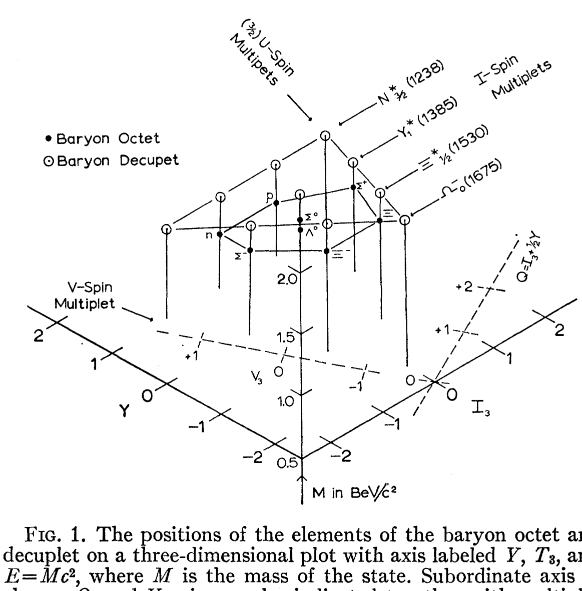


FIG. 1. The positions of the elements of the baryon octet and decuplet on a three-dimensional plot with axis labeled Y, T_3, and $E=Mc^2$, where M is the mass of the state. Subordinate axis of charge, Q, and V spin are also indicated together with multiplet arguments.

[6] R. H. Dalitz and F. Von Hipple, Phys. Letters **10,** 153 (1964).

II. PHASE SPACE

As an empirical matter it has been found useful in high-energy physics to consider certain classes of deviations of many-body production spectra from that deduced by considering only the density of relativistic phase space as serious evidence for the existence of final-state interactions. It is then useful to consider in some detail the meaning of phase space[8] and the character of the distortions we must anticipate as a consequence of effects other than specific interactions between the particles which make up the final state.

We may write the probability of producing a single final state f through the interaction of two particles making up an initial state i as

$$P(i\rightarrow f)\approx\langle f\mid s\mid i\rangle^2.$$

Neglecting spins we can define a single state as that representing a unit of four-momentum-space volume: $d^4p_1'd^4p_2, \cdots, d^4p_n'$, where p_1', p_2', $\cdots$, p_n' represent the four-moments of the n particles making up the final state. In the approximation that transition probabilities do not depend upon the values of the p_i, the transition probability to any subunit of volume of phase space within the constraints imposed by the conservation laws, is proportional to that volume and the decay spectra is determined by the properties of the available phase space. If we write the transition probability as

$$I(i\rightarrow f)\propto M_L{}^2P_L,$$

where M_L represents the matrix element and P_2 the phase-space volume:

$$\iint\cdots\int d^4p_1d^4p_2, \cdots, d^4p_n,$$

where the integration is over the region constrained by the conservation equations. The matrix element $|M_L|^2$ and the phase-space volume P_L are separately Lorentz-invariant.

This is not the phase space used by Fermi to discuss beta decay, nor the phase space used by Planck to consider blackbody radiation. In the calculations traditionally used for the purpose of these calculations it is assumed that the individual state is a unit of three-

[7] R. E. Cutkosky, J. Kalekar, and P. Tarjanne, Phys. Letters **1,** 92 (1962).

[8] R. Hagedorn, *Relativistic Kinematics* (W. A. Benjamin, Inc., Amsterdam, 1963).

momentum space volume: $d^3p_1'd^3p_2', \cdots, d^3p_n'$. The available phase space is the volume of the three-momentum space within the surfaces of constraint which result from the conservation of momentum and energy. Now we write the transition probability as $I(i\rightarrow f)=|M_F|^2P_F$. The probability is, of course, an invariant, but the matrix element M_F^2 and the phase-space volume,

$$P_F=\iint\cdots\int d^3p_1'd^3p_2', \cdots, d^3p_n',$$

are not separately Lorentz-invariant. This is not particularly relevant if, as is always the case, the evaluations are consistently made in the center-of-mass system.

There appears to be no strong fundamental reason to prefer one formulation over another. Largely because the Lorentz-invariant phase space is easier to calculate, it is used by high-energy physicists.

The two formulations are related in a simple way: in the center-of-mass system: $M_L^2=M_F^2[\Pi_n(2E_i)]$, and $P_L=P_F[\Pi_N(2E_i)]^{-1}$, where E_i is the total energy[9] of particle i. The phase-space momentum spectra for a specific number of particles is the same for each formulation if the particles are non relativistic as then $E_i\approx m_i$ is approximately a constant and the formulations differ only by unimportant normalizations. For relativistic particles, the two definitions lead to quite different results. The usual statement is that allowed β-decay matrix elements are constant and that the spectra are determined solely by phase space. If the Lorentz-invariant phase space is used, we conclude that the square of the matrix element is proportional to the product of the total energies of the three residual particles. While the choice of definition of phase space is quite irrelevant to the calculation of the transition probability for any completely defined or completely understood process, a difference in choice does lead to quite different expectations if the phase-space distribution is to be a guide to an estimate of the distribution of the momentum spectra from some process which is not understood in detail. The idea that matrix elements might well be constant is attractive, perhaps because the quality of being constant is the simplest property we can attribute to such a quantity. To see that even this constancy is not simple and indeed implies quite specific properties to the interaction is useful if it serves to consider this constancy as not much more likely or unique than other simple behaviors.

The condition that all states are equally probable cannot obtain, as a result of unitarity considerations, for most important interactions of strongly interacting particles. A simple example serves to illustrate the character of one important consideration. Consider a reaction of the type $a+b\rightarrow c+d$, where for simplicity we consider particles a, b, c, and d, without spin. If the cross section is larger than πk^{-2} the production cannot be wholly s wave, and the angular distribution cannot be isotropic. If the production is not isotropic, transition probabilities to the available final states are obviously not equally probable. While the situation is more complicated for particles with spin, restrictions of this character still exist. These are limits, resulting from unitarity, on the magnitude of the part of the matrix element which is independent of the p_i, and these limits are exceeded in most interactions which produce many-body final states.

