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SESSION B—SPECTROSCOPY WITH DIRECT REACTIONS

CHAIRMAN: *H. G. Pugh*

Information on Nuclear Strucure from (p, $2p$) and other Knock-Out Reactions

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I. GENERAL REMARKS ON (p, $2p$) REACTIONS

The first experiments at 340 MeV[1] proved that at this energy the reaction proceeds by direct interaction: angular correlations and energy spectra of high-energy outgoing protons are those expected from a quasifree scattering of the incoming protons on protons inside the target nucleus. Tyren, Hillman, and Maris[2] pointed out by their experiments at 185 MeV, that this reaction can be used for studying individual states of protons in nuclei. In these and subsequent experiments, the two outgoing protons in coincidence are detected in various directions relative to the incoming protons and their energies E_1, E_2 are measured. The summed energy spectra (E_1+E_2) are easily related to the binding energy E_B of the nuclear proton, since E_0 is the energy of the incoming proton

$$E_0=(E_1+E_2)+E_B+E_R,$$

where E_R is the recoil nucleus energy which is easily calculated for a given geometry and can be generally neglected. In fact, on the binding energy spectrum is superimposed a continuous background due to multistage processes leading to (p, $2px$) reactions which appears at $E_B>E_x$, where E_x is the separation energy of the x particle in the residual nucleus.

Most of the experiments (Uppsala, Chicago, Orsay) are carried out with the coplanar and symmetric kinematic geometry ($E_1\sim E_2$, $\theta=\theta_1\simeq-\theta_2$) which provides two advantages: the effects of multistage processes and of distortion (larger when E_1 or E_2 is lower) are minimized, and the interpretation of the results is simplified. In this case, the recoil momentum $\mathbf{k}_R=\mathbf{k}_0-\mathbf{k}_1-\mathbf{k}_2$ is colinear with k_0 and has the absolute values: $k_R=k_0-2k\cos\theta$ so that zero momentum is obtained for $\theta=\theta_0=\cos(k_0/2k)$ (44° for $E_B=0$, $E_0=155$ MeV, 34° for $E_B=34$ MeV). Using the lowest order of approximation (impulse approximation, nonrelativistic and symmetric kinematics, plane-wave Born approximation, and single-particle model for the target nucleus), the momentum of the target proton is simply: $\mathbf{Q}=-\mathbf{k}_R$ and the angular correlation distribution, for a given peak is

$$\frac{d^3\sigma}{d\Omega_1\,d\Omega_2\,dE_1}=\frac{4m}{\hbar^2}\frac{k_1k_2}{k_0}\left(\frac{d\sigma}{d\bar{\Omega}}\right)_{p,p}N_l\rho_l(Q), \qquad (1)$$

where $(d\sigma/d\bar{\Omega})_{p,p}$ is the free $p+p$ scattering cross section in their center of mass at $\bar{\theta}=90°$ for a momentum transfer $\mathbf{q}=\mathbf{k}_0-\mathbf{Q}$, i.e., for an incident energy $E_0'\neq E_0-N_l$ is the number of protons in the shell of orbital momentum l and $\rho_l(Q)$ their momentum density distribution. As the variation of $d\sigma/d\bar{\Omega}$ with θ is slow (but not negligible), the angular correlation distribution is very sensitive to the $\rho_l(Q)$ distribution around $Q=0$ ($\theta_0<44°$), up to about 1 F^{-1} (this limit does not arise from kinematical limitations but from experimental ones, the relative importance of the background giving large uncertainties for angles $\theta>60°$ or $\theta<25°$). Thus, the angular correlation distribution will exhibit a maximum near θ_0 for s state, a minimum for $l\neq0$ states and will be roughly symetric by respect to θ_0. Therefore, it will be easy to distinguish between an s state and a $l\neq0$ state, but rather difficult to distinguish between different $l\neq0$ states. Besides this, higher l states are more difficult to observe because of lower cross section due to normalization factor in $\rho(Q)$: by example one can calculate with harmonic oscillator functions the following ratios of the cross section at the maximum for completely filled states ($Z=28$):

$$1f_{7/2}:1d_{3/2}:2s_{1/2}:1d_{5/2}=3,3:2,3:12:3,5$$

which are very far from N_{lj} ratios (8:4:2:6).

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[1] O. Chamberlain and E. Segre, Phys. Rev. **87**, 81 (1962); J. M. Wilcox and B. J. Moyer, Phys. Rev. **99**, 875 (1955).

[2] H. Tyren, P. Hillman, and Th. A. J. Maris, Nuovo Cimento **6**, 1507 (1957); Nucl. Phys. **7**, 1 and 10 (1958); Phys. Rev. Letters **5**, 107 (1960); H. Tyren, P. Hillman, P. Isacsson, and Th. A. J. Maris, Proc. Intern. Conf. Nuclear Structure, reported by G. Jacob, Kingston, p. 429 (1960).

[3] T. J. Gooding and H. G. Pugh, Nucl. Phys. **18**, 46 (1960).

The most complete nonsymmetric experiment was made by Gooding and Pugh[3] at 153 MeV on C^{12} and confirms that the reaction proceeds via a direct interaction.

