Information Obtainable from $(p, 2p)$ Reactions*

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I discuss the theoretical interpretation of the $(p, 2p)$ reaction as the direct knockout of a proton from a nucleus by a single-particle interaction with an incident proton. The analysis up to date shows that there is very interesting information about the details of a nuclear reaction available from $(p, 2p)$ reactions at various incident energies, as well as information about the structure of the initial nucleus. It also leaves some interesting questions that have yet to be answered.

The analysis is carried out in the distorted-wave Born approximation. I propose to outline the derivation as completely as I can. However, in order to orient the discussion I would like first to discuss the simplest possible model, the plane-wave Born approximation. In this approximation, the matrix element $T_L{}^M$ is written in the following way:

$$T_L{}^M=\iint d^3r_1\, d^3r_2 \exp(-i\mathbf{k}_L\cdot\mathbf{r}_1) \times \exp(-i\mathbf{k}_R\cdot\mathbf{r}_2)v(|\mathbf{r}_1-\mathbf{r}_2|)\exp(i\mathbf{k}_0\cdot\mathbf{r}_1)\psi_L{}^M(\mathbf{r}_2). \quad (1)$$

We are assuming that the knocked-out proton was initially in a single-particle state $\psi_L{}^M$. The two-body interaction is approximated by a local, central interaction $v(|\mathbf{r}_1-\mathbf{r}_2|)$. The kinematic situation is defined in Fig. 1. The k's are the wave numbers corresponding to the E's.

Making the transformation

$$\mathbf{r}_1-\mathbf{r}_2=\mathbf{r}, \qquad \mathbf{r}_2=\mathbf{r}' \quad (2)$$

we may factorize the matrix element in the following way

$$T_L{}^M=\left[\int d^3r \exp[i(\mathbf{k}_0-\mathbf{k}_L)\cdot\mathbf{r}]v(r)\right] \times\left[\int d^3r' \exp[i(\mathbf{k}_0-\mathbf{k}_L-\mathbf{k}_R)\cdot\mathbf{r}']\psi_L{}^M(\mathbf{r}')\right]. \quad (3)$$

The first factor is the Fourier transform of the two-body interaction with respect to the momentum transfer suffered by the incident particle. Since we are making the Born approximation, the best choice for $v(r)$ is a pseudopotential that fits free (p, p) scattering, that is a function whose squared Fourier transform is the differential cross section for (p, p) scattering. This is a momentum space approach. We are using a model for the interaction that reproduces the right momentum components on the two-body energy shell. It is also a model for the interaction off the energy shell. Now, when $T_L{}^M$ is squared, the first factor is the free (p, p) cross section.

The second factor is the Fourier transform of the bound-state wave function with respect to the momentum transfer suffered by the residual nucleus or the supposedly inert core. It is the momentum-space wave function of the struck particle.

It is obvious, even though the plane-wave approximation may have only a rudimentary resemblance to the facts, that there are two types of information in the reaction, one about the two-body force, one about the wave function of the struck particle. How do these things affect the angular correlation?

First, the bound-state factor is familiar in plane-wave direct interaction theory. For a surface interaction it is a spherical Bessel function which is zero at zero momentum transfer if $L\geq 1$, maximum at zero momentum transfer if $L=0$. Specializing to the case $E_L=E_R$, $Q=0$, $\theta_L=\theta_R=\theta$ for simplification of the discussion, zero momentum transfer occurs when the struck particle was stationary at the moment of impact, that is when $\theta=45°$.

If θ is varied, we have two spherical Bessel functions in the angular correlation, one for $\theta<45°$ representing a collision with a particle moving away and one for $\theta>45°$ representing a collision with a particle moving towards the incident particle. For $L\geq 1$ there are two peaks. Conservation of momentum says they must be of equal height and mirror images about $K=0$ if plotted against K rather than θ. The width of the spherical Bessel function depends on L and on the rms radius of the wave function.

The first factor influences the angular correlation in the following way. The (p, p) cross section is greater for a collision with a particle moving away, that is for lower energy in the two-body center-of-mass system. Therefore the left peak is higher than the right.

This approximation thus gives us the idea that the width of the angular correlation distribution is related to the spectroscopic information, the peak-height ratio (for $L\geq 1$) is related to the information about the reaction mechanism.