There are related restrictions which will be important at high energies where the mean angular momentum of the collision is high. This will be the case when $\sigma_t\gg\pi k^{-2}$, where σ_t is the total cross section and k the wave number of the incident particles in the center-of-mass system. In the classical limit, the final-state momenta will be constrained to the plane normal to the angular momentum vector which is certainly a deviation from phase space. Such distributions[10] have been noted in analyses of very high energy events.

At high energies, where high angular momenta are important, the mean relative angular momentum of any two of the final-state particles, or the mean angular momentum of any one particle in the center of mass system, is likely to be large. If q is either the momentum, in the center of mass of the two particles, i, j, or the momentum in the center-of-mass system of a single particle k, for a production configuration in the final-state momentum space labeled by the momenta $p_1, p_2, \cdots, p_n$, we expect that for a region of momentum space near the points where $qr\ll 1$, where r is a characteristic length $\approx h/m_\pi c$, the wave functions $\Psi(p_1, p_2, \cdots, p_n)$ can be expanded in states of definite relative angular momentum l of the particles i, j or the particle k, in a form such as

$$\Psi_{ij}=\sum a_l q^{l+\frac{1}{2}} Y_l.$$

Here the a_l are weakly varying functions of the p_i and the Y_l are spherical harmonics. The factors q^l represent the influence of centrifugal barriers, $q^{\frac{1}{2}}$ is proportional to the phase-space volume. If only the phase-space volume were relevant, the a_l, for $l\geq 1$, will be very small. Their existence requires the intensity per unit phase space to be a minimum for configurations such that $q=0$.

These closely related classes of distortions from a pure phase-space spectrum explicitely result from

[9] This factor can be considered as a relativistic normalization of the wave function: ψ^2 the particle density is not Lorentz-invariant as the volume contracts by a factor γ^{-1} and the density increases by a factor γ as the system is viewed by a moving observer. $\psi^2/2E$ is however an invariant as the total energy E also changes by γ.

[10] S. Hayakawa, in *Proceedings of the 1962 Annual International Conference on High Energy Physics*, edited by J. Prentki (CERN, Geneva, 1962).

matrix elements which are not independent of p_i. Since the strong interactions cross sections are large compared to πk^{-2}, which is approximately the limit imposed by unitarity on the possible contributions to the cross section from constant matrix elements, matrix elements which are dependent on the vector momentum must be important.

Any general discription of phase-space distributions is complicated by the high dimensionality of the manifold when many particles are involved. For three particles most of the information concerning the final state can be presented simply in terms of the well-known plot introduced by Dalitz[11] to discuss K-meson decays and extended by Fabri[12] to relativistic situations. Such a Dalitz–Fabri plot is shown in Fig. 2. The ordinate and abscissa are labeled either by the center-of-mass kinetic energies of two of the particles, or by the square of the invariant mass of two different pairs of particles. The boundary of the figure, which has a simple analytic description only in particular cases, represents the constraint limit imposed by the conservation laws. At the boundary the three momenta are collinear. Typical configurations of the three momenta are shown on the figure. The position of a point on the plot represents all of the information concerning a particular event, except the polarizations of particles with spins, and except for the direction of the incoming beam.

The character of the plot which makes it particularly useful is the property that equal areas on the plot are proportional to equal volumes of invariant phase space. If the relativistically invariant matrix element is independant of the final-state momenta in the center-of-mass system, aside from an over-all orientation with

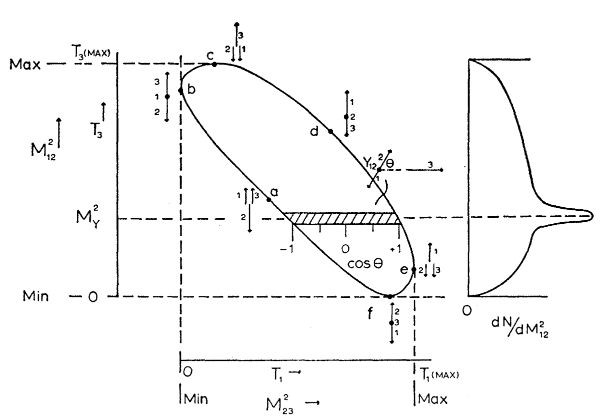


FIG. 2. A schematic Dalitz–Fabri plot representing the center-of-mass configurations of three particles, a_1, a_2, and a_3. The particles a_1 and a_2 scatter resonantly at an energy M_y. Scales of kinematic energy, invariant mass, and an accompanying plot of intensity vs mass are shown together with schematic representations of relative momentum directions corresponding to specific points on the diagram.

respect to the direction of the incoming beam, the density of events, plotted on the Dalitz–Fabri plot, will be constant. No such simple relation has been formulated for larger numbers of final-state particles.

It is convenient to discuss qualitatively the influence on distributions plotted in such a diagram of the effects we have noted resulting from the importance of high angular momentum, an importance evident from the size of the strong-interaction cross sections relative to unitarity limit imposed on the magnitude of a constant matrix element.

The well-known anisotropic angular distribution of final-state particles does not effect the Dalitz–Fabri plot distributions which present only an integration over all production angles. The restrictions resulting from the presence of angular momentum barriers might be expected to reduce the probability of events with configurations represented by points in regions near b, d, and f on Fig. 4, where one particle has zero momentum in the center-of-mass system, or near a, c, or e, where two of the particles have zero momentum in their center-of-mass system. The tendency of final-state momenta to be constrained to a plane normal to the angular momentum vector rather than be distributed in three dimensions results in a higher correlation of momenta which tends to reduce the intensity in the center of the plot and enhance the intensity near the edges where the momenta are collinear.

There is no evidence for any very strong effect from these factors in general. Angular momentum barriers may not be so important for elementary particles interactions which are relatively long tailed, as for nuclear interactions which are characterized by a comparatively sharp boundary. Also, of course, the two specific contributions to inhomogeneity of the Dalitz–Fabri plot tend to cancel out; one reduces intensity at the border, one enhances the border.