Recently, nonsymmetric results were also obtained on C^{12} using bubble chamber.[4] Strauch and Gottschalk[5] used a method which gives the energy sharing E_1/E_2 distribution at $\theta=\theta_0$ but, in this case, distortion effects are bigger.

In fact, the relation between the angular correlation distribution and the target proton momentum distribution is more complicated than indicated by the plane-wave approximation. Distorted-wave calculations have been performed by many authors for light nuclei: Berggren and Jacob[6] using WKB approximation, neglecting reflection and refraction effects, Lim and MacCarthy[7] (complete nonrelativistic treatment), Jackson and Berggren[8] (partial wave treatment for Li^6).

Calculation results, as well as experimental ones, show that the above qualitative conclusions are not too much changed:

(1) The absolute cross section decreases by a factor due to absorption. For example, Berggren and Jacob[6] calculated the following factors for $1p$ proton of O^{16}: $\frac{1}{3}$ at 450 MeV and $\frac{1}{7}$ at 170 MeV.

(2) The maximum or the minimum does not occur exactly at θ_0 and is shifted by several degrees towards large angles (i.e., $Q\neq k_R$).

(3) For $l\neq 0$, the dip is partially filled in light nuclei and almost completely in medium nuclei.

(4) After correct normalization and translation, the distorted and nondistorted distributions have approximately the same shape, except at large and low values of Q so that in first approximation, the momentum distribution width is not affected by the distortion.

(5) The distorted distributions are very sensitive to relatively small changes in the outer part of the wave function and rather insensitive to large changes in the optical potential parameters.[6]

There is no serious objection to deriving momentum distributions at low momentum transfers ($Q<1$ F^{-1}) from $(p, 2p)$ angular correlation distributions, if the energy is sufficiently high so that distortion effects are small and the impulse approximation is valid. For high momentum transfers correlations between nucleons must be taken into account as pointed out by Gottfried.[9] These correlations result in a measured momentum distribution far from the independent particle momentum distribution.

II. COMPARISON WITH OTHER REACTIONS

Therefore, the $(p, 2p)$ reactions explore the *low*-momentum component of the *proton* distribution and appear as complementary with the (p, d) reactions which look at the high-momentum component of the *neutron* distribution ($Q>1$ F^{-1} from kinematical limitation at $E_0=150$ MeV, the interpretation subject to the difficulty quoted above). The two reactions complement each other from an other point of view. The (p, d) studies, provide a better energy separation (Radvanyi *et al.*[10] obtained a resolution between 0.5 and 1 MeV at 150 MeV) than $(p, 2p)$ ones (between 2 and 5 MeV). Therefore, they are more suitable for studying states near the ground state of the residual nucleus, arising from intermediate coupling or configuration mixing in the target nucleus or the residual one. But generally they do not give information on inner shells as can be obtained from $(p, 2p)$ studies (due to the low density of high momentum and to distortion effects, s states are not observed by (p, d) reactions on light nuclei). Comparison of both results specially in light nuclei where $Z=N$ is therefore very useful. One can also think to (p, pn) reaction for looking at the low-momentum component of the neutron distribution but the experimental difficulties concerning yield and energy resolution are high and this reaction has not been used up to now.

On the other hand, the (e, ep) reaction appears now as a better tool than $(p, 2p)$ for investigating inner shells (the $1s$ protons have not been seen clearly above O^{16} by $p, 2p$). Jacob and Maris[11] have suggested to use this reaction, since nuclear matter is transparent to electrons at sufficiently high energy (500–1000 MeV), so that, the energy of the outgoing proton for symmetric kinematics ($|P_e|\sim|P_p|$) is larger than 100 MeV and distortion of the proton rather low. Potter[12] calculated the cross section for 500-MeV electrons on C^{12}. To my knowledge, two experiments were performed by detecting the two outgoing particles as it is needed for complete information:

(1) Croissiaux at Stanford and Bounin at Orsay[13] studied the $D(e, ep)$ reaction with electron linear accelerator and obtained good deuteron wave functions but emphasized the experimental difficulties due to low duty cycle in this kind of experiment.

[4] A. B. Bowden, M. R. Bowman, and T. Yuasa, Congr. Intern. de Physique Nucléaire, Paris (1964).

[5] B. Gottschalk and K. Strauch, Phys. Rev. **120**, 1005 (1960); B. Gottschalk, Harvard University, thesis (1962); B. Gottschalk, K. Strauch, and K. H. Wang, Congr. Intern. de Physique Nucléaire, Paris (1964).

[6] T. Berggren and G. Jacob, Phys. Letters **1**, 258 (1962); and Nucl. Phys. **47**, 481 (1963).

[7] K. K. Lim amd I. E. McCarthy, Phys. Rev. **133, 13**, 1006 (1964); I. E. McCarthy, Rev. Mod. Phys. **37**, 388 (1965).

[8] D. F. Jackson and T. Berggren, Nucl. Phys (to be published).

[9] K. Gottfried, Ann. Phys. (N. Y.) **21**, 29 (1963).

