Structure of He^3 and H^3 from High-Energy Electron Scattering*

T. A. GRIFFY
High-Energy Physics Laboratory, Stanford University, Stanford, California
R. J. OAKES
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California

I. INTRODUCTION

The scattering of high-energy electrons from nuclei has long been used to measure the gross properties of these nuclei such as the shape of the charge distribution.[1] In addition, high-energy electron scattering has been used as a tool for studies of nuclear spectroscopy, in particular the spins, parities, and radiation widths of excited states.[2] More recently electron scattering studies have contributed to our understanding of the few-nucleon problem with particular emphasis on the structure of the deuteron and the three-nucleon systems He^3 and H^3.

Elastic electron scattering has been used to measure the charge and magnetic moment form factors of both He^3 and H^3.[3] Theoretical analysis of these form factors has yielded new information concerning the structure of the three-nucleon system.[4–7] In the present paper we show that recent measurements of the electron–proton coincidence cross section from He^3 and H^3 [8] can be used to obtain further information concerning these nuclei.[9]

The three processes we wish to consider are:

$$e+\mathrm{He}^3\rightarrow d+p+e', \tag{A}$$

$$e+\mathrm{He}^3\rightarrow (n+p)_{J=0}+p+e', \tag{B}$$

$$e+\mathrm{H}^3\rightarrow (n+n)_{J=0}+p+e'. \tag{C}$$

The inelastically scattered electron and the ejected proton are to be measured in coincidence. We treat this process in the impulse approximation, i.e., we assume that the electron interacts with the ejected proton and neglect the interaction of the proton with the residual two-nucleon system. In Sec. II we analyze the cross section in terms of nonrelativistic wave functions for the three-nucleon systems. In order to do this we make use of the nonrelativistic reduction of the electron–nucleon interaction given by McVoy and Van Hove.[10] In Sec. III we give an alternative description based on a pole approximation to a dispersion relation. The results are compared and discussed in Sec. IV.

II. ANALYSIS IN TERMS OF WAVE FUNCTIONS

We assume for the moment that the wave function for He^3 or H^3 is given by the dominant S-state wave function

$$\psi_i(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = u(r_{12}, r_{13}, r_{23})\phi_0. \tag{1}$$

The spatial wave function u is completely symmetric under the interchange of any pair of nucleons while ϕ_0 is the completely antisymmetric spin–isospin ($J^\pi=\frac{1}{2}^+$, $T=\frac{1}{2}$) function defined by Schiff.[6]

In the final three-body wave function we describe the motion of the proton by a plane wave and assume that the final two-nucleon system is left in either the $^3S_1$ or $^1S_0$ state. The final wave function is then

$$\psi_f(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \sqrt{3}\varphi_J(r_{23}) \exp[-i\mathbf{p}_i\cdot(\mathbf{r}_2+\mathbf{r}_3)/2+i\mathbf{p}_f\cdot\mathbf{r}_1] \cdot\chi_J(2,3)\chi_{\frac{1}{2}}(1)\eta_T(2,3)\eta_{\frac{1}{2}}(1). \tag{2}$$

The quantities J and T are the spin and isospin of the final two-nucleon system and χ_J and η_T are the appropriate spin and isospin functions. The spatial part of the two-nucleon wave function is denoted by $\varphi_J(r_{23})$. In Eq. (2) $\mathbf{p}_f$ is the final proton momentum and $\mathbf{p}_i$ is the initial proton momentum which is the negative of the total momentum of the recoiling two-nucleon system.

Using these wave functions and the electron–nucleon interaction given by McVoy and Van Hove the co-

* This work was supported in part by the U. S. Office of Naval Research and the U. S. Air Force through Air Force Office of Scientific Research contract AF 49(638)-1389. Computation was supported by National Science Foundation Grant NSF-GP948.