Let us now consider the derivation of as exact an

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approximation as possible for the reaction. The transition amplitude for a reaction due to two potentials U and V is

$$T=\langle\Psi^{(-)}\mid V\mid\Psi^{(+)}\rangle+\langle\Phi^{(-)}\mid U\mid\Psi^{(+)}\rangle, \quad (4)$$

where $\Psi^{(\pm)}$ are wave functions calculated in the "distorting" potential U. $\Phi^{(-)}$ is the noninteracting wave function for the final state. If the Hamiltonian of the system is the same before the interaction as afterwards, the second term is zero. This is approximately true for $(p, 2p)$, so there is some *a priori* justification for the neglect of the second term. (This is not the case for rearrangement collisions.)

What do we use for $\Psi^{(+)}$ and $\Psi^{(-)}$? $\Psi^{(+)}$ represents an ingoing particle interacting with a nucleus. The distorted wave approximation for this is familiar. It is

$$\Psi^{(+)}=x^{(+)}\psi_L{}^M, \quad (6)$$

where $x^{(+)}$ is an optical model wave function, $\psi_L{}^M$ is the bound-state wave function discussed before.

$\Psi^{(-)}$ represents a three-body state where two of the bodies have mass 1 and one has mass A. The wave function for this state may be separated into one representing the motion of the center of mass and a two-body wave function which is the solution of the following Schrödinger equation

$$[-(2\mu)^{-1}\nabla_L{}^2+V_L-(2\mu)^{-1}\nabla_R{}^2+V_R-A^{-1}\nabla\cdot\nabla_R]\Psi^{(-)}=E\Psi^{(-)}. \quad (7)$$

The interaction term $A^{-1}\nabla_L\cdot\nabla_R$ may be treated as a perturbation. In the first order the equation separates into two one-body equations which may be thought of as optical model equations.

$$[-(2\mu)^{-1}\nabla^2+V]x^{(-)}=E'x^{(-)}.$$

The effective energy is, putting $\alpha=A^{-1}$,

$$E'=\frac{1}{2}\left(\frac{1+\alpha}{1+2\alpha}E_{\text{lab}}+Q\right)\times\left\{1-\frac{\alpha[(4\alpha^2+4\alpha+2)\cos^2\theta-1]}{(1+\alpha)^2(1+2\alpha\cos^2\theta)}\right\}. \quad (8)$$

The distorted-wave matrix element is now

$$T_L{}^M=\iint d^3r_1\,d^3r_2 x^{(-)*}(\mathbf{k}_L,\mathbf{r}_1)x^{(-)*}(\mathbf{k}_R,\mathbf{r}_2)\times v(|\mathbf{r}_1-\mathbf{r}_2|)x^{(+)}(\mathbf{k}_0,\mathbf{r}_1)x_L{}^M(\mathbf{r}_2). \quad (9)$$

I describe a calculation by Lim and myself[1] starting from this matrix element. Unless I specifically say otherwise, I refer to the coplanar symmetric case. We attempted to use models for every factor in the matrix element which were as realistic as possible and which were directly related to simpler experiments so that there are no free parameters in the theory.

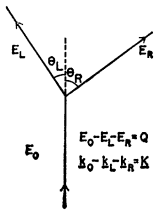

FIG. 1 Definition of the symbols used in the description of the $(p, 2p)$ reaction.

The optical model potentials were found by interpolation in the available optical model literature. Spin–orbit coupling was omitted, since we are not describing polarizations. This approximation is good for momentum transfers which are not too large. The reactions we are describing have in fact quite small momentum transfer. (I discuss this later.) The parameters for the entrance and exit channels, respectively, were V_0, W_0, and V_1, W_1 with an Eckart form factor described by r_0 and b.

The bound-state wave function description is most important. The information we have, assuming a single-particle model, is the binding energy from the $(p, 2p)$ experiment itself, and the rms radius of the charge distribution. This gives us a strong set of constraints since for light nuclei we have both the s- and p-state angular correlations so that we must fit the electron-scattering radii.

In fact calculations with potential wells of different shapes have shown that with a given binding energy and rms radius the matrix element is essentially independent of the well shape, provided it has a fairly flat bottom. We used a square well for simplicity with parameters V_B and a.