[10] P. Radvanyi, J. Genin, and C. Detraz, Phys. Rev. **125**, 295 (1962); D. Bachelier, M. Bernas, C. Detraz, J. Genin, J. Haag, and P. Radvanyi, Proc. Conf. on Direct Interactions, Padua (1926); D. Bachelier, M. Bernas, I. Brissaud, C. Detraz, N. K. Ganguly, and P. Radvanyi, Congr. Intern. de Physique Nucléaire, Paris (1964); and C. Detrax, Thesis, Université de Paris (1964).

[11] G. Jacob and T. A. Maris, Nucl. Phys. **31**, 139, 152 (1962).

[12] J. Potter, Thesis, Université de Paris (1964); and Nucl. Phys. (to be published).

[13] M. Croissiaux, Phys. Rev. **127**, 613 (1962); and P. Bounin, Thesis, Université de Paris (1964).

TABLE I. Experimental parameters in $(p, 2p)$ studies.

	E_0 (MeV)	Energy detector	Energy resolution (MeV)	Horiz. opening angle(°)	Nuclides studied
Uppsala (2)	185	Range telescopes	4.4	10	Li^7, Be^9, B^{11}, C^{12}, N^{14}, O^{16}.
Harwell (3) (16)	150	Total absorption plastic scintillators	6	6.5	Li^7, Be^9, C^{12}, O^{16} and prelim. results on *d–s* nuclei.
Orsay (17)	155	Total absorption NaI scintillators	5	5	Li^6, Li^7, Be^9, B^{10}, B^{11}, C^{12}.
Harvard (5)	160	Total absorption NaI scintillators	7 to 4	3	Li^7, Be^9, B^{11}, C^{12}, O^{16}, F^{19}, Sc^{45}, V^{51}, Co^{59}, Ni^{58}.
Uppsala (18) (19)	185	Range telescopes	3	3.3 8.8	Li^6, Li^7, Be^9, B^{10}, B^{11}, Mg^{24}, Al^{27}, Si^{28}, P^{31}, Ca^{40}.
Chicago (20)	460	Magnetic spectrometers with 4+4 plastic telescopes	3	3.6	He^4, Li^6, Li^7, Be^9, B^{10}, B^{11}, C^{12}, N^{14}, O^{16}, Al^{27}, Si^{28}, P^{31}, S^{32}, A^{40}, Ca^{40}, V^{51}, Co^{59}.
Orsay (21)	155	Magnetic spectrometers with 30+8 plastic detectors	4 to 2	3.5	Ca^{40}, Sc^{45}, Ti^{48}, V^{51}, Cr^{52}, Mn^{55}, Fe^{56}, Ni^{58}, As^{75}.

(2) Amaldi *et al.*[14] studied C^{12} and Al^{27} (e, ep) reaction at 500–600 MeV with the Frascati synchrotron and obtained binding energy spectra (with a resolution of about 10 MeV) confirming the $(p, 2p)$ results in the case of C^{12} and giving the binding energies of the $(2s\text{–}1d)$, $1p$ and $1s$ states in Al^{27} (respectively: 14.5, 32, and 59 MeV). These first results show that this new method is very promising and it is hoped that experimental progress will be made in intensity and duty cycle of electron beams and also in detecting devices [the cross section is lower by a factor of 10 in comparison with the $(p, 2p)$ cross section] so that momentum distribution determination and comparison with $(p, 2p)$ results will be feasible. This would be of great importance for a better knowledge of both distortion effects and proton wave functions.

For the inner shells there is, however, a serious limitation as pointed out by Maris[15] due to the fact that a hole in an inner shell has a very short lifetime, so that the corresponding peak is very broad. This is confirmed by experimental widths Γ of the $1s$ holes.

	Li^6	C^{12}	O^{16}	Al^{27}
E_B in MeV	21	34	44	59
Γ in MeV	0.08 (Width of the 16.69-MeV excited state in He^5)	9	$\sim$14	$\sim$20

However, the last result on Al^{27} gives some hope that the width rises sufficiently slowly as to permit observation of the $1s$ shell in heavier nuclei.

[14] V. Amaldi, G. Campos Venuti, G. Cortellessa, C. Fronterotta, A. Reale, P. Salvadori, and P. Hillman, Congr. Intern. de Physique Nucléaire, Paris (1964); and Phys. Rev. Letters **13**, 341 (1964).

[15] Th. A. J. Maris. Proc. of the Conf. on Direct Interaction, Padua (1962), p. 31.

III. EXPERIMENTAL APPARATUS

The experiments above 100 MeV were performed with synchrocyclotron extracted beams. Most experiments were performed at 150–185 MeV but an energy as 460 MeV (Chicago) or towards 300 MeV, seems a better choice because the distortion effects rise rapidly when the energy of one outgoing proton is below 100 MeV. However, the energy must not be too high if one wants good separation of the peaks, specially in medium nuclei.

Table I summarizes the experimental parameters used by the different groups[2,3,5,16–21] and shows the progress performed in energy and angular resolution.

The main difficulty in these experiments arises from accidental coincidences. It is necessary to use prompt coincidences between the two protons and to improve beam duty cycle. Auxiliary acceleration at about 10 kc before extraction can provide duty cycle up to about 50% instead of a few percent without it.