[1] R. Hofstadter, Ann. Rev. Nucl. Sci. **7**, 231 (1957).
[2] C. Barber, Ann. Rev. Nucl. Sci. **12**, 1 (1962).
[3] H. Collard, R. Hofstadter, E. B. Hughes, A. Johansson, M. Yearian, R. B. Day, and R. T. Wagner, *Proccedings of the 1964 International Conference on High Energy Physics at Dubna* (to be published), and earlier references cited therein.
[4] J. S. Levinger, Phys. Rev. **131**, 2710 (1963).
[5] B. K. Srivastava, Phys. Rev. **133**, B545 (1964).
[6] L. I. Schiff, Phys. Rev. **133**, B802 (1964).
[7] D. A. Krueger and A. Goldberg, Phys. Rev. **135**, B934 (1964).
[8] A. Johansson, Phys. Rev. **136**, B1030 (1964).
[9] T. A. Griffy and R. J. Oakes, Phys. Rev. **135**, B1161 (1964).
[10] K. W. McVoy and L. Van Hove, Phys. Rev. **125**, 1934 (1962).

incidence cross section for processes (A), (B), and (C) are[9]

(A) $e+He^3\rightarrow d+p+e'$

$$d^3\sigma/dE_f d\Omega_f d\Omega_p=\tfrac{3}{2}\sigma_0\,|\,I_1\,|^2, \tag{3a}$$

(B) $e+He^3\rightarrow(n+p)_{J=0}+p+e'$

$$d^3\sigma/dE_f d\Omega_f d\Omega_p=\tfrac{1}{2}\sigma_0\,|\,I_0\,|^2, \tag{3b}$$

(C) $e+H^3\rightarrow(n+n)_{J=0}+p+e'$

$$d^3\sigma/dE_f d\Omega_f d\Omega_p=\sigma_0\,|\,I_0\,|^2, \tag{3c}$$

where

$$\sigma_0=\sigma_{\text{Mott}}\frac{|\,\mathbf{p}_f\,|\,(\mathbf{p}_f^2+M^2)^{\frac{1}{2}}}{(2\pi)^3\,|\,\mathbf{k}_i/E_i-\mathbf{p}_i/M\,|}\left\{F_{1p}^2-\frac{q^2}{4M^2}\kappa_p^2F_{2p}^2\right.$$
$$+\frac{\mathbf{q}^2}{2M^2}\tan^2\tfrac{1}{2}\theta(F_{1p}+\kappa_pF_{2p})^2+\tan^2\tfrac{1}{2}\theta\frac{F_{1p}^2}{4M^2}(2\mathbf{p}_f-\mathbf{q})^2$$
$$+\frac{F_{1p}^2}{4M^2}\sec^2\tfrac{1}{2}\theta[\hat{k}_i\cdot(2\mathbf{p}_f-\mathbf{q})\hat{k}_f\cdot(2\mathbf{p}_f-\mathbf{q})$$
$$\left.-2M(\hat{k}_i+\hat{k}_f)\cdot(2\mathbf{p}_f-\mathbf{q})]\right\} \tag{4}$$

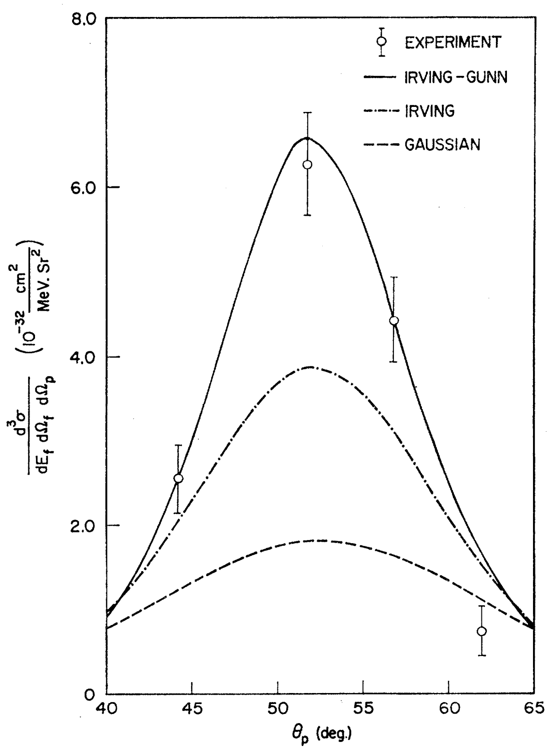

FIG. 1. The cross section $d^3\sigma/dE_f\,d\Omega_f\,d\Omega_p$ for the process $e+He^3\rightarrow d+p+e'$ as a function of the proton scattering angle θ_p for the conditions $E_i=549.1$ MeV, $E_f=443.4$ MeV, and $\theta=51.68$ deg. The curves shown are the results obtained using Gaussian, Irving, and Irving–Gunn three-body wave functions having parameters α of 75.9 MeV, 250 MeV, and 152 MeV, respectively. The normalization is absolute.