Indeed the fit, in general, of certain types of final-state elementary-particle distributions to the prediction of the Lorentz-invariant phase space is extraordinarily good. Figure 3 shows a typical distribution of invariant mass[13] together with the spectrum predictions from the Lorentz-invariant phase space and the spectra from the Fermi phase space. While the fit is better with the invariant phase space,[14] the difference is not great. Since the Fermi phase-space distribution is equivalent to a Lorentz-invariant phase-space distribution with a matrix element proportional to $(\Pi E_i)^{\frac{1}{2}}$, the difference between predicted intensities in particular regions of phase space is very great. It seems that very often, as in this particular case, invariant mass distributions represent averages over regions of

[11] R. H. Dalitz, Phys. Rev. **94,** 1046 (1954).
[12] E. Fabri, Nuovo Cimento **11,** 479 (1954).

[13] C. Baltay, T. Ferbel, M. Gailloud, A. H. Bachman, *et al.* in *Nuclear Structure, Proceedings of the International Conference at Stanford University,* edited by R. Hofstadter and L. I. Schiff (Stanford University Press, Stanford, California, 1962).
[14] Professor Jack Sandweiss (private communication).

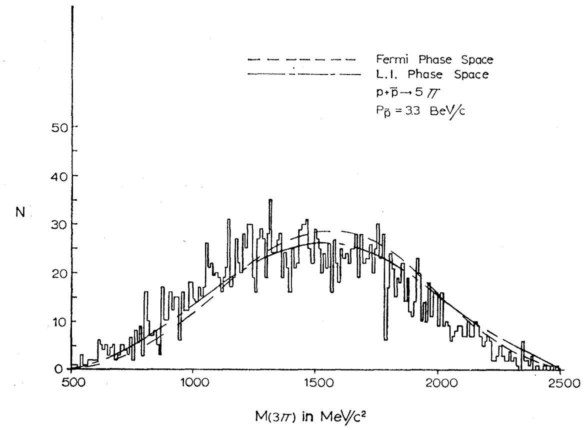


FIG. 3. The histogram represents the measured invariant-mass distribution of three π mesons from the reaction $p+\bar{p}\rightarrow 5\pi$. The other lines represent the distributions expected from Fermi phase space, and from Lorentz-invariant phase space. The deviation remaining between measured values and the Lorentz-invariant phase space can be understood quantitatively as resulting from a small contamination from misidentified events of the type: $p+\bar{p}\rightarrow 5\pi+\pi^0$.

phase space in such a way as to reduce the effect of energy-dependent matrix elements. Effects of the existence of angular momentum barriers may, then, often have little effect on invariant-mass distributions for the same reason.

III. FINAL-STATE INTERACTIONS—KINEMATIC EFFECTS

In the limit that the width, or imaginary part of the mass, of the state Y which decays into particles a_1 and a_2 is small compared to the region of invariant mass of a_1 and a_2 allowed kinematically in an interaction which results in a production of $a_1, a_2, \cdots, a_n$, and in the limit that the intensity of production of the state Y is large compared to the nonresonant production of a_1 and a_2, both the dynamic properties and symmetry properties of the state can be studied as if the state is pure. The dynamic studies can proceed as if the state can be isolated from its production process and in its decay process. If this is the case the simple Breit–Wigner form $I_Y(E)\propto \Gamma_Y/[(E_\lambda-E)^2]$ will be valid where Γ_Y is the partial width for the decay into particles a_1 and a_2, Γ is the total width, E_λ is the resonance energy and $I_Y(E)$ is the intensity of the (a_1, a_2) state as a function of the invariant mass E. Symmetry properties—spin, parity, isotopic spin, etc.—can then be examined from analyses of angular distributions, and polarizations of a_1 and a_2, and from branching ratios, again as if the state Y were isolated so as to be independent of the production process and the environment of other particles in the final state.

The degree of validity of the approximation that the resonance state can be considered isolated depends in detail on the properties of the state and on the production process. It is important to obtain some understanding of the character of the deviations from the assumption that the state is pure, isolated from its environment, both for the purpose of understanding the relation of the experimental quantities to the properties of the pure or isolated state, which are of primary theoretical interest, but also to gain some insights which might suggest experimental designs which would minimize such complications.

It appears that a useful division of the total effects of the local environment can be made into two classes: the class of effects which result from the coherence of the resonant amplitude with nonresonant amplitudes, and a class of effects resulting from the interaction of the state Y and its decay products with other final state particles. It is convenient to label this second class of effects as dynamic and the first class, which does not manifestly involve forces, as kinematic[15]—it is recognized that the use of the term is not completely conventional.

In the limit of small width or long lifetime, the decay of a state takes place so far, in space and time, from the production event, and the other particles associated with the production event, that any interaction with that part of the environment can be ignored. If the mass of the decaying particle is constrained by the measuring process the decay will proceed exponentially in time and we can write the amplitude as a function of time as: $A(t)\propto \exp(-i\Gamma t/2\hbar)$, where Γ is the width of the state. A Fourier transform gives the amplitude as a function of the mass E; $A(E)\propto (E_\lambda-E+i\Gamma/2)^{-1}$. The properties of this expression, represented as a vector in the complex plane, may be seen more clearly when written in the equivalent form:

$$A(E)=A\exp(i\delta)\sin\delta,$$

where $\delta=\tan^{-1}(\frac{1}{2}\Gamma/(E_\lambda-E))$. Here A is complex and E_λ is the resonance energy equal to Mc^2 where $M+i\Gamma$ is the complex mass of the state Y. This vector traverses a circle in the complex plane moving counterclockwise with increasing energy. A clockwise motion would represent a state with intensity increasing with time, violating causality.

Our viewpoint is essentially that of considering the real world as a perturbation on a world in which the state Y is stable, the decay interaction turned off. For some purposes a complementary view is more useful. The real world is a perturbation on a world in which the final-state particles do not interact and no Y particle exists. Then the particle–particle interaction in the final state, which may be strong enough to result in a Y resonance, is the perturbing effect. It is this view which has led to the name of final-state interactions for the resonant behavior which we also can

[15] R. K. Adair, Rev. Mod. Phys. 33, 406 (1961).

consider as resulting from the production of unstable particles.

