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SESSION C—SPECTROSCOPY WITH SEQUENTIAL REACTIONS

CHAIRMAN: *G. M. Temmer*

Sequential Decay of ^{12}C into Three α Particles

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I. INTRODUCTION

This paper describes experiments carried out at Rice University by Bronson, Simpson, Jackson, and Phillips on the reaction ^{11}B(p, 3α). A complete description of these experiments and their detailed interpretation will be published elsewhere. The present paper summarizes the principal results that are of interest to this conference. Earlier results have been reported previously.[1]

When ^{11}B is bombarded with protons of energy of a few MeV, resonant states in ^{12}C are formed. The level diagram[2] of ^{12}C is shown in Fig. 1. The experiments summarized in this paper were carried out at the energies indicated by the arrows.

A number of experiments on the reactions $^{11}\text{B}+p\rightarrow 3\alpha$ have been carried out; however, most of these experiments did not detect two of the α particles in coincidence. Such experiments do not completely determine the kinematics of the final state and, of course, do not allow any unique determination of the reaction mechanism. Only the experiments of Dehnhard *et al.*,[3] and the present experiments, are not subject to this criticism since two of the three final-state α particles were detected and their energy measured.

The first purpose of the investigations reported here was to develop experimental and theoretical techniques for treating nuclear reactions that lead to three-particle final states. The first question to be answered in examining such a problem is whether the reaction is *simultaneous* or *sequential*. It will be shown that all the Rice measurements require a sequential interpretation and that the measurements of Dehnhard *et al.* are compatible with such an interpretation. Secondly, if the reaction is sequential it becomes possible to use such reactions to deduce the spectroscopic properties of intermediate states. This is an especially attractive possibility since it may allow the study of otherwise inaccessible (often metastable) systems such as $^8\text{Be}+{}^8\text{Be}$, neutron+neutron, pion+pion, etc. However, to be certain of the proper interpretation of such spectroscopic studies it seems necessary first to carry out measurements on sequential reactions where the final-state interactions are well understood. Thus the three α-particle final state is ideal because the α–α interaction has been well studied[4] experimentally. In addition it is, of course, necessary to interpret the data with an accurate theory. Since three-particle final states produce reaction amplitudes that do not occur in two-particle states, proper account must be taken of these effects. In particular, before the experiments were initiated it was realized that interference effects (a) due to the identity of the α particles, and (b) sensitive to the order of emission of the α particles would be expected to be important. The results to be reported have shown that these effects do occur, can be accounted for by theory, and allow such measurements to be used as a new spectroscopic tool.

A. Three-Body Breakup—Simultaneous

If it is assumed that a nuclear reaction occurs so as to produce simultaneously three nuclear particles then it may be expected that the energy of any one of the three particles will be totally dispersed since the energy is conjugate to the time. Thus it is natural to suppose that the spectrum of the particles will be dominated by the phase space available to each. A particle i, with momentum between p_i and p_i+dp_i, and emitted into solid angle $d\Omega_i$ at angles θ_i, ϕ_i, has a phase-space

[1] J. D. Bronson, W. D. Simpson, W. R. Jackson, and G. C. Phillips, Bull. Am Phys. Soc. **9**, 406 (1964); G. C. Phillips, Rev. Mod. Phys. **36**, 1085 (1964).

[2] T. Lauritsen and F. Ajzenberg-Selove, *Nuclear Data Sheets* (National Academy of Science—National Research Council, Washington, D. C., 1961), Sets 5 and 6.

[3] D. Dehnhard, D. Kamke, and P. Kramer, Z. Naturforsch. **16A**, 1245 (1961); Phys. Letters **3**, 52 (1962).

[4] T. A. Tombrello and L. S. Senhouse, Phys. Rev. **129**, 2252 (1963); J. L. Russell, G. C. Phillips, and C. W. Reich, *ibid.* **104**, 135 (1956).

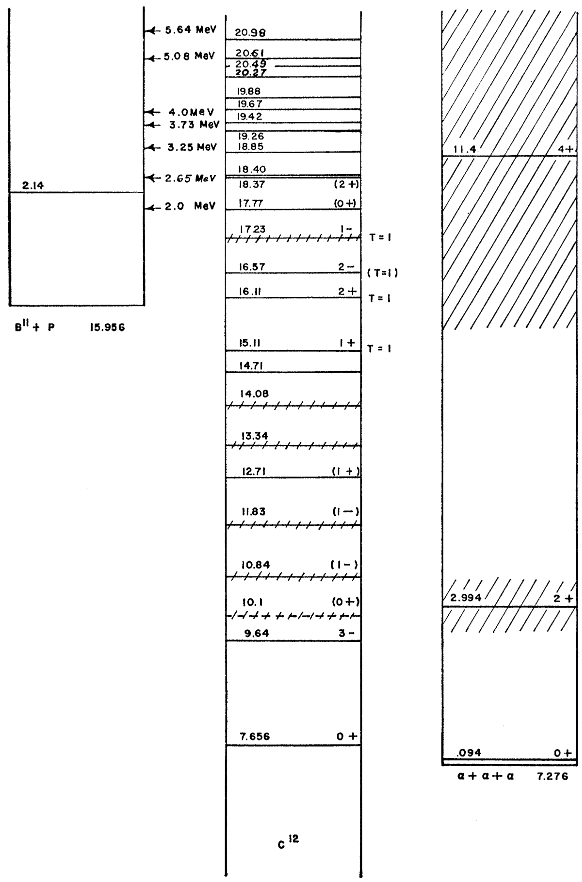

FIG. 1. Energy-level diagram of ^{12}C with the systems $p+^{11}B$ and $\alpha+^{8}Be$ shown for comparison. The arrows indicate c.m. energies in ^{12}C and the number beside the arrow the proton bombarding energy where data were obtained.