Magnetic spectrometers can provide better energy resolution, up to about 2 MeV, and higher counting rate, because each detector receives only a little part

[16] H. G. Pugh and K. F. Riley, Proc. Rutherford Jubilee. Intern. Conf. Manchester (1961), p. 195.

[17] J. P. Garron, J. C. Jacmart, M. Riou, and Ch. Ruhla, J. Physique **22**, 622 (1960); J. P. Garron, J. C. Jacmart, M. Riou, Ch. Ruhla, J. Teillac, C. Caverzasio, and K. Strauch, Phys. Rev. Letters **7**, 261 (1961); J. P. Garron, J. C. Jacmart, M. Riou, C. Ruhla, J. Teillac, and K. Strauch, Nucl. Phys. **37**, 126 (1962); J. P. Garron, Ann. Phys. (Paris) **7**, 301 (1962).

[18] G. Tibell, O. Sundberg, and U. Miklavzic, Phys. Letters **1**, 172 (1962); and **2**, 100 (1962); Proc. of the Conference on Direct Interaction, Padova (1962), p. 1134.

[19] G. Tibell, O. Sundberg, and P. U. Renberg, Arkiv Fysik **25**, 433 (1963).

[20] H. Tyren, S. Kullander, and R. Ramachandran, Proc. of the Conf. on Direct Interaction, Padova (1962), p. 1109; H. Tyren, S. Kullander, R. Ramachandran, and O. Sundberg, Congr. Intern. de Physique Nucléaire, Paris (1964).

[21] C. Ruhla, M. Riou, R. A. Ricci, M. Arditi, H. Doubre, J. C. Jacmart, M. Liu, and L. Valentin, Phys. Letters **10**, 326 (1964); and Congr. Intern. de Physique Nucléaire, Paris (1964).

TABLE II. Protons binding energies (MeV) from (p, $2p$) summed-energy spectra and separation energy E_s(MeV): He^4 and $1p$ nuclide. ? Assignment uncertain. ?? Existence uncertain.

Nuclide	E_s	$l\neq 0(1p_{1/2})$	$l\neq 0(1p_{3/2})$			$l=0(1s_{1/2})$	Ref.
He^4	19.813					20.4±0.3	20
Li^6	4.655		4.5±1.5 ?			20.3±1.5	17
			4.8±0.3			22.4±0.7	18, 19
			4.9±0.3			22.7±0.3	20
Li^7	10.006		10.5±1.6			24.9±1.6	2
			10.2±1.6			23±1.5	17
			11.5±2.5			25±2.5	5
			10.1±1.4			24.1±1.5	16
			11.3±0.5			25.8±0.6	18, 19
			11.8±0.3			25.5±0.4	20
Be^9	16.885		17.8±1.6			25.8±1.6	2
			17.2±1.5			26±1.5	17
			17±2.5			26±3	5
			18.2±1.5			27.2±1.9	16
			18.6±1.0			28.7±1.5	18, 19
			16.4±0.3			25.4±0.5	20
						(32.3±0.6 ?)	20
B^{10}	6.585		7±1.1	13±1.2		31.5±1.5	17
			8.3±0.6		17.5±0.7 ?	35±2	18, 19
			6.7±0.5	11.9±0.5	17.1±0.6	30.5±0.6	20
B^{11}	11.237			13.7±1.6		35.6±1.6	2
			10.4±1.6	15.2±1.7		34±3	17
				13±3		34±3	5
			11.1±0.9	15.2±0.9	20.9±0.9	40±5	18, 19
			10.9±0.4	14.6±0.5	21.2±0.5		20
C^{12}	15.958		17.3±1.6			34.2±1.6	2
			15.8±1.2			34.5±1.5	17
			16±2			33±3.5	5
			16.4±1.4			34.5±1.9	3, 16
			14.7±0.8			34.2±2	20
N^{14}	7.546	7.3±1.6	15.0±1.6				2
		7.5±0.5	15.3±0.5	19.8±0.6		42	20
		(11.5±0.6 ??)					
O^{16}	12.113	12.8±1.6	19.5±1.6				2
		13±2	18±2.5			34±3.5	5
		13.1±1.4	18.7±1.4				16
		12.4±1.0	19±1			44±2 ?	20

of the momentum spectrum. However, a limitation occurs up to now by target effects which give a spread of about 1 MeV or more, due to straggling in prismatic targets (where incident and outgoing protons lose the same energy, about 10 MeV) or to the difference of the energy lost of incident and outgoing protons in thin targets.

Ion source development (providing more intense proton beam with energy spread lower than 1 MeV) is hoped for an improvement of this situation, allowing the use of thinner targets with reasonable counting rate.

IV. EXPERIMENTAL RESULTS ON PROTON BINDING ENERGIES

Riou[22] and Tibell *et al.*[19] have collected published values of binding energies E_B corresponding to peaks observed in (p, $2p$) summed-energy spectra. In Tables II, III, IV we have summarized all these results, added of the values recently given by Tyren *et al.*,[20] Gottschalk *et al.*,[5] and Ruhla *et al.*[21] at the Congres International de Physique Nucléaire (Paris, July 1964).[23] Furthermore, we give the l-assignment deduced by the authors from angular correlation or energy sharing distributions (generally, one can distinguish only between $l=0$ and $l\neq 0$), the shell-model state (j–j coupling) assignment which appears the most probable and the proton separation energies E_S deduced from mass differences, corresponding to the ground state of the residual nucleus. There is general agreement between values and assignments from different authors. They can now be used with some confidence for general systematics of proton binding energies and location of the shells. The following preliminary remarks can be made.