In the following detailed discussions, for reasons of simplicity, final-state systems of three particles will usually be considered where two of the three particles are the delay products of a particular quasistable state. Figure 2 shows a Dalitz plot of the final-state configurations of particles a_1, a_2, a_3 and an accompanying invariant mass plot of a_1, a_2 where a sharp resonance is observed in the presence of a background characterized by uniform density in phase space. Equal intensities along the line of constant invariant mass on the Dalitz plot represent equal intensities of $\cos\vartheta$, where $\cos\vartheta$ in the angle of the decay of the Y with respect to the direction of production of the Y in the center-of-mass system.

It is almost always more useful to discuss resonances in terms of energy or invariant-mass distributions, as on a Dalitz plot, rather than use a complementary description of direct or immediate production of a long lived state, which later decays, together with direct production of a non resonant character. A two-dimensional Fourier transform of the energy distributions of the Dalitz plot of Fig. 2, will show long time components, M_{12}, generated by the sharp peak in the M_{12} invariant-mass distribution together with short time components from the contribution of the broad background distribution of M_{12} invariant mass. Even the resonant part of the distribution will contribute only to short time components τ_{13} and τ_{23}, the time particles a_1, a_3 and a_2, a_3 spend together, consistent with the simple description in the classical limit that particles a_1 and a_2 separate a long time after the interaction while a_3 separates from a_1 and a_2 immediately.

Usually it is important to consider interference between resonance and background, or between two resonances. Since energies, not times, are invariably measured in pertinent experiments, the interference behavior is usually studied as a function of energy. When intensities over small energy intervals $\Delta E \lesssim \Gamma$ are studied, the smallest meaningful time interval is $\approx\hbar/\Delta E$ and any operational concept of sequence must be discarded.

The state Y, shown in Fig. 2, may also decay into other modes, perhaps two-body modes b_1, b_2. Then a Dalitz plot of the energy distribution of b_1, b_2, a_3 should also show an intensity ridge along a line corresponding to the loci of b_1, b_2 invariant mass is equal to M. For a situation as presented in Fig. 2, where the peak is narrow and the background small, the invariant mass and total width of the state are just the central mass of the distribution and the width of the distribution for each decay mode. The branching ratio for decays to a_1, a_2 to b_1, b_2 are just the ratios of the intensities under these peaks.

Such simple interpretations of the data are not so reliable if the resonance is broad and if the background is substantial. Interference with background nonresonant production can result in complications similar to those observed in total-cross-section measurements in nuclear physics. We consider this in some detail writing for the production amplitude: $\Psi = \sum \mu_{jn} k_j y_n$, where y_n represents the angular part of the wave function of the two particles a_1 and a_2, n representing the total angular momentum, the z component in the beam direction, and the parity; k represents the angular part of the wave function of particle a_3 together with the center of mass of the y-system consisting of the pair a_1, a_2. Again j is an index representing angular momentum and the z component. Dalitz-plot distributions or invariant-mass plots derived from bubble chamber experiments generally represent integrations over production angles. Then states y produced with different angular momenta j are effectively orthogonal. Since the intensities also represent integrations over all decay angles there is no interferences between y states of different angular momentum. The only important interference effecting intensity is interference between the resonance amplitude and the background amplitude with the same quantum numbers. There is a close parallel with the behavior of resonance scattering in total cross section measurements; only interference with the background amplitude of the resonance quantum numbers need be considered.

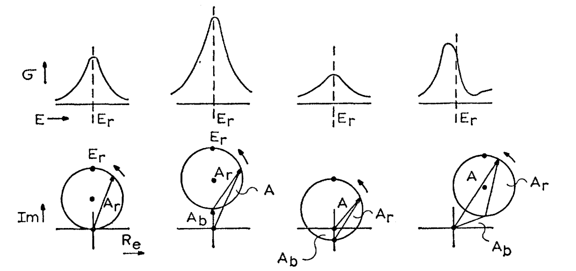


FIG. 4. Amplitudes and intensities resulting from the addition of a resonance amplitude, A_r to a background amplitude A_b under various conditions of relative magnitude and phase.

There is then reason to feel that intensity measurements and branching ratios determined from bubble-chamber measurements without cutoffs or restrictions in the angular acceptances of events may not be too much troubled by interference with backgrounds. There may be larger effects from this source of interference, from data taken from select angular intervals.

The variety of consequences of such interference is shown in the diagrams of Fig. 4 which show the behavior of background amplitudes and resonant amplitudes as a function of energy near resonance. The resonant amplitude is the same in the four cases; in Fig. 4(a) there is no background, Fig. 4(b) shows a large constructive interference, Fig. 4(c) a large destructive interference, Fig. 4(d) an interference which shifts the peak energy. While 4(b) and 4(c) represent

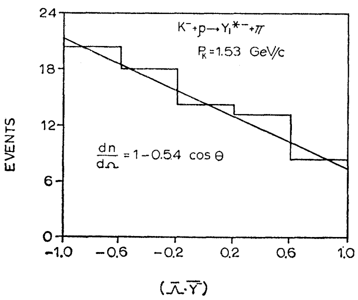


FIG. 5. Angular distribution of the Λ from $Y_1^*(1385)$ decays from the reaction $K^-+p\rightarrow Y_1^{*-}+\pi^+$. The laboratory momentum of the K is 1.53 BeV/c. J. Button-Shafer, D. Huwe, and J. J. Murray (Ref. 18).

pathological extremes they do service to illustrate that such resonance parameters as resonance width, resonance energy, and intensity and hence branching ratios, are sensitive to interference with the background.