volume $p_i^2 dp_i d\Omega_i$ so that the probability of the decay occurring may be specified by the differential probability $dP(p_1, p_2, p_3)$, where

$$dP(p_1, p_2, p_3) = \prod_i p_i^2\, dp_i\, d\Omega_i \Big/ \int \prod_i p_i^2\, dp_i\, d\Omega_i.$$

This condition is subject, in the center of mass of the whole system, to the conservation of energy

$$0 = \sum_i p_i^2/2m_i - E,$$

where E is the total energy, and to the conservation of momentum

$$0 = \sum_i \bar{p}_i.$$

Now this problem has been considered by a number of authors[5] and it has been shown that the general shape of the distribution function of one of the particles is that of a semi-ellipse. Thus the expected spectrogram of one of the three simultaneously emitted particles is similar to that of a beta-ray spectrum except that a vertical tangency of the spectrum is expected at the kinematic maximum and minimum. For similar reasons to that of beta-ray spectra this shape must be modified by other considerations.

In particular, when the observed particle has near maximum energy the other two particles must have zero energy: since each of the two unobserved (low-energy) nuclear particles must always suffer some inward reflection at the edge of the attractive nuclear forces of the other particles (and perhaps also due to centrifugal and Coulomb barriers), their emission is suppressed and, of course, the (simultaneous) emission of the observed particle is also suppressed. In the same way, if the observed particle has a low energy, it will be reflected at the edge of the forces acting on it due to the other two particles. These effects combine to change the phase-space, semi-elliptical distribution into a bell-shaped curve.

The above arguments are probably quite important for the three-body decay of particles interacting *via* nuclear forces because they imply that near either the maximum or the minimum energy, the purely phase-space effects may be weakened, while the effects of the details of the nuclear-force interactions may be expected to be dominant. Thus near the maximum or minimum energy of the continuous spectrum of a particle emitted from a three-body reaction it may be expected that the reaction may proceed sequentially rather than simultaneously.

B. Sequential Breakup

1. Time Delay

When a nuclear system decays into three particles it may be represented by the following reactions:

$$a+A \xrightarrow[\tau_1=\tau_2=0]{} b+c+C; \qquad \text{(1a)}$$

$$a+A \rightarrow {}^*D \xrightarrow[\tau_1>0 \quad \tau_2=0]{} b+c+C; \qquad \text{(1b)}$$

$$a+A \rightarrow {}^*D \underset{\tau_1>0}{\rightarrow} b+{}^*B, \qquad {}^*B \underset{\tau_2>0}{\rightarrow} c+C; \qquad \text{(2a)}$$

$$a+A \underset{\tau_1=0}{\rightarrow} b+B^*, \qquad {}^*B \underset{\tau_2>0}{\rightarrow} c+C. \qquad \text{(2b)}$$

Here τ_1 and τ_2 represent the time delays (if any) of the two steps of the reaction. Reaction (1a) may be called a direct–direct or simultaneous reaction and is discussed in A above. Reaction (1b) may be called a delayed–direct reaction since in the first step a compound nucleus $^*D$ is produced which after a time delay τ_1 simultaneously breaks up into three particles. The spectra should also be as discussed above. Reactions (2a) and (2b) may be, respectively, termed delayed–delayed or direct–delayed reactions. These latter two types of reactions are expected to show a peaked energy structure in their spectra since there will be

[5] See for example: G. E. Uhlenbeck and S. Goudsmit, *Verhandelingen Van, Peter Zeeman* (Martimus Nijhoff, The Hague, 1935).

preferential emission of particles b to produce the semistable states of $^*B$. Several authors[6,7] have discussed these latter two types of reactions.

An important point to note is that the order of emission may not be determined by certain types of measurements. Rather than reactions (2a) and (2b) the reaction may proceed by direct-delayed or delayed-delayed reactions such as:

$$a+A \underset{\tau_1 \geq 0}{\rightarrow} c+{}^*F; \qquad {}^*F \underset{\tau_2 > 0}{\rightarrow} b+C, \tag{3}$$

or

$$a+A \underset{\tau_1 \geq 0}{\rightarrow} C+{}^*G; \qquad {}^*G \underset{\tau_2 > 0}{\rightarrow} b+c. \tag{4}$$

Since reactions (1) through (4) all end in the same final state it is necessary to discuss whether or not they can be distinguished. As will be seen later, with examples, it is possible, in some cases, to determine the order of emission of three particles. In general, however, the order of emission may not be determined and there is a reaction amplitude for all processes with delay in the final state: Thus, if the reaction amplitude for reactions (2), (3), and (4) are defined as M_2, M_3, M_4, respectively, then the cross section is proportional to $|M_2+M_3+M_4|^2$ and cross terms are generated that, in general, have not been treated in most theories of such processes. Dalitz[8] has recognized the possible importance of these terms; however, only Bronson[9] has obtained experimental evidence for these effects and has discussed them. This will be discussed in detail below.

2. *Watson's Method*

In an early paper[6] on the treatment of sequential decay Watson considered reactions such as (2a) and (2b). His arguments are not given here in detail, but his methods and his results are summarized. Watson considered a reaction such as (2) proceeding backwards in the time: the particle c bombards C and produces a (metastable) nucleus $^*B$

$$c+C \rightarrow {}^*B \tag{5a}$$

which serves as the target particle for the next step of the reaction

$$b+{}^*B \rightarrow a+A. \tag{5b}$$

Watson argued that the probability of the whole reaction (5a), (5b) proceeding should be proportional to the formation cross section of $^*B$ by reaction (5a). This is expected to be true, in particular, when the interactions of the nucleons in $^*B$ are such as to produce a resonant, metastable state. Thus his arguments are applicable to narrow resonant states in $^*B$ produced by strong, short-range interactions.