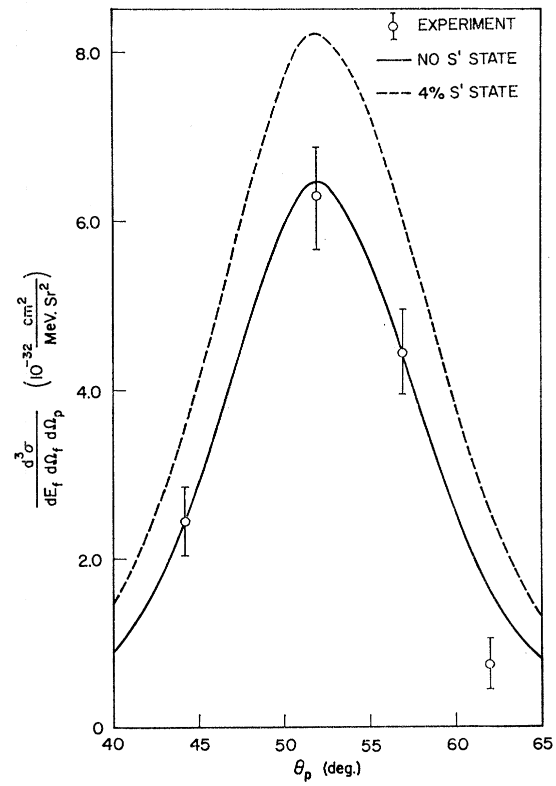

FIG. 2. The cross section $d^3\sigma/dE_f\,d\Omega_f\,d\Omega_p$ for the process $e+He^3\rightarrow d+p+e'$ as a function of the proton scattering angle θ_p for the conditions $E_i=549.1$ MeV, $E_f=443.4$ MeV, and $\theta=51.68$ deg. The curves are the results obtained using the Irving–Gunn wave function ($\alpha=152$ MeV) with no S' state and with a 4% admixture.

and

$$I_J(\mathbf{p}_i)=\int d^3\boldsymbol{\varrho}\int d^3\mathbf{r}\varphi_J(\boldsymbol{\varrho})\,\exp[i(\mathbf{q}-\mathbf{p}_f)\cdot\mathbf{r}]u(\boldsymbol{\varrho},\mathbf{r}). \tag{5}$$

The vectors $\boldsymbol{\varrho}$ and $\mathbf{r}$ are related to r_{12}, r_{13}, and r_{23} through the equations $\mathbf{r}_{23}=\boldsymbol{\varrho}$, $r_{12}=\mathbf{r}-\boldsymbol{\varrho}/2$, $\mathbf{r}_{13}=\mathbf{r}+\boldsymbol{\varrho}/2$ while

$$\sigma_{\text{Mott}}=\frac{e^4\cos^2\theta/2}{4E_i^2\sin^4\theta/2}. \tag{6}$$

Note that since $\mathbf{q}-\mathbf{p}_f=\mathbf{p}_i$ is the initial momentum of the ejected proton, the cross-section factors into the cross section for scattering from a proton of momentum $\mathbf{p}_i$, times the probability of finding a proton with momentum $\mathbf{p}_i$ in the initial nucleus. The angular distribution of the coincidence proton clearly provides a sensitive test of the initial three-body wave function.