Information concerning the spin and parity of a nearly stable baryon state can be obtained from the measurement of the angular distribution and polarization distribution of the two-body decay products. Most of the information concerning spin and parity has been obtained from methods of analysis which serve primarily to exclude specific possibilities, particularly low spin values, rather than allow a specific identification of spin and parity. For example, the decay spectrum of a state cannot have an angular distribution more complicated than $\cos^{2j-1}\vartheta$, where j is the spin.[16] A determination of the existance of a $\cos^{2n}\vartheta$ term in any decay distribution then excludes the possibility of spin j such that $j<n+\frac{1}{2}$. At the other extreme a state Y produced in a reaction such as $K^-+p\rightarrow Y+\pi$, where the target proton is polarized and the K^- is captured in an s state, are in the definite state $\Psi_j^{\frac{1}{2}}$ where j is the spin and the m component is in the initial polarization direction. Decay and polarization distributions from the decay of a state so defined are completely and discretely determined by the values of the spin j and the parity of Y.

There is a considerable literature concerning analyses of isobar decay data for the purpose of spin and parity determinations. A brief, but general, review of these techniques has been presented by Dalitz.[17] While these, sometimes elaborate, procedures are exact for isolated long lived states, their results, for most final-state resonances, must be regarded as useful approximations of varying validity. The interference of these states with background intensity, or with other resonances very much obscures simple interpretations. There is ample evidence that such interference can be important. Often the decay angular distributions from a resonant state Y show a fore–aft asymmetry. The expectation value of $(\bar{p}_Y\cdot\bar{p}_\Lambda)$, where $\bar{p}_Y$ is the momentum of the Y and $\bar{p}_\Lambda$ is the momentum of one of the decay products, is not zero as it must be, by parity conservation for a pure Y state. Figure 5 illustrates experimental[18] results which show the presence of a fore–aft asymmetry indicating the presence of strong interference effects.

For the purpose of illuminating the general problems which arise through interference between resonant states and background we consider a particular reaction where we give the particles the mnemonic labels: $K+p\rightarrow Y+\pi^-$, where the Y is an unstable baryon decaying as: $Y\rightarrow\Lambda+\pi^+$. We write for the amplitude of all $\Lambda+\pi$ pairs, including as a particular case the pairs with the quantum numbers of the Y_1 produced by interactions with unpolarized target protons:

$$\Psi=\sum_{jm\pi}A_{jm\pi}(\epsilon_++(-1)^k\epsilon_-)\psi_{j\pi}{}^m,$$

where $k=m/2-\pi/2$; π is the parity, $+1$, or -1; the ϵ are orthonormal unit vectors such that $\epsilon_s\epsilon_t=\delta_{st}$; the $\psi_{j\pi}{}^m$ represents $\Lambda-\pi$ states of definite total angular momentum j, parity π, and component of angular momentum m in the direction $n=(\bar{p}_b\times\bar{p}_\Lambda)/(\bar{p}_b\times\bar{p}_\Lambda)$, where $\bar{p}_b$ and $\bar{p}_\Lambda$ are the beam momentum and momentum of the $\Lambda-\pi$ system. In this discussion we assume that for fixed center of mass energy and production angle, the resonant amplitudes vary with invariant mass E/c^2 of the Λ–π system, as the simple Breit–Wigner form written above.

The experimental measurements on three body decays generally exhibit, non resonant backgrounds of the order of, or greater than 10% of the peak intensity. Observed intensities of nominally nonresonant processes substantiate such an estimate. As a result of interference with this background, which is coherent with the resonant amplitude, appreciable anisotropies will occur even if the Y has spin $\frac{1}{2}$. Consider, for example, the decays of a Y of spin $\frac{1}{2}$ and odd parity. Anisotropies in the angular distribution with respect to the $\bar{n}$ axis result only from interference with background states of the same parity and result in decay angular distributions of the form $1+a\cos^2\theta+b\cos^4+\cdots$, where $\cos\theta=(\bar{k}_\Lambda\cdot\bar{n})/|k_\Lambda|$. Assuming, to be definite, that b, c, $\cdots=0$, and considering only states with angular momentum components in the positive $\bar{n}$ direction, we find $a=(2A_b+A_r)^2/(\frac{1}{2}A_b-A_r)^2-1$, where A_b is the background amplitude and A_r the resonance amplitude. The odd parity background state, chosen such that $b=0$, is $\psi_{\frac{3}{2}}{}^{\frac{1}{2}}$. The relative phases are taken as 0° at resonance and the $\psi_j{}^m$ are normalized states of total angular momentum j and component m in the direction $\bar{n}$. A background intensity of 5% then leads to a value of a of 1.5!

[16] E. Eisner and R. G. Sachs, Phys. Rev. **72**, 680 (1946).
[17] R. Dalitz, Ann. Rev. Nucl. Sci. **13**, 399 (1963).
[18] J. Button-Shaffer, D. Huwe, and J. J. Murray, *Proceedings of the 1962 Annual International Conference on High Energy Physics*, edited by J. Prentki (CERN, Geneva, 1962).

In general, the relative phases of the background and resonance amplitudes vary as the value of the invariant mass passes through resonance and the anisotropy will also vary. The absence of any strong effect of this kind in some experiments has been considered evidence that the anisotropy is not a result of interference, but is solely the result of the high spin of the Y. This variation of anisotropy with energy is of course a quantitative question and the variation is not, in fact, necessarily large. The curve of Fig. 6 shows the anisotropy parameter a as a function of invariant mass for the particular case of a spin $\frac{1}{2}Y$ with 5% background intensity discussed above.

The interference which result in the anisotropies also produces Λ-polarizations in a direction perpendicular to the $\bar{n}$ axis. The particular combination discussed above results in spin expectation values in this direction proportional to $\sin\theta\cos\theta$. Interference between the $\Psi_{\frac{1}{2}}$ state and a $\Psi_{\frac{3}{2}}^{\frac{1}{2}}$ background of the same parity, adjusted so as to give a decay anisotropy in the same direction, results in a spin expectation value proportional to $-\sin\theta\cos\theta$, and simulates the polarization angular distribution pattern of a spin-$\frac{3}{2}$ even-parity state.

While counter examples can be constructed it seems probable that most decay angular distributions of particles with spin $\frac{3}{2}$ or greater decaying without interference can be counterfeited by the decay of a particle of smaller spin together with small appropriate background amplitudes. While the choice of background amplitudes is always special and narrow, it must be recognized that the choice of amplitudes and phases of the magnetic substates of the pure state is also narrow and particular.