Under these conditions the result was obtained that the cross section for reactions (5) and thus for reactions (2) should be proportional to the cross section for reaction (5a):

$$\sigma_{(2)} \propto \sigma_{(5a)} \propto \sin^2 \delta / P,$$

where δ is the scattering phase shift for $c+C$ and P is a barrier penetration factor.

As was discussed above, these effects should be especially important when the observed particle has an energy near the extrema of its available continuous distribution.

Watson's treatment may be criticized on three grounds: Firstly, it supposes that the interactions are quite strong in the final state. In fact, there are many interesting cases (such as the singlet-deuteron, the dineutron, and states of ^{8}Be) where the interactions are not strong and yet it is important to treat these problems; secondly, Watson's treatment omits phase-space dependence of the probability of the over-all reaction. Thirdly, this treatment neglects the fact, discussed above, that the order of emission is important.

Nevertheless Watson's treatment is, in general, the starting point of most interpretation of experimental data of three-body spectra.

3. *The Generalized Density of States Method*

Other authors have considered this problem of sequential decay and arrived at slightly different results.[7] The generalized density of states is defined in analogy to a two body reaction

$$a+A \rightarrow b+{}^*B,$$

where a spectrum of particles b is produced with a set of sharp energies that leave $^*B$ in a set of particle-bound states of energies $E=E_n$. The cross section, of course, is

$$\sigma(E) = (\mu_a \mu_b k_b / 4\pi^2 \hbar^4 k_a) \mid \langle B(E), bH' \mid a+A \rangle \mid^2 \times \sum \delta(E-E_n),$$

where H' is the reaction perturbation Hamiltonian, and this cross section has a nonzero value only for the allowed energies E_n of B. The generalized density of states for this case is defined as

$$\rho(E) = \sum_n \delta(E-E_n).$$

As in Watson's treatment consider the reaction proceeding in reverse. Then $\rho(E)$ is seen to represent the probability that the nucleons which constitute $^*B$ can exist (in a nuclear volume) to be available as a target for c.

The quantity ρ is generalized to continuum states of $^*B$ by the following arguments: (a) The probability that the reaction occurs via a first emission of b to pro-

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[6] K. M. Watson, Phys. Rev. **88**, 1163 (1952).
[7] G. C. Phillips, T. A. Griffy, and L. C. Biedenharn, Nucl. Phys. **21**, 327 (1960).
[8] R. H. Dalitz, Ann. Rev. Nucl. Sci. **13**, 339 (1963).
[9] J. D. Bronson, Jr., Ph.D. thesis, Rice University, 1964 (unpublished).

duce a continuum state of B^* should be proportional to the probability that $c+C$ be localized as the metastable nucleus $^*B$ within a nuclear volume V *that includes the interaction volume of b with* $^*B$. Again, short-range nuclear forces are supposed and the time-reversed process may be considered. (b) This probability may be calculated, for scattering states of $c+C$, in terms of their wave function of relative motion:

$$\psi=N(E)[U(r)/r]Y_l^m(\theta,\phi),$$

where $U(r)/r$ is the relative radial function and is normalized inside the volume V of radius a. The normalization factor $N(E)$ then reasonably defines the generalized density-of-states function:

$$\rho(E)\equiv\rho_0(E)N^2(E),$$

where $\rho_0=\mu R/(\pi\hbar^2 k)$ is the usual density-of-states function determined by normalizing ψ in a large sphere of radius R. Thus since

$$\int_0^R |\psi|^2\,dv=1,$$

and

$$\int_0^a |U|^2\,dr=1,$$

$$N^2(E)=\int_0^a|\psi|^2\,dv$$
$$=\int d\Omega\,|Y_l^m|^2\int_0^a|W|^2\,dr=\int_0^a|W|^2\,dr,$$

where $W(E,r)=N(E)U(r)$. From the Wronskian it follows that

$$\int_0^a|W|^2\,dv=\frac{\hbar^2}{2\mu}\left[\frac{\partial W^*}{\partial r}\frac{\partial W}{\partial E}-W^*\frac{\partial^2 W}{\partial r\partial E}\right]_a$$

so that N^2, and thus $\rho(E)$, can be determined *versus* excitation energy, E, of $^*B$ provided the external wave functions (phase shifts) are known. In the case of a resonance this results in

$$\rho(E)=(2\mu a/\pi\hbar^2\xi^2)[\sin^2(\delta+\phi)]/P,$$

where $\xi=U(a,E)=$constant and P is a penetration factor. This result is similar to the result of Watson but differs by the term ϕ (a hard-sphere phase shift) in the argument of the sine function.

This method has attempted to remove the objections of considering only strongly interacting particles in the final state and the neglect of phase-space effects. However, it suffers from the same objection that it also neglects the effects of the possibility of other orders of emission.

4. *Interference Effects*

Interesting interference effects are expected to occur for three-particle breakup that are peculiar to the three-particle final state. One of these was mentioned above in regard to the order of emission of a sequential three-body reaction. Another interference effect occurs whenever two or three of the particles are identical.