We have evaluated the cross section given in Eq. (3a) for some specific three-nucleon wave functions. We choose to discuss process (A) in which the two-nucleon final-state forms a deuteron since the deuteron

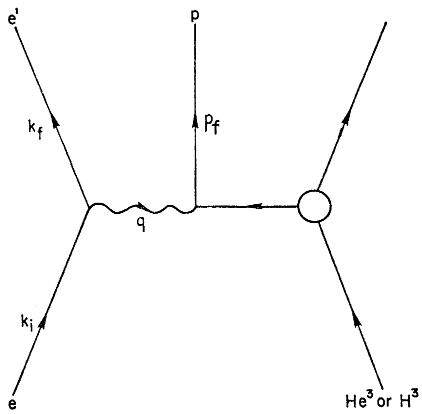

FIG. 3. Nucleon–pole diagram.

wave function is relatively well known, allowing the three-nucleon system to be investigated without the additional uncertainties in one's understanding of the two-nucleon system. The results of the calculation are shown in Fig. 1. We have used a Hulthén wave function for the deuteron[11] and have used Gaussian,

$$u(r_{12}, r_{13}, r_{23}) = A \exp\left[-\alpha^2(r^2+3\rho^2/4)\right], \tag{7a}$$

Irving,

$$u(r_{12}, r_{13}, r_{23}) = A \exp\left[-\tfrac{1}{2}\alpha(2r^2+3\rho^2/2)^{\frac{1}{2}}\right], \tag{7b}$$

and Irving–Gunn,

$$u(r_{12}, r_{13}, r_{23}) = \frac{A \exp\left[-\frac{1}{2}\alpha(2r^2+3\rho^2/2)^{\frac{1}{2}}\right]}{(2r^2+3\rho^2/2)^{\frac{1}{2}}}, \tag{7c}$$

wave functions for the three-nucleon system. In Eqs. (7) A is the appropriate normalization constant.[9] Also shown in Fig. 1 are the recent experimental results of Johansson[8] for the coincidence cross section, which agree best with the Irving–Gunn predictions.

We have also included in the analysis an admixture of the S-state of mixed symmetry (called S') as suggested by Schiff.[6] The results of the calculation with a 4% S' state are shown in Fig. 2, for the Irving–Gunn wave function. From a comparison with the experimental points it seems that an upper limit of about 2% can be placed on the amount of S' state admixture. This conclusion is supported by the recent calculations of the radiative capture of neutrons by deuterium.[12]

III. ANALYSIS IN TERMS OF THE POLE APPROXIMATION

The electron–proton coincidence cross section can also be analyzed by means of the dispersion relation techniques so commonly applied to higher energy phenomena. The important singularity to consider is the nucleon–pole contribution shown in Fig. 3.[13] This pole is quite near to the physical region so that off-mass-shell effects reasonably can be neglected.

The cross sections for processes (A), (B), and (C), up to terms of the order of the binding energy are given in terms of the nucleon–pole term by

(A) $e+\text{He}^3 \to d+p+e'$

$$\frac{d^3\sigma}{dE_f\, d\Omega_f\, d\Omega_p} = \frac{3}{2}\sigma_0\left[\frac{\gamma_A}{(q-p_f)^2-M^2}\right]^2, \tag{8a}$$

(B) $e+\text{He}^3 \to (n+p)_{J=0}+p+e'$

$$\frac{d^3\sigma}{dE_f\, d\Omega_f\, d\Omega_p} = \frac{1}{2}\sigma_0\left[\frac{\gamma_B}{(q-p_f)^2-M^2}\right]^2, \tag{8b}$$

(C) $e+\text{H}^3 \to (n+n)_{J=0}+p+e'$

$$\frac{d^3\sigma}{dE_f\, d\Omega_f\, d\Omega_p} = \sigma_0\left[\frac{\gamma_C}{(q-p_f)^2-M^2}\right]^2, \tag{8c}$$

where the γ's are related to the reduced widths for the appropriate breakup.[14] Comparing Eqs. (8) with