In conclusion it seems that no one experiment is likely to conclusively establish the spin of a baryon state Y. However, the weight of a large amount of evidence can decisively settle the question.

IV. DYNAMIC DISTORTIONS OF FINAL-STATE RESONANCES

Both the mass and width, as well as other properties, of a particle state are affected by local conditions including the presence of matter. We are especially concerned with the properties of states under the idealization, or in the limit, that the rest of the universe is absent. Our formulations of the properties of particles are generally made in the spirit of this idealization. It is usually possible to perform experimental measurements on such states in a manner so that the disturbing effects are unimportant or so that extrapolations of the actual values of the measurements to values predicted for an isolated state are easily performed. As an example we note that the position and widths of the energy levels calculated for an isolated hydrogen atom are not quite the same as those measured experimentally. The differences, resulting largely from interactions with other atoms, have been extensively studied under the general name of line broadening.

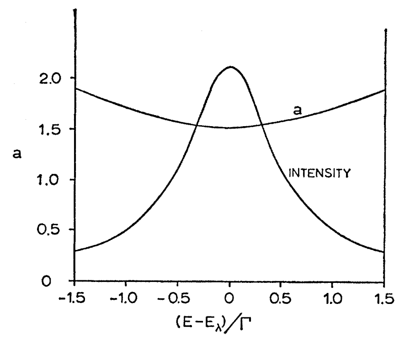


FIG. 6. Decay asymmetry vs energy for the decay of an odd-parity spin-$\frac{1}{2}$ state Y, plus 5% background intensity of spin $\frac{3}{2}$, odd parity. The total intensity is also shown. The phase relation and axis direction are such as discussed in the text.

Many of the short-lived elementary-particle states can be studied only in final-state interactions. Often they are produced in a manner such that they spend a time which is an appreciable fraction of their mean life within the characteristic interaction distances of other particles. If we consider, for the sake of simplicity and definiteness, a reaction leading to the production of an unstable isobar Y plus a recoil particle π, and consider that the reaction proceeds through central collisions or states of low angular momentum, we can establish a qualitative criteria for the importance of dynamic effects. If, in the center-of-mass system, the mean decay distance of the isobar is much greater than a length, $a\approx\hbar/m_\pi c$, characteristic of the radius of interaction, we can assume that the dynamic effects distorting the isobar are not very important. A typical value of the ratio l/a, where $l=(p/M)(\hbar/\Gamma)$, p is the momentum, M the mass, and Γ the width of the state Y_1, may be taken from the reaction $K^-+p\rightarrow Y^*+\pi$, where the K^- lab momentum is 1100 MeV and we use 50 MeV for Γ. Then $l/a\approx 1$, and dynamic distortions may be large. Such a criterion can only be a guide. The effects might be expected to be much smaller in peripheral collisions dominated by the exchange of one particle. In the limit of extrapolation to the pole the reaction can be considered as two-body scattering or reaction, the spectator particle is very far away, and there are no dynamic distortions at all.

It is informative to classify the effects of the interaction of the Y with the environment at production in analogy with effects concerning measurements of the properties of an excited atomic state in a gas. While the use of this parallel will not provide answers to the problems of dynamic distortion, it may help us to assemble the proper questions.

The photon emitted during the decay of an excited atom will have a certain probability of being scattered such that the wavelength is changed, or being otherwise absorbed by the gas, before reaching the detector. Likewise, the decay products of an isobar Y may be scattered in the environment immediate to its production such that the kinematic relation corresponding to the specific invariant mass of the decay products is destroyed. The photon emitted by the atom is removed from the detector very many wavelengths from the source. The accident can be considered, then, to be incoherent, reducing the intensity at the detector. This will not be the case for the scattering of the Y decay products. That accident will occur very close to the source, the finite aperture and resolution of the detector will not average over many phases and the removal of the Y from the detector must be considered coherently as a reduction of the amplitude of the Y production.

We can regard two obvious consequences of the scattering of the Y decay products separately. The removal of the Y from the detector reduces the amplitude of Y decay at short times. The amplitude as a function of time will not be precisely an exponential and the Fourier transform will not be precisely a Breit–Wigner form. The system of scattered particles is, in general, still coherent with the unscattered amplitudes. Though for scatterings involving high momentum transfers changes in relative angular momenta will be important resulting in incoherence with Y decays upon averaging over all angles as in a Dalitz plot or in the case of most invariant-mass distributions. Small-angle scattering, corresponding to small momentum transfers, results in relatively small changes in the invariant mass of the two decay particles, a smaller probability of angular momentum change, and a coherent contribution which is likely to broaden the width of the Y state.

Line broadening is the best known effect on excited atoms which results from the presence of other atoms. At its simplest, line broadening can result from a de-excitation of the atomic state by collisions of the second kind in a time short compared to the natural or radiative mean life of the state. The reduced lifetime results in a complementarily broader linewidth.

We might expect similar damping of the Y amplitude to take place resulting in a similar line broadening. Other reaction channels would be filled by the interaction of the Y and the recoil particle, an interaction which can be considered, in itself, a final-state interaction. However, final-state interactions in other channels produce Y states. In a gas at equilibrium, excited atoms extinguished by final-state interactions are balanced by collisions which excite the atoms. Since their actions are incoherent the cutoff of the radiation of individual atoms still results in line broadening. There is probably no such general conclusion in coherent situations, but it would seem that in isobar production some line broadening would likely occur.

The wave function of an ionized-gas atom is disturbed upon the approach of another atom. This disturbance can be considered as a polarization of the atom resulting from the effects of the incompletely shielded electric field of the neighboring atom and the perturbation of the wave function which results in effects very much the same as that induced by the Stark effect. In particular, energy-level shifts are observed and the angular momentum and parity of the excited state is no longer a good quantum number. (The total angular momentum and parity of the two atoms is conserved, of course.) As a result of the breakdown of symmetry, radiative transitions which were otherwise forbidden are greatly enhanced.