To examine the effects of these causes consider the case of the decay of continuum states of tritium into two neutrons and a proton:

$$^3\mathrm{H}\rightarrow n+n+p. \qquad (6)$$

This reaction can proceed by the sequential mechanisms

$$^3\mathrm{H}\rightarrow\text{di-neutron}+P, \qquad (7)$$

$$^3\mathrm{H}\rightarrow\text{singlet deuteron}+n, \qquad (8)$$

or

$$^3\mathrm{H}\rightarrow\text{triplet deuteron}+n. \qquad (9)$$

If the matrix elements are labeled by the reaction equation number then the total matrix element for the sequential decay is

$$M_7+M_8+M_9.$$

However, the identity of the two neutrons requires that each matrix element be antisymmetric. If the two neutrons are labeled a and b, the $M_i=(1/\sqrt{2})\times[M_i(a,b)-M_i(b,a)]$ so that the total matrix element is

$$M=(1/\sqrt{2})[M_7(a,b)+M_8(a,b)+M_9(a,b)$$
$$-M_7(b,a)-M_8(b,a)-M_9(b,a)],$$

and the cross section will be

$$\sigma=\alpha\Big[\sum_i|M_i(a,b)|^2+\sum_i|M_i(b,a)|^2$$
$$-2\,\mathrm{Re}\sum_i M_i(a,b)M_i^*(b,a)$$
$$+2\,\mathrm{Re}\sum_{i\neq j,j}M_i(a,b)M_j^*(a,b)$$
$$+2\,\mathrm{Re}\sum_{i\neq j,j}M_i(b,a)M_j^*(b,a)\Big].$$

Consider the significance of these terms: The first and second terms correspond to the (antisymmetrized) result expected for the final-state interaction—in this case, decay *via* the di-neutron or a singlet or triplet deuteron; the third term is an interference term due only to the antisymmetrization required for the identical fermions; the fourth and fifth terms arise from a combination of the results of antisymmetrization and what may be called an "order-of-emission" interference effect.

The first two terms are treated, to some approximation, in the treatments of Watson or of Phillips, Griffy, and Biedenharn. The remaining terms are not. The existence of these terms was recognized by Dalitz[8] and

Phillips[10] and experimental verification has been obtained by Bronson *et al.*[9]

Iàn Duck[11] has developed a theory to account for all the terms in such a decomposition and his results are compared later to the measurements of Bronson *et al.*

II. EXPERIMENTAL METHOD

The study of the $^{11}B(p, 3\alpha)$ reaction was carried out using the Rice University 6-MeV Van de Graaff accelerator. Thin, self-supporting foil targets of isotopically enriched ^{11}B were bombarded with protons. A special scattering chamber was used that employed two Si solid-state surface-barrier detectors. One of the detectors could be oriented at any angle about the target in a plane containing the beam; the other detector could be placed at any point upon a sphere about the target.

A slow–fast coincidence arrangement was employed and 25–50 nsec was the coincidence resolution time. The two pulse sizes were recorded in a Nuclear Data 32 by 32, two-parameter analyzer. All data shown below have been corrected for accidental coincidences. Complete details of the experimental method will be published elsewhere.

III. TYPICAL EXPERIMENTAL RESULTS

A. Two-Parameter Pulse-Height Spectra

The reaction $^{11}B(p, 3\alpha)$ has been studied at a variety of bombarding energies and angles as shown in Fig. 1. Typical spectra are shown in Figs. 2 and 3. In the upper right-hand corner of each of these diagrams the energies T_1 and T_2 of the two coincident α particles are the axes of the plot. The solid curve is the calculated kinematic locus of coincident events. The assorted symbols, representing the number of true coincidences for each pair of values (T_1, T_2) are seen to fall along this locus.

Below and to the left of the (T_1, T_2) diagram are histograms of intensity of coincident counts vs respectively, either T_1 or T_2. Other curves are drawn on these diagrams and their significance is given in the figure captions.

It is seen that all the peaks in the (T_1, T_2) diagram, or in the histograms, correspond (at least roughly) to the first two states of ^{8}Be: the 0^+ ground state at 94 keV $\alpha+\alpha$ c.m. energy or the 2^+ first excited state at about 3 MeV of $\alpha+\alpha$ c.m. energy.

In general there are three manifestations of the ground state of ^{8}Be that may be uniquely interpreted in each such diagram as in Figs. 2 and 3. These correspond to: (a) the most energetic and narrow peak of intensity vs. counter energy, T_i, corresponds to the detection of first-emitted α particles, by counter (i), that is, α particles that have been emitted from ^{12}C to form the ground state of ^{8}Be; (b) the two lower energy, narrow, peaks correspond to the detection in counter (i) of an α particle emitted from the ground state of ^{8}Be, that is, second-emitted α particles. Thus there is a unique identification of which (identical) α particles, *first-emitted* or *second-emitted* went into a given counter. This is not too surprising since the ground state of ^{8}Be has a rather long lifetime.[2]