For the two-body pseudopotential we used

$$v(r)=-83\left(\frac{\exp(-0.73r)}{0.73r}-5\frac{\exp(-1.5r)}{1.5r}+20\frac{\exp(-3r)}{3r}\right)\text{MeV}. \quad (10)$$

The squared Fourier transform of this $v(r)$ gives the fit to the 90° differential cross section for (p, p) scattering shown in Fig. 2.

Figure 3 shows the result of the calculation for 155-MeV incident protons on C^{12} compared with experiment.[2]

There are two things to be noticed about the result. First, the magnitude of the left peak is fairly well reproduced. Less exact single-particle direct inter-

[1] K. L. Lim and I. E. McCarthy, Phys. Rev. Letters **13,** 446 (1964).

[2] J. P. Garron, J. C. Jacmart, M. Riou, C. Ruhla, J. Teillac, and K. Strauch, Nucl. Phys. **37,** 126 (1962).

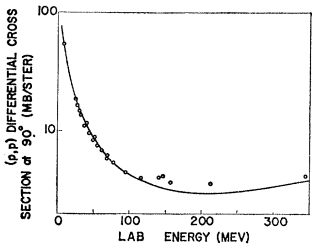

FIG. 2. The differential cross section for (p, p) scattering at 90° calculated from the 3-Yukawa pseudopotential[10] plotted against experimental data.

action theories have not previously reproduced magnitudes anywhere near correctly and this is a very hopeful indication that we are on the right track. Second, the size of the right peak is underestimated by a factor of about 2. Our intuition from the plane-wave model would tell us that we have not enough high momentum transfer components in the two-body interaction.

We have varied the factors in the integrand of the matrix element to see if the correct angular correlation can be produced without using hopelessly unrealistic values of the parameters. Our experience with zero-range calculations[3] has shown that, although distortion does reduce the right peak relative to the left peak, the large observed effect certainly cannot be reproduced with any believable optical model parameters.

The binding energy and rms radius of the bound-state wave function are fixed, but we have artifically added a 20% admixture of a $2p$ wave function in an infinite square well of the same radius in order to see if additional high momentum components in the bound state will help. This admixture changed the differential cross section by about 1%.

However, as expected, the ratio of peak heights turned out to be very sensitive to the amount of one pion component in the interaction. Reducing the ratio of one pion component by a little less than a factor of 2 produces a reasonable fit to the experiment. The fit for a factor of 2 is shown in Fig. 4.

It might be asked whether the spin dependence of the two-body force might not change the result appreciably. We have carried out an explicitly spin-dependent formulation of the problem. In the cross section there are terms which contain space–symmetric and space–antisymmetric matrix elements.

In the zero-range DWBA, the space–antisymmetric terms vanish for the case of symmetric coplanar scattering. With our pseudopotential the antisymmetric cross section calculated without spin–orbit coupling is 1% of the symmetric cross section at 155 MeV and 20% at 50 MeV. We can neglect it at 155 MeV. The size of the antisymmetric cross section is a measure of how far off the energy shell we are. Spin–orbit coupling is small at 155 MeV.

A preliminary formulation of the problem has been made with a velocity-dependent two-body potential. In this case, the separation approximation is not equivalent to the impulse approximation and it is possible for the angular correlation to be different at 155 MeV from the impulse approximation value. It is shown later that the impulse approximation is quite close for the local pseudopotential at this energy. A calculation using simplified optical model wave functions will be performed with spin–orbit coupling and velocity dependence. Of course it must be realized that the local pseudopotential is chosen in such a way as to simulate the velocity dependence in free (p, p) scattering.

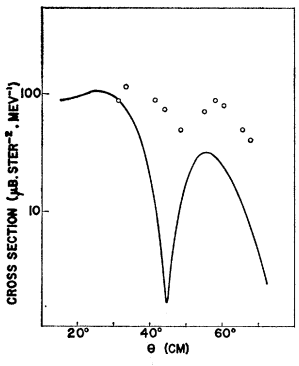

FIG. 3. 155 MeV $(p, 2p)$ angular correlation for the p state of C^{12} compared with theory.

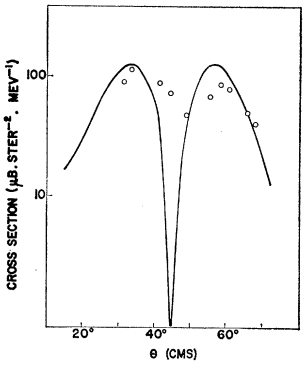

FIG. 4. The angular correlation for 155 MeV $(p, 2p)$ on the p-state of C^{12} with the core terms in the interaction increased by a factor of 2.