(1) The $1s$ state binding energy rises in function of

[22] M. Riou, Proc. of the Conf. on Direct Interaction, Padua (1962), p. 18.

[23] Ruhla *et al.* obtained new values with thin targets leading to some significative changes. We give here these values for $l=0$ states.

Table III. Binding energies (MeV) $2s$, $1d$ nuclides.

Nuclide	E_s	$l\neq0$	$l=0(2s_{1/2})$		$l\neq0$				Ref.
F^{19}	7.964		6.9±1.2				11.2±2.0	19.4±1.4	16
			8				12	19	5
							$(1p_{1/2})$	$(1p_{3/2})$	
Na^{23}	8.790				10.4±1.7	15.4±1.7 ?	21.2±1.9 ?	29.6±2.0 ?	16
Mg^{24}	11.693				17.2±1.6				
Al^{27}	8.272	9.0±1.7	13.4±1.4		19.4±1.4			30.5±1.9	16
			14.1±1.1		19.8±1.0			~34 ?	18, 19
		13 ??	15.6±0.3		20.0±0.5 ?				20
Si^{28}	11.581		14.3±1.5		19.0±1.4			29.5±3.5 ?	16
			16.3±0.6		21.4±1.1				18, 19
			13.2±0.3		17.0±0.5		27 ??	36 ??	20
P^{31}	7.286		8.4±1.7		12.5±1.5			35.5±1.9	16
			7.8±1.4	13.2±1.1	19.6±1.3		26±3		18, 19
			7.3±0.3	13.9±0.3 ?	19.9±0.8 ?		29 ??		20
S^{32}	8.863		7.2±1.5		12.4±1.7			33.5±4.2	16
			9.1±0.3		12.5 ??	16.1±0.3 ?	26.6±1.1 ?	33.2±1.1 ?	20
Cl^{35}	6.367		11.7±1.5		18.4±1.7			42.8±3.2 ?	16
Ar^{40}	12.523		13.1±0.3	16.4±0.3	19.9±0.5				20
Ca^{40}	8.336				14.0±2.7			36.8±4.2 ?	16
		8.6±0.8	10.6±0.7		15.0±1.5				18, 19
		8.3±0.6	11.6±0.5		15.5±0.6				20
		8.4±0.5	11.0±0.5		14.9±0.5	19±1	24.5±1.5 ?		21
Assignment for Ca^{40}		$1d_{3/2}$	$2s_{1/2}$		$1d_{5/2}$	$1d_{5/2}$?	?		

TABLE IV. Binding energies (MeV) $1f_{7/2}$ nuclides.

Nuclide	E_s	$l=0(2s_{\frac{1}{2}})$	Ref.
Sc^{45}	6.888	12.1±1	21
		12	5
Ti^{48}	11.436	13±1	21
V^{51}	8.044	13.6±0.5	21
		15	5
		15.7±0.5	20
Cr^{52}	10.515	12.8±0.8	21
Mn^{55}	8.058	13.3±1(?)	21
Fe^{56}	10.196	12.8±0.5	21
Co^{59}	7.366	12.1±0.8	21
		14	5
		13.9±0.8	20
Ni^{58}	8.173	11±0.8	21
		11	5

A (Fig. 1) with a slope of about 2 MeV per nucleon between He^4 and O^{16} and somewhat less (~1.4) between O^{16} and Al^{27}. The slope can be physically described as the mean interaction energy between the 1s proton and an external nucleon if one supposes that the interaction energy of the 1s nucleons is not affected.

The binding energy difference between B^{10} and B^{11}, of several MeV (~2 to ~5), suggests that the interaction between the 1s proton and an external neutron is the most important. The binding energies in Li^6 and Be^9 appears as relatively lower than others and can indicate a special structure.

(2) $1p_{\frac{3}{2}}$ (or $1p_{\frac{1}{2}}$ for N^{14} and O^{16}) binding energies in $1p$ nuclides correspond to energy separation except for B^{10} and B^{11} where three peaks are observed indicating clearly that these nuclei must be described by intermediate coupling (two peaks would be expected) or configuration mixing. For other nuclei j–j coupling appears generally as a good description. However, it must not be forgotten that (p, d) spectra for C^{12} and O^{16} [10] show strong peaks corresponding to j–j coupling, many others corresponding to intermediate coupling and in some cases configuration mixing. Balashov and Boyarkina[24] have calculated by the intermediate coupling approximation, the excitation spectrum of C^{13} obtained by the $N^{14}(p, 2p)$ reaction and obtained good agreement with the experimental spectrum of Tyren *et al.*[2,20]