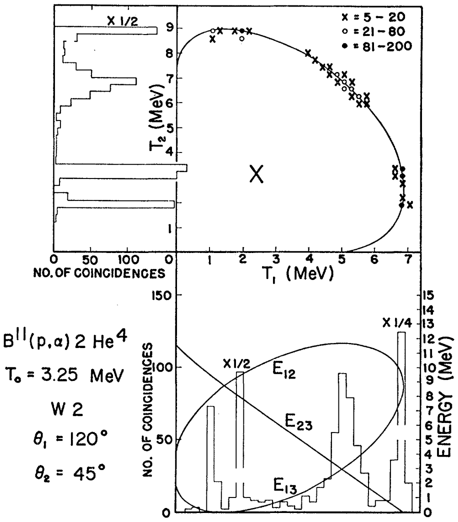

FIG. 2. Two-parameter spectra for $^{11}B(p, 2\alpha)^4He$. The coincident α-particle energies, T_1 and T_2, measured in two solid-state detectors are shown in the top-right diagram. T_0 is the bombarding energy, and θ_1 and θ_2 are the counter angles. The smooth curve is the calculated locus of coincidences. The symbols give the actual number of counts observed. The histograms below and to the left of the T_1, T_2 diagram are obtained from projecting the same data upon the T_1 or T_2 axes, respectively. The curve labeled E_{23} has the significance that a first emitted α-particle, depositing T_1 in counter 1, will leave ^{8}Be at an excitation E_{23} measured along the ordinate and with the mass energy of two α particles taken as the zero. Thus the ground state of ^{8}Be is to be read $E_{23}=0.094$ MeV. The curves E_{12} and E_{13} have a similar interpretation; however, they refer to the detection, in counter 1, of a second-emitted α particle, i.e., one emitted from a state in ^{8}Be.

The interpretation of the broader peak, or peaks, that correspond approximately to the ^{8}Be 2^+ first excited state is not so simple and will now be discussed.

Examination of Fig. 2 shows that the peak at about 5.3 MeV energy in the histogram of counter 1 occurs at an excitation energy (in ^{8}Be) of about 3 MeV for both E_{23} and E_{13}. This seems very reasonable since the first (2^+) excited state of ^{8}Be is at about that energy. However, it is noted that the half-width of the peak is only about 0.5 MeV; the state is known to have a width of about 1 MeV,[4,12] when studied by α–α scattering or by the $^{11}B(p, \alpha)$ ^{8}Be reaction. This situation is clearly anomalous.

Reference to Fig. 3 reveals an equally anomalous

[10] G. C. Phillips, Bull. Am. Phys. Soc. **9**, 389 (1964).
[11] I. Duck, Bull. Am. Phys. Soc. **9**, 417 (1964); also paper in this conference, Rev. Mod. Phys. **37**, 418 (1965).
[12] G. D. Symons and P. B. Treacy, Nucl. Phys. **46**, 93 (1963).

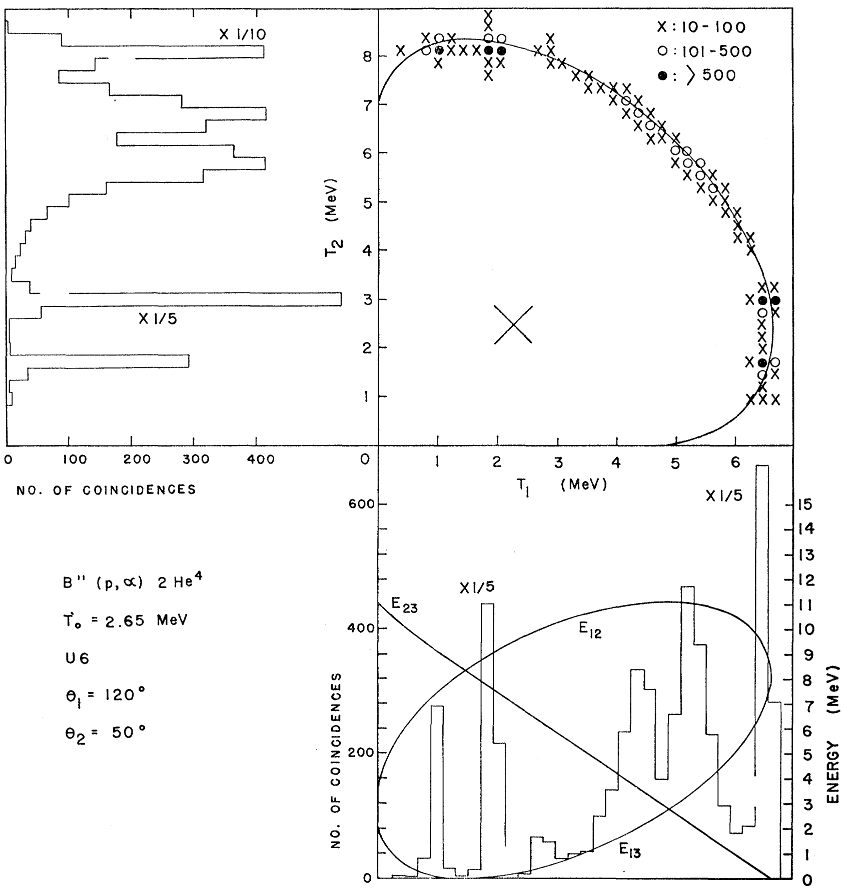

Fig. 3. Two-parameter spectra for $^{11}B(p, 2\alpha)^4He$. See caption for Fig. 2.

situation: the peaks at about 5.3- and 4.4-MeV α-particle energy might be interpreted as indicating possible states in ^8Be at about 2.0- or 3.6-MeV excitation, or at both energies. It is known that states of ^8Be do not exist at these energies. It is important to note also that *the minimum of cross section occurs at the known energy of the first excited state of* ^8Be. Equally interesting is the fact that the two peaks in Fig. 3 (at 4.4 and 5.3 MeV in counter 1) have energy widths considerably smaller than the known width of the first excited state of ^8Be.

Finally, it is important to note that these effects for the 2^+, first excited state seem to occur when the curves labeled E_{13} and E_{23} cross at about the energy of excitation of the first excited state. This means, kinematically, that the experiment does not ascertain whether the α particles detected in counter 1 are *first-emitted* or *second-emitted* and allows strong interference terms of the type described above to occur.