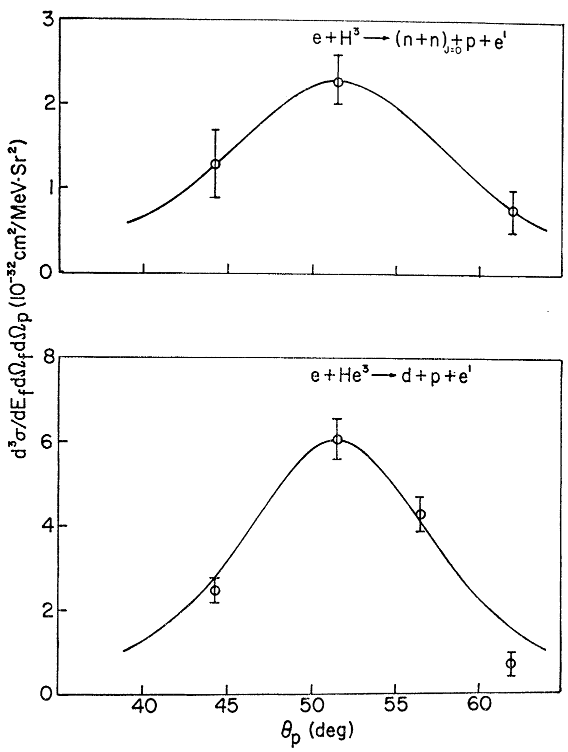

FIG. 4. The cross sections $d^3\sigma/dE_f\, d\Omega_f\, d\Omega_p$ as given by the nucleon–pole approximation.

[11] L. Hulthén and M. Sugawara, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39.

[12] N. T. Meister and T. K. Radha, Phys. Rev. **135**, B769 (1964).

[13] This approach has been applied to the analysis of electron-deuteron scattering by L. Durand, III, Phys. Rev. **123**, 1393 (1961).

[14] The dimensionless coupling constant that would appear in an effective Lagrangian is $g=\gamma(2\pi M)^{-\frac{1}{2}}$.

the previous results [Eqs. (3)] we see that effectively the overlap integrals $I(\mathbf{p}_i)$ have been replaced by the pole terms,

$$\gamma[(q-p_f)^2-M^2]^{-1}\simeq\gamma(\tfrac{3}{2}\mathbf{p}_i^2+2M\epsilon)^{-1},$$

ϵ being the appropriate binding energy. Consequently, the γ's are given by

$$\gamma_i=2M\epsilon_i I_i(0). \qquad (9)$$

Figure 4 shows the results obtained from this pole approximation for processes (A) and (C). [The cross section for process (B) is reported to be approximately $\frac{1}{5}$ times the cross section for process (A).[8]] In each case the constant γ has been used to normalize the calculations to the data at the peak ($\mathbf{p}_i=0$), giving values of $\gamma_A=105$ MeV$^{\frac{1}{2}}$, $\gamma_B=115$ MeV$^{\frac{1}{2}}$, and $\gamma_C=121$ MeV$^{\frac{1}{2}}$. (The corresponding dimensionless coupling constants are $g_A=1.37$, $g_B=1.50$, and $g_C=1.57$.)

IV. DISCUSSION AND SUMMARY

We have computed the cross section for the knockout of a proton by an inelastically scattered electron in He^3 and H^3 in two approximations: (i) the nonrelativistic approximation in which the three-body nucleus is described by a wave function, and (ii) the nucleon–pole approximation. In the former case the Irving–Gunn wave function was found to best predict the shape of the cross section as shown in Fig. 1. In the latter case the shape is given by the nucleon propagator and agrees quite well with the data as indicated in Fig. 4. In both approximations essentially one parameter enters, $I(0)$ in the former and γ in the latter. Their relation is given by Eq. (9). Empirically, it was found that $\gamma_A \lesssim \gamma_B \simeq \gamma_C$ as one might expect from charge independence and the fact that the nucleon–nucleon force is slightly stronger in the $^3S_1$ state then the $^1S_0$ state. In both cases it would be interesting to compare the values of the parameters with those found in the analysis of other processes in which the same vertices occur.

Discussion

PHILLIPS: I wasn't clear on whether or not the actual experiments differentiated between the case of the emission of a proton to leave a bound deuteron, or whether it was in fact a four-body final state.

GRIFFY: If you look at the proton energy, there is a peak corresponding to leaving a deuteron, and this tails off. It is not a very clean separation, but there is an approximately 2.2-MeV separation between leaving the deuteron and an unbound proton.

O'CONNELL: If you were to make a more straightforward three-body breakup calculation, would you treat the reaction as sequential, or quasifree scattering?

GRIFFY: If you want to say a strong correlation between the two final neutrons, in the case of tritium, is the same as saying it is a sequential decay, then I would call it that. There certainly is a strong correlation, since there is a strong, singlet interaction between two neutrons at low energies, and these things are going off at relatively low energies.