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Problems in the Analysis of Quasifree Scattering*

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I. INTRODUCTION

The value of the quasifree $(p, 2p)$ reaction in elucidating the shell structure of nuclei is now well established, and theoretical analyses of the data have reached a fairly high degree of accuracy, particularly in the treatment of the distorted waves (D62, B63, M64, J64). Similarly, quasifree scattering in such reactions as (p, pd), $(p, p\alpha)$, $(\alpha, 2\alpha)$ should yield information on the cluster structure of nuclei. The data on the (p, pd) and $(p, p\alpha)$ reactions (R62, R63) have so far been analyzed using plane waves and neglecting antisymmetrization (R62, R63, S63) despite the fact that one of the outgoing particles is strongly absorbed and antisymmetrization is essential in the cluster model, or using distorted waves but ignoring the three-body nature of the final state (S64).

We discuss here the problems involved in an accurate analysis of the (p, pd) reaction. We consider the transition from an initial state consisting of an incident proton 0 and a nucleus A, to a final state of an outgoing proton $0'$, an outgoing deuteron d and a residual core C of $A-2$ nucleons. We use the relative coordinates $\mathbf{r}_{0A}=\mathbf{r}_0-\mathbf{r}_A$, $\mathbf{r}_{dC}=\mathbf{r}_d-\mathbf{r}_C$, $\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2$, where $\mathbf{r}_A$, $\mathbf{r}_C$, $\mathbf{r}_d$ are position vectors of centers of mass. We first neglect antisymmetrization and assume that nucleons 1 and 2 form a *permanent* deuteron cluster. The corresponding matrix element will be called the *direct* term.

Following the usual method of direct interaction theory, we separate the interaction of the incident proton with the nucleus, $\sum_i v_{0i}$, into a direct interaction with the deuteron cluster and an optical potential. The distorted wave functions for the proton or deuteron must then be obtained by solving, either approximately or exactly, the Schrödinger equation for the motion of the particle relative to the origin of the optical potential, and are therefore functions of $\mathbf{r}_{0A}$ and $\mathbf{r}_{dC}$. This means that the total kinetic energy operator must be expressed in terms of the momenta $\mathbf{p}_{ij}$ *conjugate to the relative coordinates*, and denoting the new k.e. operators by T_{ij} we find (J64, J64a)

$$T_0+T_d+T_C=T_{0A}+T_{dC}+T_{A+1}.$$

We work in the center-of-mass system so that $T_{A+1}=0$, and denote the conjugate momenta $\mathbf{p}_{0A}$, $\mathbf{p}_{dC}$ by $\mathbf{k}_0$, $\mathbf{k}_0'$, $\mathbf{k}_d'$ for the initial proton and the final proton and deuteron, respectively. The formalism of the cluster model is particularly convenient for this analysis (J64a), and the wavefunctions for the initial and final states are then given by

$$\Psi_i=\phi_i(\mathbf{r}_{12})\psi(\mathbf{r}_{dC})\,\Phi\text{ (core) }\chi_p^+(\mathbf{k}_0, \mathbf{r}_{0A})$$

$$\Psi_f=\phi_f(\mathbf{r}_{12})\,\Phi\text{ (core) }\chi_p^-(\mathbf{k}_0', \mathbf{r}_{0A})\chi_d^-(\mathbf{k}_d', \mathbf{r}_{dC}),$$

where $\chi^{\pm}$ are incoming and outgoing distorted wave functions and ϕ_f is the internal wave function for a free deuteron.

II. THE DIRECT TERM

We use zero-range approximation and consider two forms for the direct interaction. First, we take $V_{\text{int}}=t_D(q)\delta(\mathbf{r}_{0d})$, (I), where $\mathbf{q}$ is the proton momentum transfer $\mathbf{k}_0-\mathbf{k}_0'$, and we assume that the interaction occurs at the center of mass of the deuteron cluster. With this interaction, the matrix element for elastic scattering of protons by deuterons is proportional to $t_D(q)F(0)$, where we define the deuteron form factor as

$$F(q)=\int\phi_f^*(\mathbf{r})\phi_f(\mathbf{r})\exp\left(-\tfrac{1}{2}i\mathbf{q}\cdot\mathbf{r}\right)d\mathbf{r},\qquad F(0)=1.$$

If we use impulse approximation, the direct matrix element for quasifree scattering becomes

$$T_{if}(\mathrm{I})=t_D(q)\,\langle\chi_p^-(\mathbf{k}_0', \mathbf{r}_{0A})\chi_d^-(\mathbf{k}_d', \mathbf{r}_{dC})\phi_f(\mathbf{r}_{12})$$
$$\times\mid\delta(\mathbf{r}_{0A}-g\mathbf{r}_{dC})\mid\phi_i(\mathbf{r}_{12})\psi(\mathbf{r}_{dC})\chi_p^+(\mathbf{k}_0, \mathbf{r}_{0A})\,\rangle$$

where $g=(A-2)/A$. Thus the matrix element separates into two three-dimensional integrals. Using plane waves, the integral over $\mathbf{r}_{dC}$ reduces to

$$G^{\text{PW}}(Q)=\int\psi(\mathbf{r}_{dC})\exp\left(-i\mathbf{Q}\cdot\mathbf{r}_{dC}\right)d\mathbf{r}_{dC},$$

where $\mathbf{Q}=-(g\mathbf{k}_0-g\mathbf{k}_0'-\mathbf{k}_d')$ is the momentum of the deuteron cluster in the laboratory system. Thus $G^{\text{PW}}(Q)$ is the momentum distribution function for the deuteron cluster. We define a distorted momentum distribution