In summary, the data of Figs. 2 and 3, typical of all the data, indicate that: (a) the sharp ground state of ^8Be is strongly populated and peaks in the spectra are observed at the expected places; (b) there is little evidence for simultaneous breakup since there are no broad distributions. In fact, the data show that a simultaneous mechanism can account for no more than 5% of the cross section and that at least 95% of the cross section must be ascribed to sequential decay; (c) there are anomalous departures from simple expectations for the 2^+, first excited state of ^8Be when the kinematics do not allow a distinction between

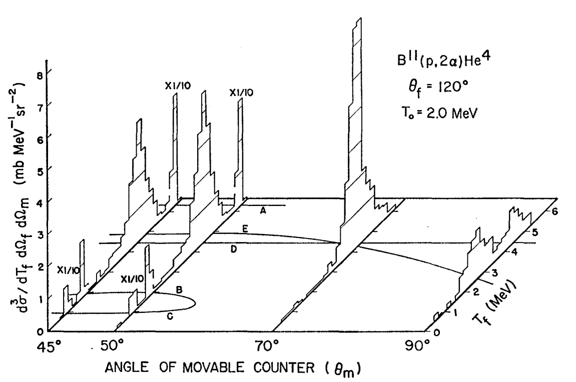

Fig. 4. Three-parameter spectra $^{11}B(p, 2\alpha)^4He$. Differential cross sections are shown *versus* the energy, T_f, deposited in a fixed counter held at θ_f, and *versus* the angle θ_m of the other detector. The significance of the curves A through E are described in the text.

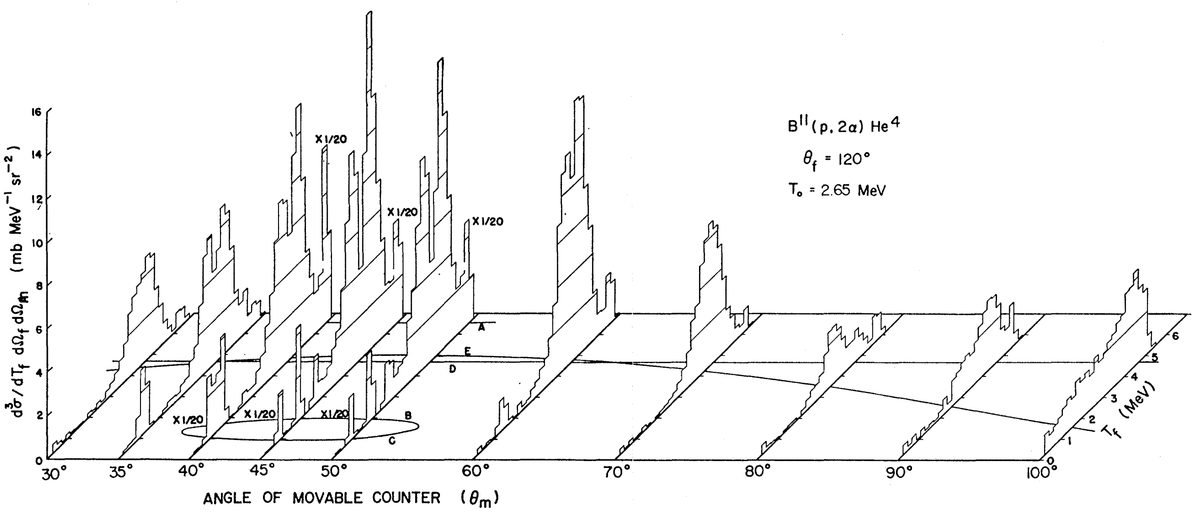

FIG. 5. Three-parameter spectra for ^{11}B(p, 2α)^{4}He. See caption for Fig. 4.

first-emitted and *second-emitted* particles. These anomalies consist of energy shifts of the expected positions of the cross section peaks and narrowing of the widths of peaks.

B. Three-Parameter Pulse-Height Spectra

The observations discussed above can be clarified by considering other dynamical variables. Consider families of histograms of cross section vs T_1, such as are shown in Figs. 2 and 3. If the bombarding energy is held fixed and the angles of one detector are held constant while the other detector angle is varied, one may obtain a set of histograms that may be plotted as shown in Figs. 4 and 5. These isometric projections allow an examination of a large amount of experimental data. The significance of the curves A, B, C, and D, E are as follows: A, B, C refer to the ground state of ^{8}Be; A is the expected position of the ground state peak when the fixed counter detects the first-emitted α particle; B and C correspond to the detection, in the fixed counter, of one of the second-emitted α particles. These three curves are in one-to-one correspondence to the three narrow peaks of Figs. 2 and 3. Note that all the T_f, θ_m plane is not available to the coincident detection of the α particles. The curves D, E are similar curves calculated for the 3.0 MeV, 2^+ first excited state of ^{8}Be; D corresponds to the detection of a first-emitted α particle in the fixed detector while E corresponds to the detection of a second-emitted α particle.

Note for Figs. 4 and 5 that there are regions of the T_f, θ_m plane where D and E are closer together than the half-width of the first excited state of ^{8}Be. In these regions it is kinematically impossible to ascertain whether the α particle detected in the fixed counter is a first or a second-emitted α particle. As a result, the order-of-emission interference effect discussed above can occur.

Reference to the angular region $45° \leq \theta_m \leq 70°$ in Fig. 4 indicates that *constructive* interference occurs. At the angle of 90° the two loci, D and E, are sufficiently separated so that the two matrix elements apparently do not interfere strongly and the peaks at about 4.4 and 2.6 MeV for T_f can be assigned respectively to first-emitted and second-emitted α particles producing the first excited state of ^{8}Be.