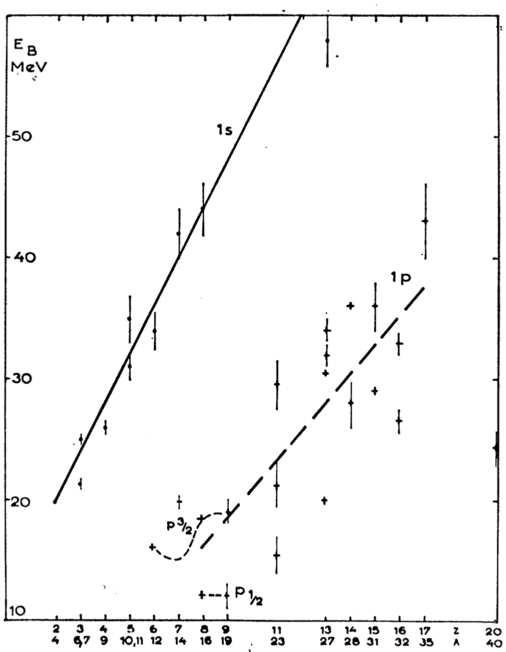

FIG. 1. Proton binding energies E_B of the closed 1s and 1p shells.

With the exception of F^{19} it is rather difficult to obtain $1p$ binding energies in $2s$, $1d$ nuclides. However, the systematic observation of high binding energy peaks with $l \neq 0$ (listed in last column of Table III) suggests the $1p$ assignment (the assignment for the last but one column is more doubtful). If this is true, one sees (Fig. 1) that the $1p$ state binding energy rises

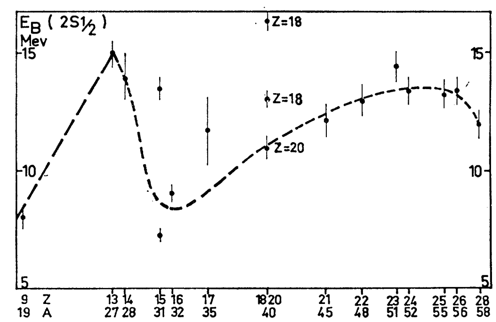

FIG. 2. Proton binding energies E_B of the $2s\ \frac{1}{2}$ shell.

with A in a manner rather similar to the 1s state. The $1s$–$1p(1p_{\frac{3}{2}})$ energy difference appears as roughly constant (~25 MeV) except for some light nuclei: C^{12} (18 MeV) Be^9 (9 MeV) Li^7 (15 MeV) Li^6 (17 MeV). The $1p_{\frac{3}{2}}$–$1p_{\frac{1}{2}}$ seems also constant (6 MeV).

(3) The $2s$–$1d$ filling appears as rather complex: the $2s$ state appears for all nuclides except Na^{23} and Mg^{24}. Figure 2 gives binding energy E_B for the $2s_{\frac{1}{2}}$ state (which is the best characterized) in $2s$, $1d$, and $f_{7/2}$ nuclides (the curve is drawn only for guiding the eyes). The binding energy curve shows strong fluctuations when $2s$ shell is filling and a general rise with A after ^{32}S but the slope is considerably lower than for 1s and 1p shells: it is about 0.2 MeV/nucleon instead of about 2 MeV.

In spite of these fluctuations, it appears that the energy differences $1d_{\frac{5}{2}}$–$2s_{\frac{1}{2}}$ and $2s_{\frac{1}{2}}$–$1d_{\frac{3}{2}}$ (first peak) does not vary very much before Ca^{40} and certainly above when all the shells are closed (the two are about 3

[24] V. V. Balashov and A. N. Boyarkina, Nucl. Phys. **38**, 627 (1962).