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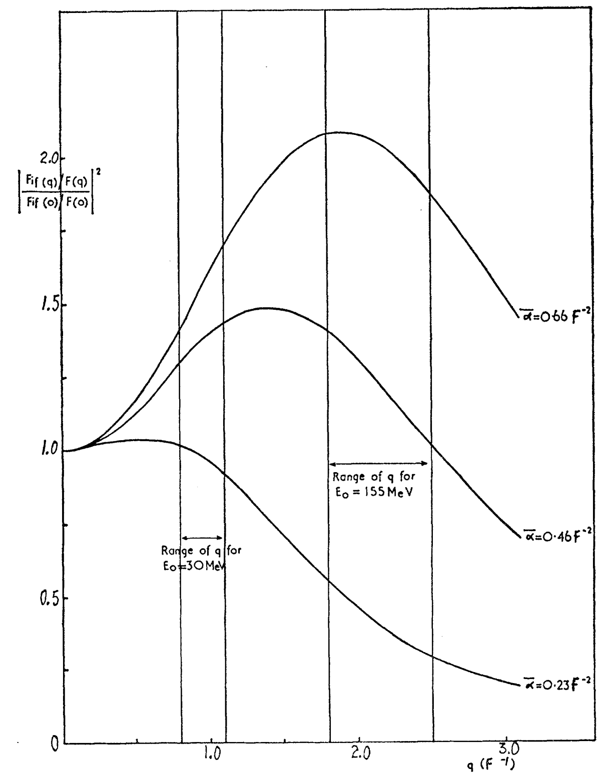

FIG. 1. The variation of $|F_{if}(q)/F(q)|^2$ with q compared with the value at $q=0$. α is the length parameter of the wave function for the deuteron cluster inside the nucleus.

$G^{\mathrm{DW}}(Q)$ in a similar way, and also a mixed form factor

$$F_{if}(q)=\int\phi_f^*(\mathbf{r})\phi_i(\mathbf{r})\exp\left(-\tfrac{1}{2}i\mathbf{q}\cdot\mathbf{r}\right)d\mathbf{r}$$

so that the matrix elements can be written as

$$T_{if}(\mathrm{I,\,PW})=t_D(q)G^{\mathrm{PW}}(Q)F_{if}(0);$$

$$T_{if}(\mathrm{I,\,DW})=t_D(q)G^{\mathrm{DW}}(Q)F_{if}(0).$$

As an alternative form for the direct interaction we take $V_{\mathrm{int}}=t(q)[\delta(\mathbf{r}_{01})+\delta(\mathbf{r}_{02})]$, (II), where $t(q)$ is now the t matrix for nucleon–nucleon scattering averaged over the spin and I spin of the deuteron. The matrix element for free p–d scattering is proportional to $2t(q)F(q)$ and the matrix element for quasifree scattering is given by

$$T_{if}(\mathrm{II})=2t(q)\,\langle\chi_p^-(\mathbf{k}_0',\mathbf{r}_{0A})\chi_d^-(\mathbf{k}_d',\mathbf{r}_{dC})\phi_f(\mathbf{r}_{12})$$
$$\times\mid\delta(\mathbf{r}_{0A}-g\mathbf{r}_{dC}+\tfrac{1}{2}\mathbf{r}_{12})\mid\phi_i(\mathbf{r}_{12})\psi(\mathbf{r}_{dC})\chi_p^+(\mathbf{k}_0,\mathbf{r}_{0A})\rangle.$$

With plane waves this separates to give

$$T_{if}(\mathrm{II,\,PW})=2t(q)G^{\mathrm{PW}}(Q)F_{if}(q)$$

but with distorted waves a six-dimensional integral remains. The cross section for quasifree scattering is obtained by replacing the t matrix by the free cross section (S64, J64a), which gives

$$\frac{d^3\sigma(\mathrm{II,\,PW})}{d\Omega_p\,d\Omega_d\,dE}\propto\frac{d\sigma}{d\Omega_{pd}}\mid G^{\mathrm{PW}}(Q)\mid^2\left|\frac{F_{if}(q)}{F(q)}\right|^2;$$

$$\frac{d^3\sigma(\mathrm{I,\,PW})}{d\Omega_p\,d\Omega_d\,dE}\propto\frac{d\sigma}{d\Omega_{pd}}\mid G^{\mathrm{PW}}(Q)\mid^2\left|\frac{F_{if}(0)}{F(0)}\right|^2$$

and similarly for $d^3\sigma(\mathrm{I,\,DW})$.