In Fig. 5, the angles of 30° and 70° seem to show little interference effects while the angles $35° \leq \theta_m \leq 60°$ clearly show a minimum that may be interpreted as a *destructive* interference term arising from the two types of emission: first and second emission.

IV. DISCUSSION AND CONCLUSIONS

Ian Duck[11] and Peter Swan[13] discuss theoretical calculations that attempt detailed fitting of these data. Although exact fits have not been attained to date it seems clear that the theory outlined above may be expected to account for the experimental results. Thus, a number of conclusions can be drawn: (1) The reaction ^{11}B(p, 3α) proceeds (for the energies studied) *via* a sequential mechanism involving the ground state and first excited state of ^{8}Be. Less than 5% of the decay can be ascribed to simultaneous decay. This result contradicts the interpretation of other experiments,[12] and shows the danger inherent in attempting to draw conclusions about three-particle final states from incomplete experiments. (2) It is possible, in some experimental circumstances, to ascertain the order of emission of the (identical) α particles. In other cases the order is not kinematically determinable. (3) Strong interference effects due to (a) particle identity and (b) order-of-emission effects are expected theoretically and are observed experimentally when the kinematic

[13] Peter Swan, paper in this conference, Rev. Mod. Phys. **37**, 336 (1965).

situation does not allow a determination of the order of emission. These interference effects must be carefully accounted for if three-body final states are to be employed as a spectroscopic tool. For example, consider Fig. 5: If only a single histogram, such as that at $\theta_m=45°$ were used to deduce the level structure of ^{8}Be, one would incorrectly conclude that ^{8}Be has two excited states. In the same way any statement about the angular momentum properties of states of ^{8}Be could be improperly interpreted unless correct account is taken of these interference effects. (4) The angular variables, in the three-particle final state, are intimately coupled to the energy variables and thus the angular variables provide a means of determining other dynamical quantities such as the excitation energies of levels and their widths. These facts provide a potential new method of measuring lifetimes, or widths, of excited states that has been previously discussed.[14]

All the above discussion would apparently contradict the interpretation of the experiments of Dehnhard *et al.*[3] who have studied the ^{11}B$(p, 3\alpha)$ reaction in the energy range 0.1–0.2 MeV and have interpreted the coincident two α-particle spectra on the 163-keV resonance as being due to a simultaneous decay. Off resonance, the yields tend to proceed, as in the Rice experiments, *via* sequential decay through ^{8}Be states. To reconcile this apparent dilemma the following model is proposed. Assume that the resonance $T=1$, 16.11-MeV state of ^{12}C studied by Dehnhard *et al.* has a wave function with a cluster component consisting of an α particle coupled to one, or both, of the lower $T=1$ states of ^{8}Be (at about 16.08- and 16.67-MeV excitation in ^{8}Be).[2] The ^{12}C state would be bound by about 7.34 and 7.93 MeV, respectively, against decay to the center of gravity of these states. However, these states have nonzero α-particle decay widths of about 0.3 and 0.2 MeV, respectively,[2] and the state of ^{12}C would decay by α emission to the "tails" of these resonance. Decay to "tails" of broad resonance in ^{8}Be have been observed for both weak interactions[15] and strong interactions.[16] This proposed mechanism accounts for the broad, unpeaked spectra observed for the $T=1$, 16.11-MeV ^{12}C resonance, but accounts for these effects by a sequential decay to the "tails" of higher $T=1$ ^{8}Be states rather than by invoking a simultaneous decay. It must be remarked, however, that such a mechanism as proposed here is not unique; nevertheless this argument indicates that the assignment of a simultaneous mechanism to this resonance is not unique either and that the question must await further investigation.

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[14] G. C. Phillips, Rev. Mod. Phys. **36**, 1085 (1964).
[15] T. A. Griffy and L. C. Biedenharn, Nucl. Phys. **15**, 636 (1960).
[16] R. R. Spencer, G. C. Phillips, and T. E. Young, Nucl. Phys. **21**, 310 (1960).

Discussion

DONOVAN: I think your remarks about the $T=1$ impurity are quite true in principle. However, one is surprised, I think, at the intensity of the effect. The studies of the beta decay of Li^8 forming Be^8 states done by examining the emitted alpha-particle spectrum which we did a couple of years ago with fairly high precision, have shown that in the region of the first-excited state of Be^8 the shape of this alpha spectrum can be matched with very high accuracy with the $l=2$ alpha–alpha phase shifts. There is no indication of any appreciable impurity in that region. One could probably pick out a percent, or so; so I am a little surprised at the magnitude of the effect.

PHILLIPS: Yes; I think perhaps that is true, and of course one has to put numbers in it to be absolutely certain.

I would remind you, though, of the fact that that problem was examined a number of years ago, and also by Biedenharn and Griffy, and they concluded that the high-energy beta spectrum and low-energy alpha spectrum could not be accounted for by the scattering phase shifts of Be^8 (all $T=0$), and one had to invoke a contamination of $T=1$, the tails of these very states I am talking about, to describe the data.

One other thing one must remember is that the explanation that I gave in terms of contamination of $T=1$ in the Be^8 state is not necessarily the only way one could do this, because one could have a smaller contamination of $T=0$ in the resonant C^{12} state. And then further, to make it awfully bad, one could possibly have mixtures. So I think at the present time we certainly need to investigate this further, but my point is that we have an enormous amount of data, only one datum of which seems to require a simultaneous mechanism, and it is at the lowest energy and that doesn't seem right, either.

So I suggest a uniform description of all of these data should be sought.