We might consider that similar but more general polarizations or breakdowns in the symmetry of the Y state may take place as a result of the perturbing field of the recoil particle. While the isolated Y will be an eigenstate of angular momentum parity and hypercharge, and less exactly, an eigenstate of isotopic spin and element of a particular representation of SU_3, none of these conclusions need be undisturbed by the presence of the recoil particle. We can expect, in general, that the perturbed Y will have an admixture of states with opposite parity, different angular momentum, different strangeness, different isotopic spin, and different SU_3 representation.

As a consequence a number of rather esoteric effects may occur besides the level shift or change in resonance energy. As a result of the admixture of opposite parity, the expectation value of $(\sigma_y^- \cdot \bar{p}_\Lambda)$ need not be zero. This can be partially simulated by kinematic interference with background, however the phase between the parity conserving amplitude and the parity nonconserving amplitude will usually be near 0 or π and will not vary much over the resonance, while for kinematic interference the phase between resonance and background will typically change by nearly 180°

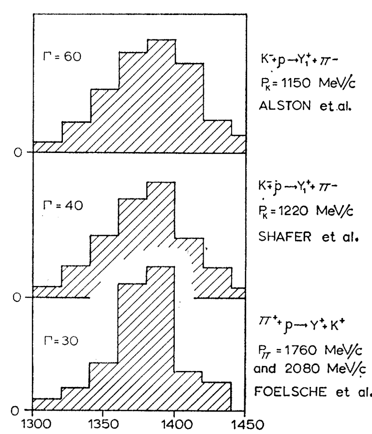


FIG. 7. Invariant-mass distributions for $\Lambda+\pi^+$ near 1385 MeV/c^2 from various experiments. M. Alston, L. Alvarez, P. Eberhard, M. Good, W. Graziano, *et al.*, Phys. Rev. Letters **5**, 520 (1960); J. Button-Shafer, D. Huwe, and J. J. Murray (Ref. 18); H. Foelsche, A. Lopez-Cepero, C. Y. Chien, and H. L. Kraybill (to be published).

through the resonance. Transitions otherwise forbidden will now be slightly allowed. Strangeness-changing transitions can occur, e.g., there may be a small intensity of the parity violating transition $Y \rightarrow n + \pi$. Since the intensity is proportional to the square of the amplitude and the square of the perturbation it seems unlikely that the intensity of such transitions will be large. The admixture of an amplitude of different isotopic spin or different SU_3 representation can have much larger effects on the branching ratios. While the decay of a pure $N^*(\frac{3}{2})$ neutral state with isotopic spin $\frac{3}{2}$ will go to $(\pi^0, n)/(\pi^-, p)$ in the ratio of 2:1, an admixture of 10% amplitude (1% intensity) of isotopic-spin-$\frac{1}{2}$ state with zero phase difference changes the ratio to 1.3:1.0. A mixture of SU_3 representations also can result in changes in decay branching ratios. The Y_1^* is nominally a state of the SU_3 representation (10), and the branching ratio $(\Sigma\pi)/(\Lambda\pi)$ is calculated to be about 0.13 including the effects of the symmetry breaking interaction. A small mixture of (10*) representation of about 4% in intensity will reduce this branching ratio to about 0.05 in better accord with experimental observation. Such a mixture would increase the total width by a factor of about $\frac{1}{3}$ by increasing the portion of the wave function in the Λ, π channel which is energetically free to decay. The inverse effect could also occur; the wave function in the open channel could be reduced, the decay probability reduced and the linewidth reduced.

There is now evidence concerning the simplest of the distortions, the line broadening. There are a large number of measurements of the $Y_1^*(1385)$ width which suggest a width of about 50 MeV.[18,19] This class of events is generally characterized by low center of mass velocities and central collisions. More recently a number of measurements[20] of the Y^* width have been made at somewhat higher center-of-mass velocities and smaller four-momentum transfers. These results show a Y^* width of less than 30 MeV. The curves of Fig. 7 show a comparison of typical results. It would appear that line broadening, as a result of dynamic effects, probably occurs and that under the conditions that many of the Y_1^* measurements have been made these distortions are quite large. Furthermore it appears that the natural width of the Y_1^* is less than 30 MeV rather than 50 MeV. No measurements of the Σ/Λ branching ratio have been made for Y_1^* production under favorable conditions such that the measured width is small. The large differences between theory and experiment may arise from the same dynamic effects which appear to broaden the natural width of the Y_1^*.

[19] M. Roos, Nucl. Phys. **52**, 1 (1964).
[20] C. Baltay, J. Sandweiss, H. D. Taft, B. B. Culwick, W. B. Fowler *et al.*, Phys. Rev. Letters **7**, 346 (1963; L. J. Curtis, C. T. Coffin, D. I. Meyer, and K. M. Terwilliger, Phys. Rev. **132**, 1771 (1963); H. J. Martin, L. B. Leipuner, W. Chinowski, F. T. Shively, and R. K. Adair, Phys. Rev. Letters **6**, 283 (1961).

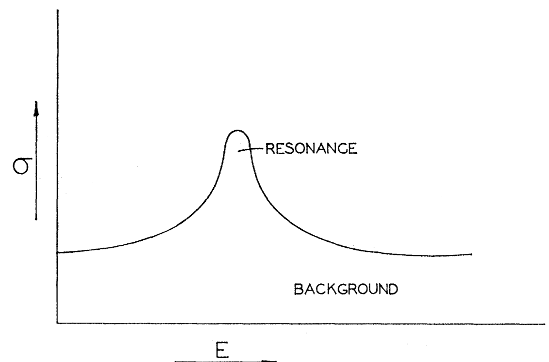
Discussion

McCARTHY: I didn't understand your argument about energy, time, and coherence. It seems to me there is no time resolution in the scattering experiment. You talk as though you can change time resolution and increase the energy resolution.

ADAIR: Yes; I did.