III. FEASIBILITY OF CALCULATING THE DIRECT TERM

It is evident that interaction I leads to simpler formulas than interaction II. Unfortunately, the use of I implies that the particles 1 and 2 are closely correlated and behave as a deuteron throughout the scattering process,[1] hence it *presupposes just the condition we wish to investigate.* To test this point we compare the two formulas obtained using plane waves. We have made calculations on the $\mathrm{Li}^6(p,\,pd)$ reaction using the cluster model wave function of Tang *et al.* (T61) which gives $\phi_i(\mathbf{r}_{12})=\exp\left(-\frac{1}{4}\bar{\alpha}r_{12}^2\right)$ with $\bar{\alpha}=0.66\ \mathrm{F}^{-2}$, and using a Hulthen wave function for ϕ_f. Data on this reaction are available for incident protons of 155 MeV (R62, R63) and 30 MeV (D64). It can be seen from Fig. 1 that the variation of $|F_{if}/F|^2$ with q is such that significant errors will arise if this quantity is replaced by its value at $q=0$, particularly for the high-energy experiment.

The conclusion of the previous paragraph has a serious consequence for distorted-wave calculations since it implies that we should calculate not $G^{\mathrm{DW}}(Q)$

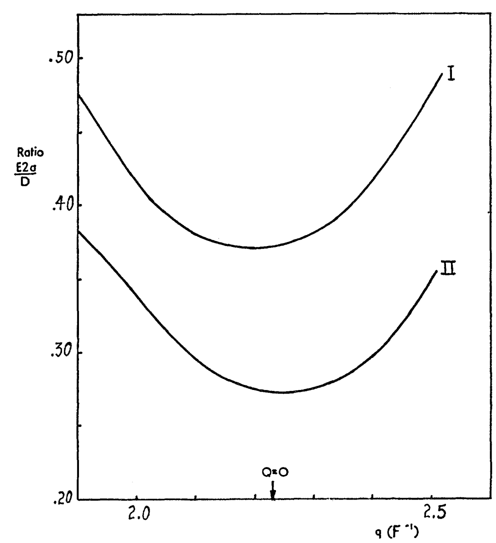

FIG. 2. The ratio of the exchange term $E2a$ to the direct term D as a function of q. (The ordinates should be multiplied by $\frac{1}{2}$.)

[1] Thus it would be more appropriate to the $(p,\,p\alpha)$ reaction since the α cluster is tightly bound.

but the six-dimensional integral occurring in T_{if}(II, DW). It is certainly possible to compute the latter using the methods developed in finite-range theory (A64, P64), but simple interpretations of the reaction in terms of the properties of $G(Q)$ will no longer be possible.

IV. THE EXCHANGE TERMS

Exchange between the incident proton and the deuteron cluster has been included to the extent that we have used the experimentally determined free cross section. We now consider the exchange terms which arise from antisymmetrization of the nuclear wave functions. These terms may be represented symbolically as follows.

Direct

$$D = \langle 0;12;345\,..\,A \mid \textstyle\sum v_{0i} \mid 0;12;345\,..\,A \rangle$$

Single exchange

$$E1 = \langle 0;14;325\,..\,A \mid \textstyle\sum v_{0i} \mid 0;12;345\,..\,A \rangle$$

Double exchange

$$E2 = \langle 0;34;125\,..\,A \mid \textstyle\sum v_{0i} \mid 0;12;345\,..\,A \rangle.$$

We have estimated the contribution to the $Li^6(p, pd)$ reaction from the following typical exchange terms,

$$E1a = \langle 0;14;3256 \mid V_{\rm int}(14) \mid 0;12;3456 \rangle,$$

$$E2a = \langle 0;34;1256 \mid V_{\rm int}(34) \mid 0;12;3456 \rangle,$$

$$E2b = \langle 0;34;1256 \mid V_{\rm int}(12) \mid 0;12;3456 \rangle,$$

using interactions I and II and plane waves. In order to evaluate the single exchange terms we have made the drastic assumption that the free deuteron may be regarded as a point, and the results given here can be regarded as illustrative only. For q in the range for the high-energy experiment, the ratio of terms $E2b$, $E1a$ to the direct one are $<10^{-3}$, whereas $E2a$ yields the large ratio shown in Fig. 2.

V. CONCLUSION

We have shown that owing to distortion and exchange effects, the (p, pd) reaction on light nuclei is more complex, both in interpretation and computation, than has previously been supposed. Simplification will result if the cluster has a relatively small size in the nucleus in the initial state, and is strongly absorbed in the final state so that the reaction is localized. These conditions may well be satisfied for medium and heavy nuclei.

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Discussion

PUGH: I think this is a very valuable contribution to introduce a note of caution to the analysis of some of these experiments. The simple analysis seems to work very well qualitatively, and this encourages us to make stronger conclusions than we might otherwise do. Are there any comments or questions?

JACKSON: I might just add that I would feel a lot happier about using interaction I, that is, an interaction at the center of mass of the cluster, in the case of the $(p, p\alpha)$ experiment. One feels that an alpha cluster is a little more of a cluster than perhaps the deuteron is.