BROWNE: I wish to remark on the isobaric spin purity of the 15.11- and 16.11-MeV states of C^{12}. Recent data on the N$^{14}(d,\alpha)$C^{12} reaction to these states indicates they are quite pure $T=1$ states with probably less than 1% of $T=0$ admixture.

I would also point out that the 16.6- and 16.9-MeV states of Be^8 have widths of 95- and 85-keV, respectively. The first of these is a $T=1$ state and the second has $T=0$. The impurities here may be large.

KRAMER: Two questions. The first one, did you apply the symmetrized theory to the data on this decay, and what would be the effect of the interferences between the different states?

The second question is if you have a resonance, $T=1$, $\Gamma=200$ keV, at 16 MeV in Be^8 and you go down to about 3 MeV, this gives you a factor in the cross section of about 10^{-4}. I think it is difficult to explain, then, the cross section of ^{11}B$(p, a)^8$Be from the 16.1-MeV state in ^{12}C.

PHILLIPS: I believe you asked two questions. The first one was about the theoretical interpretation of these interference terms between the different states. I would like to defer that until after Dr. Duck's talk, because he will lecture on that.

In regard to your second question, I agree. It seems to me quite surprising. I would point out, though, that most of that width of that resonance state in ^{12}C is due to the re-emission of a proton. It does have a small total width, and in fact it has a measurable gamma-ray width. So I think one has to examine this entirely in terms of just the numbers.

HOLMGREN: That last statement of yours about the total width being largely the proton width I don't think is quite correct. In the slide presented by Dr. Waggoner yesterday we see this particular level in the ^{10}B(^{3}He, p) reaction, and then alpha decay. We see a constant proton energy line in the two-dimensional, $p\alpha$, energy spectrum from the ^{10}B(^{3}He, $p\alpha\alpha\alpha$) reaction which corresponds to the formation of the 16.11-MeV level of ^{12}C and its subsequent α decay. She finds that this level decays primarily by α emission and that the decay takes place to a large extent by a sequential process through the 2.9-MeV level of ^{8}Be. However, the spectrum in the region of this level does have more low-energy α-particle events than the spectrum in the region of the 16.57-MeV level of ^{12}C. I believe the difference between Waggoner's measurements and those of the preceding paper is that the large center of mass motion in Waggoner's measurements tends to separate the events corresponding to decays through the 2.9-MeV level of ^{8}Be from those corresponding to decays proceeding to the tail of the very broad 12-MeV level of ^{8}Be. Events corresponding to decays to the tail of this level would tend to fill in the center regions of the Dalitz plots shown in the preceding paper and would not be separated from events corresponding to decays through the 2.9-MeV level. We believe that the only difference between the 16.11- and 16.57-MeV levels of ^{12}C is that the 4+ to 2+ branching ratio is much larger for lower level.

PHILLIPS: Yes. I was struck by the slides that Dr. Waggoner showed yesterday, and it seems to me that perhaps there could be made a very careful comparison of your data, taken by an entirely different technique, and the data of the German group.

JACKSON: I would just like to observe that, in high-energy physics, and so also here, there is an opertional way of deciding the relative amounts of resonance formation (sequential reaction) and nonresonant uncorrelated events (direct interaction, as the term is used here). If the data on the Dalitz plot show more or less uniform variations in density of population, the reaction is a direct or uncorrelated process. If there is a resonant band of relatively high density, resonance formation is the dominant process. With reasonable assumptions about the direct processes (e.g., probability according to phase space) and resonant line shapes for the reasonances, one can make a least-squares fit to the data on the Dalitz plot and quote percentages for the various processes. When identical particles or overlapping resonances are involved, care must be taken, but in many cases meaningful results can be obtained, as has been demonstrated by Dalitz and Miller.

PHILLIPS: In regard to the Dalitz plots, where these bands cross each other is the region where the sort of interference terms I discussed today and showed experimental examples of, became important. In these regions, if you examine a lot of data, taken from different angles, you might get a picture that you have perhaps two states, where indeed you might only have one; or you might get a result that the width of the state is a factor of two narrower than the width of the state actually is. I think some of this should be of interest to high-energy physicists.

JACKSON: I think it is important to make a distinction between effects due to the identity of the three alpha particles and overlap bands when the particles are distinguishable. The point is that the phenomena are described by one amplitude that is a function of two momenta in the center of mass. You project out the sequential parts of that amplitude and get probabilities, but there is still a possibility that there is a contribution that is neither sequential nor direct, corresponding to the three particles close together in configuration space. One must be very careful in the language one uses in describing this. In other words, even though particles are distinguishable the overlap region on the Dalitz plot can in fact be some strange object.

PHILLIPS: That is exactly the point I was trying to make. That one has two types of interference terms coming through there. There is also an effect due to the fact that one has three orders of emission for three distinguishable particles for a sequential process, and these amplitudes must be added together, and it is this order of emission type of interference effect that I think is especially interesting.

It is possible in some kinematical circumstances to say which of these three alpha particles, even though they are identical, comes out first. In most angles, most energies, it is possible to do that.

But there are regions of the multidimensional energy-angle space where it is not possible to do so, and when it is not possible to assert which alpha particle came out first, then these interference terms are very large, and as I showed, they make one state look like two states. They make the width of the state appear to be half of what it should be.

DONOVAN: I would like to point out again a remark yesterday made by Professor Kamke. He stated there is another resonance in ^{12}C, a slightly higher $T=1$ resonance, which yields a Dalitz diagram which is qualitatively different from the lower energy $T=1$ resonance. It's going to be hard, on the basis of any uniform theory including one postulating isotopic-spin impurity in Be^8 to explain this qualitative difference, and I think perhaps this is a very important point.