The questions we are left with are: Is it necessary to use a better approximation then the distorted-wave Born approximation to describe the reaction? If a better approximation does not give a significantly different result, can the difference between the two-body interactions inside and outside nuclear matter be explained by a theory of nuclear matter? These questions are left for the future, but it is possible to make one qualitative observation.

Momentum transfers under discussion here are not much greater than 1 F^{-1}. This is the reciprocal of the healing distance at which, at least for infinite nuclear matter, the two-body wave function becomes like the

[3] K. L. Lim and I. E. McCarthy, Phys. **133**, B1006 (1964).

wave function with strong short-range correlations. This at least indicates that the strong correlations are not likely to upset a simple two-body theory like the distorted-wave Born approximation for the momentum transfers we are concerned with.

The present theory is sufficiently complete to answer some more questions of principle. It may be asked why it is useful to use a pseudopotential which is a model for the two-body interaction both on and off the energy shell, when we only know the two-body interaction on the energy shell. Is not the impulse approximation adequate? We can answer this question by comparing our result with the impulse approximation for the same situation.

We first make the separation approximation. That is we use Eq. (3) with the second factor replaced by the zero-range distorted-wave matrix element. The first factor is replaced by the appropriate expression on the two-body energy shell. Figure 5 shows how the impulse approximation compares with the more exact theory at 155-MeV and 50-MeV incident energy.

Although the impulse approximation is not bad at 155 MeV, it is very far off at 50 MeV. This leads to the useful conclusion that $(p, 2p)$ experiments at 50 MeV contain a large amount of information about the effective two-body force in nuclear matter off the energy shell. $(p, 2p)$ experiments are a very good way of getting this information because they are particularly simple experiments with a three-body final state. The fact that one of the bodies is heavy enables us to use the separable perturbation expansion of Eq. (6).

So far I have not said much about the information that can be obtained about bound states. Our early zero-range calculation[3] showed that the rms radii of the s and p states in light nuclei can be well enough determined to show that, if the square-well parametrization of the wave functions is used, the s state well is narrower and deeper than the p state well. This state dependence of the single-particle potential also arises from the finite nucleus calculations of Brueckner, Lockett, and Rotenberg.[4] The finite range calculation bears this out. The s-state rms radius appears to be a little larger than that of the α particle.

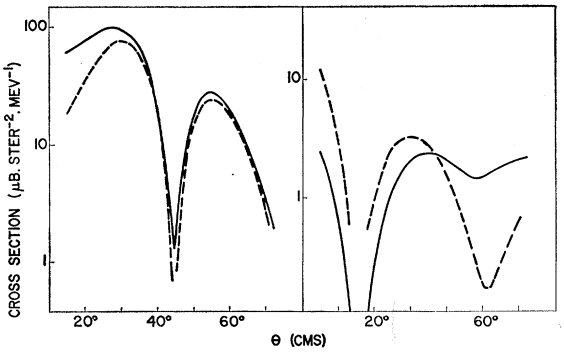

FIG. 5. Comparison of the impulse approximation (broken line) with the more exact theory at 155 MeV (left) and 50 MeV (right).

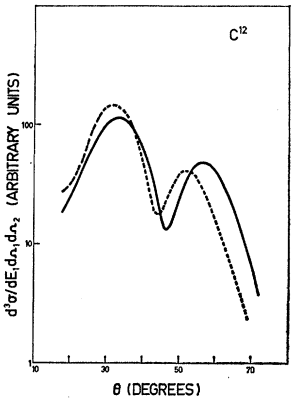

FIG. 6. Comparison of the angular correlation for 155-MeV protons on the p state of C^{12} for $E_L=E_R=69.5$ MeV (full line) with that for $E_L=74.5$ MeV, $E_R=64.5$ MeV (broken line), showing how poor energy resolution can fill in the minimum.

The calculation also contains information about the adequacy of simpler methods of analysis. The 155-MeV results show that at higher energies the impulse approximation should be quite good enough for obtaining spectroscopic information. Jackson and Berggren[5] have compared a simple zero-range distorted wave calculation at 170 MeV for a very light nucleus, Li^6, with a plane wave calculation which is modified by introducing space weighting factors related to the WKB approximation. The agreement is good enough to show that at very high energies, say greater than 200 MeV, a modified plane wave approximation in conjunction with the impulse approximation is sufficient to tell us about the bound-state properties.