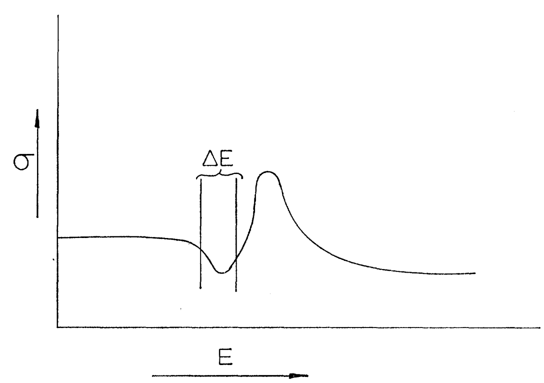
McCARTHY: If you measure time, you can treat the problem with the theory of wave-packet scattering. The statement usually made (e. g., by Friedman and Weisskopf) is that the wave packets from compound and direct scattering do not overlap (interfere) if they are well-enough defined in time. In fact, if you work out the interference terms, you find that it does tend to zero as the time resolution is increased, but at the same rate as the compound scattering term. You are left, for very good time (and correspondingly bad energy) resolution, only with the direct term.

ADAIR: Let me restate, I hope in a clearer fashion, what I meant. Let us consider a measurement of total elastic scattering cross section such that one gets a cross section like this:



You might calculate the lifetime of the resonant state from the width in energy and conclude that the lifetime is such that the state lived 20 or 30 characteristic nuclear times, while the background was the result of an immediate reaction: the two particles just bounced off.

Now the statement is often made, and correctly in the proper context, that the resonant state decays are incoherent with the background because it has lived so long. If it is incoherent we expect no interference between resonance and background. But there is interference behavior if one looks at the behavior over small energy intervals as in the sketch where a destructive interference



between resonance and background is shown. The seeming contradiction results from our cavalier disregard of complementarity. In discussing a very small energy interval, ΔE, we have lost our time resolution and can no longer consider that the resonance decays after the direct background scattering.

The first use of this with which I am familiar—and it was illuminating to me as I was working on the same problem and missed the point completely—was in the work of Feshbach, Porter, and Weisskopf, who showed that narrow elastic-scattering resonances could be considered as inelastic, absorptive factors if one averaged over broad energy intervals. You are then using good time resolution and the scattering by the long lived, narrow, resonances is incoherent with the incident beam, just as inelastic processes.

PHILLIPS: The point I tried to make this morning is that interference effects occur if you are not absolutely sure that a particle is first-emitted, or whether it is second-emitted—and I think that is the probability in this reaction—and also this effect if you don't know the angles of the detectors. I don't know how you can talk about these resonant states, if you don't say that one of them is first-emitted.

ADAIR: The experiments I presented are not troubled by overlapping bands in the Dalitz plot which can lead to interferences which distort widths. When one studies interference effects in a small region of energy on the Dalitz plot, time information is lost and the concept of time order has no meaning. Perhaps we are saying the same thing in complementary ways. It is customary, in high-energy physics, to discuss these reactions in terms of energy, which is the measured quantity, and not in the complementry, but superfluous, and often misleading, terms of time ordering.

DONOVAN: I think it is a misinterpretation of what you mean by good energy resolution, and what Professor Phillips means by good energy resolution. In absolute units it is very different in low-energy and high-energy physics. The important point is whether the energy resolution is comparable with the width of what you are looking at; not whether it is good or bad. And if you are looking at very narrow states, like gamma emitting states, and examining the particle resolution, then it is never good.

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Peripheral Production and Decay Correlations of Resonances*

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1. QUASI-TWO-BODY CHANNELS IN MULTIPARTICLE FINAL STATES

In reactions produced by the bombardment of nucleons with pions, K mesons, nucleons, and antinucleons, final states involving three or more particles become increasingly important as the bombarding energy is raised. For example, in K^+p collisions at an incident laboratory momentum of 1.96 GeV/c, the total cross section of 19.5 mb is divided among the various final states as follows[1]:

$$K^+p \to \begin{cases} K^+p & 7.6 \text{ mb} \\ K^0\pi^+p & 4.6 \\ K^+\pi^0p & 2.0 \\ K^+\pi^+n & 1.6 \\ K^+\pi^+\pi^-p & 1.7 \\ K^0\pi^0\pi^+p & 1.3 \\ K^0\pi^+\pi^+n & 0.33 \\ K^+p(3\pi) & 0.1 \end{cases}.$$

Another example involves 2.9-GeV/c π^+ incident on hydrogen,[2] where some of the final states are

$$\pi^+p \to \begin{cases} \pi^+p\pi^0 & 3.6 \text{ mb} \\ \pi^+n\pi^+ & 2.1 \\ \pi^+p\pi^+\pi^- & 3.1 \\ \pi^+p\pi^+\pi^-\pi^0 & 4.1 \\ \pi^+n\pi^+\pi^+\pi^- & 0.56 \end{cases}.$$

One sees from these examples that 3, 4, and even 5 particles in the final state is a common occurrence.[3]

A prominent feature of these reactions is the presence of mesonic and baryonic resonances. Such resonances (the nine vector mesons, ρ, ω, K^*, $\bar{K}^*$, ϕ; the decuplet of isobars, N^*, Y_1^*, Ξ^*, Ω^-; and others) are, with few exceptions, dynamically unstable and are observed only through kinematic correlations of their decay products. A typical example is shown in Fig. 1. The Dalitz plot for the reaction

$$K^+p \to K^0\pi^+p,$$

* Supported in part by the U. S. Office of Naval Research under contract ONR 1834(05).

[1] S. Goldhaber, in *Proceedings Athens Topical Conference on Recently Discovered Resonant Particles* (26–27 April 1963), edited by B. A. Munir and L. J. Gallaher (Ohio University Press, Athens, Ohio, 1963), p. 92.

[2] C. Alff, D. Berley, D. Colley, N. Gelfand, U. Nauenberg *et al.*, Phys. Rev. Letters **9**, 322 (1962).

[3] The rather peculiar fact that one of the five particle states ($\pi^+p\pi^+\pi^-\pi^0$) in the π^+p reactions has a greater probability than any of the three- or four-particle states, and almost an order of magnitude greater frequency than another five-particle state has its explanation in the presence of the quasi-two-body process, $\pi^+p \to \omega N^*$.