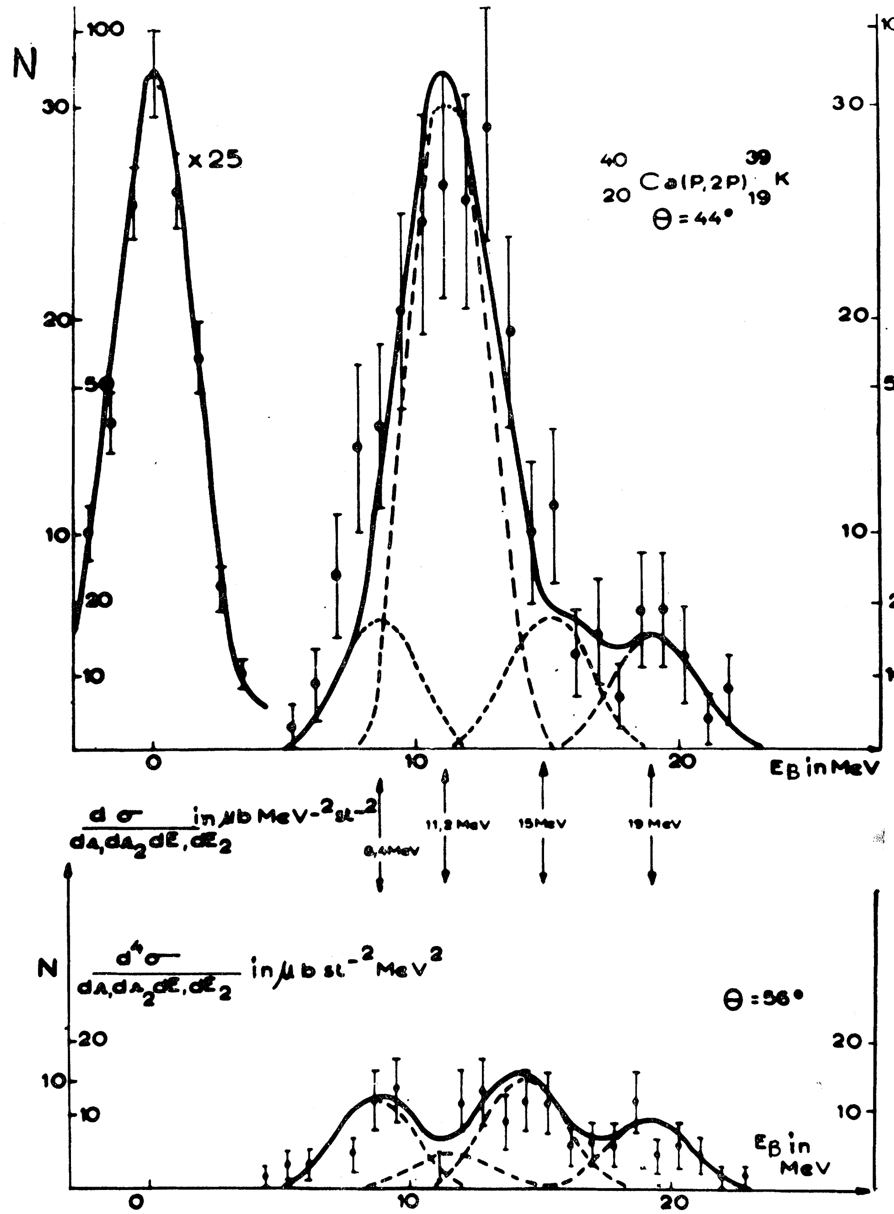

FIG. 3. Binding energy spectra from $Ca^{40}(p, 2p)$ reaction (N: number of events).

MeV). It follows that Ca^{40} can be considered as a core for heavier nuclei. The binding energy spectra of Ca^{40} obtained by Ruhla *et al.*[21] are shown on Fig. 3.

The $d_{\frac{3}{2}}$ state gives two peaks in Ca^{40} and heavier nuclei (and even three in Ca^{40} if one excludes the $1p$ assignment for the last peak) with a constant energy difference of about 3 MeV. This complexity must be compared with one observed by Ca^{40} (p, d) spectra[10,25] showing many states at about 6- and 8-MeV excitation energy.

In summary, it seems that the energy difference between the $1s$–$1p_{3/2}$–$1p_{1/2}$ shells (about 25 and 6 Mev) on the one part, $1d_{5/2}$–$2s_{1/2}$–$1d_{3/2}$ (all about 3 MeV) on the other part, are roughly constant. These values are in general agreement with the calculated values of Brueckner *et al.*[26] for O^{16} and Ca^{40}. The mean slope of the curves $E_B=f(A)$ are very different: between $\sim$1.4 and 2 MeV/nucleon for $1s$–$1p$ states (as determined up to Al^{27}) in good agreement with the value given by Brueckner calculations[26] (about 1 MeV/nucleon between O^{16} and Ca^{40}) and 0.2 MeV/nucleon for $2s$ (as determined between S^{32} and Ni^{58}. The mean value of

[25] C. D. Kavaloski, G. Bassani, and N. M. Hintz, Phys. Rev. **132**, 813 (1963).

[26] K. A. Brueckner, A. M. Lockett, and M. Rotenberg, Phys. Rev. **121**, 255 (1961).

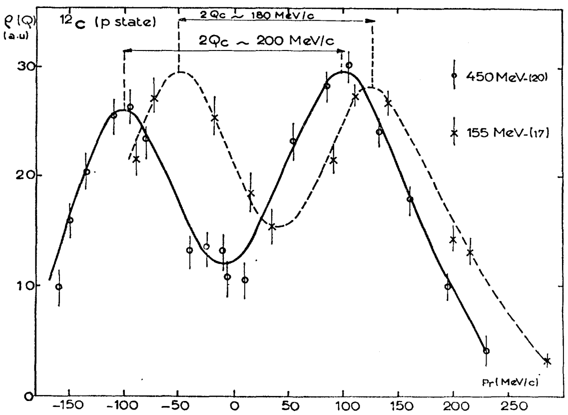

FIG. 4. Momentum distribution $\rho(Q)$—arbitrary unit—for $1p$ protons in C^{12} from $(p, 2p)$ angular correlation distributions at 155 MeV (Garron *et al.*, Ref. 17) and 450 MeV (Tyren *et al.*, Ref. 20).

this slope is significantly lower than expected from Brueckner results (~0.5 MeV between Ca^{40} and Zr^{90}).

V. PROTON MOMENTUM DISTRIBUTION

As we have seen before (in Sec. I), the momentum distribution width is not too much affected by distortion and, in first approximation, it can be useful to characterize the experimental distributions:

$$\rho(Q)\alpha\frac{d^3\sigma}{d\Omega_1\,d\Omega_2\,dE_1}\bigg/\left(\frac{d\sigma}{d\bar{\Omega}}\right)_{pp}$$

by a single parameter Q_c. For s distributions we choose the half-width at $1/e$ of the height and for p distributions the half-distance between the two maxima. These distributions parameters deduced from various experimental distributions,[5,17–20] on Li^6, Li^7, Be^9, and C^{12} agree rather well, within the experimental uncertainties of about 10%. Figure 4 shows the agreement of the experimental distributions at 155 MeV[17] and 460 MeV[20] for the p state of C^{12}. The shift of 40 MeV/c for the 155-MeV distributions is due to distortion.