Finally, I would like to mention one experimental point. It is noticeable that the low momentum transfer minimum in the angular correlation is much deeper in theory than in experiment at 155 MeV (although at 50 MeV the distortion turns the minimum into a maximum just as it does for 2+ excitations by proton inelastic scattering at low energies). Good angular resolution does not change this. However, if only the sum of the final energies is well resolved, there is an explanation. Unequal final energies with the same sum can shift the minimum considerably as is shown in Fig. 6. Here the curve for $E_L=E_R$ in the zero-range approximation is compared with that for $E_L=74.5$ MeV, $E_R=64.5$ MeV. We conclude from this that individual resolutions of at least 1 MeV are needed.

Discussion

ZAMICK: I can't see from your expression for your distorted-wave scattering amplitude that it would involve, at all, a two-body scattering amplitude. Can you show how you do that?

MCCARTHY: Yes. It involved a two-body scattering pseudo-

[4] K. A. Brueckner, A. M. Lockett, and M. Rotenberg, Phys. Rev. **121**, 255 (1961).

[5] D. F. Jackson and T. Berggren (preprint).

potential which, if you believe it, gives you a two-body scattering amplitude.

ZAMICK: In what way?

MCCARTHY: This pseudopotential is not taken outside the integral, as you do in the impulse approximation. Therefore, there are virtual momentum components of the T matrix which you have to integrate over, which do not conserve energy. The third body can take away some energy.

ZAMICK: Can you show in an operational way how one can get off the energy shell?

MCCARTHY: Yes; the operational way I suggest is to make a pseudopotential and put it into the calculation. It has to be done through a pseudopotential, the way I am talking about it. This may not be the best way. You start with a good pseudopotential on the energy shell and get into a situation where you know you are off the energy shell. Now the calculation done this way has shown that you can get off the energy shell given a certain not thoroughly correct pseudopotential. This shows you where you have to look to be off the energy shell. Then the only operational way I can suggest is that you find a phenomenological pseudopotential that fits. It may be that there is no pseudopotential which fits both (p, p) and $(p, 2p)$ data. Then we have a many-body force.

ZAMICK: Is the difference between the impulse approximation and your approximation that it simply replaces the potential by two-body scattering amplitude? Is that correct?

MCCARTHY: No. What you have to do to our approximation to get the impulse approximation is first of all make the separation approximation. This is exact for plane waves. You separate the double integral into two single integrals. The term on your right is the zero-range plane wave calculation. You change this to a zero-range distorted-wave calculation. The term on your left, which is the Fourier transform of the two-body wave function, is still off the energy shell slightly because of kinematics. You get to the nearest place on the energy shell and use the corresponding two-body scattering cross section, after squaring.

ZAMICK: Is there any way one can investigate off the energy shell p-p scattering without going into such complications?

PUGH: There is one way to study off-energy-shell p–p scattering; I believe it was suggested by Marshak a number of years ago. One can look at free p–p scattering accompanied by bremsstrahlung. The γ-ray spectrum is expected to have a peak at the high-energy end, where both protons are emitted in the same direction at a rather small angle to the beam.

ZAMICK: Since your expression for the T matrix is no longer factorable how do you know that you are still extracting the momentum distribution of nucleons in the nucleus?

MCCARTHY: This is a point of contention which I have had a few pointed arguments about. You don't know this. In fact you are not even allowed to use the word! The momentum distribution of particles in the nucleus is a plane wave Born-approximation concept. It doesn't bear any relationship to an experiment. You have to talk about the momentum transfer distribution. They are inextricably mixed up, as you say.

Let me say one more thing: It turns out, from curve fitting, you can uniquely extract a wave function which, if you like, gives you a momentum distribution, but this is a kind of roundabout way of saying things.

PHILLIPS: Would you care to comment any about what would have to be done to such a theory in interpretations of the data, such as Dr. Riou showed us, if there were some time delay in the emission of the two protons? If there were a significant nonzero time interval between the emission of the two protons?

MCCARTHY: No; I'm sorry. I wouldn't care to comment about that; I have been thinking about this, and I haven't thought about it enough to comment.