The results quoted in a previous review[22] remain valid. Table V gives the momentum distribution parameters Q_c in MeV/c ($1F^{-1}=197$ MeV/c) from He^4 to C^{12} and the ratio Q_c/α_B, where

$$\alpha_B=|(1-A^{-1})1876E_B|^{\frac{1}{2}}$$

is the momentum corresponding to binding energy of the corresponding peak, or the mean binding energy when many states are involved as in B^{10} and B^{11}.

One sees that the parameters for s and p states are very different as it was pointed out first by Garron *et al.*[17] for C^{12}. The extreme case is Li^6 where the p distribution is very narrow and the dip not pronounced,[17,18,20] so that one can consider the possibility of $1p+2s$ mixing in the ground state of Li^6. The Q_c values for $1s$ states are not very far from the value for He^4. One sees, on the other hand, that the ratio Q_c/α_B is approximately constant for each state and this result can be used as a test for choosing a realistic single-particle wave function and the corresponding potential.

The infinite harmonic oscillator (H.O.) well $V=-V_0+\frac{1}{2}M\omega^2r^2$ is widely used in the theoretical analysis of experimental results, as electron scattering (where different states are not separated). It gives orbital wave function in r and Q spaces for $1l$ states.

$$\psi_{1l}(r)\propto r^l\exp|-\tfrac{1}{2}(r/a)^2|$$

$$\psi_{1l}(Q)\propto Q^l\exp|-\tfrac{1}{2}(Q/Q_a)|^2, \quad (2)$$

with

$$Q_a=\hbar/a=(M\hbar\omega)^{\frac{1}{2}} \quad \text{and} \quad \rho(Q)=\psi^2(Q).$$

With the choice of the experimental parameters Q_c, these would be equal to Q_a and one sees that in the large momentum area explored here, it is necessary to use different values of the Q_a parameters for $1s$ and $1p$ states. Miss Jackson[27] has shown that the fact of choosing two different H.O. parameters for $1p$ nuclides, is equivalent to using a finite potential. This was used for analysis of electron scattering on Li^6, but $(p, 2p)$ results show that this is more general. In fact, the *infinite* H.O. well appears as rather inadequate for giving proper weight to the low-momentum component studied in $(p, 2p)$ reaction, i.e., for giving a correct symptotic behavior in space.

The asymptotic function for a well with a finite range

$$\psi_{1l}(Q)\propto Q^l\alpha_l^2/Q^2+\alpha_l^2, \quad (3)$$

with $\alpha_l=\alpha_B$ give too high values of the ratio Q_c/α_B (0.8 for s state and 1 for p state). It means that the range of the well is too large for using only asymptotic forms.

Alternatively one can use a well with a long range. In their W.K.B. analysis, Berggren and Jacob[6] used H.O. and exponential wave functions. These functions are (α_l is in F^{-1})

$$\psi_{1l}(r)\propto r^l\exp(-\alpha_l r),$$

$$\psi_{1l}(Q)\propto Q^l\alpha_l^{2(l+2)}/(Q^2+\alpha_l^2)^{l+2} \quad (4)$$

corresponding to a state-dependent potential with infinite range,

$$V=-(\hbar^2/M)\alpha_l(l+1)(1/r)$$

and leading to Q_c/α_B values (0.53 for s state and 0.45 for p state) which are in good agreement with experimental ones but appear rather minimum values.

The situation can be still improved by a choosing a potential with a finite range. This can be done with the square well potential for which exact solutions can be found. Y. Sakomoto[28] used it for the analysis of $1p$-state Li^6 distribution. But analytical form are rather complicated and this potential is not the more realistic.

[27] D. F. Jackson, Proc. Phys. Soc. (London) **76,** 949 (1960).
[28] Y. Sakamoto, Phys. Letters **1,** 256 (1962).

TABLE V. Momentum distribution parameters Q_c and ratio Q_c/α_B, where: $\alpha_B = |(1-A^{-1})\ 1876\ E_B|^{1/2}$.

		He^4	Li^6	Li^7	Be^9	B^{10}	B^{11}	C^{12}
1*s*-state	Q_c(MeV/*c*)	105	110	115	110	120	120	150
	Q_c/α_B	0.63	0.61	0.60	0.53	0.50	0.50	0.62
1*p*-state	Q_c(MeV/*c*)		~40	65	65	90	90	75
	Q_c/α_B		0.50	0.50	0.40	0.60	0.60	0.45

Jean[29] suggested wave functions $\psi_{1l}(r)$ given by a linear combination of Hankel's functions and wave functions $\psi_{1l}(Q)$ with a simple analytical form,

$$\psi_{1l}(Q)\alpha Q^l \frac{\alpha_l^2}{q^2+\alpha_l^2}\cdot\frac{|\alpha_l+\mu|^2}{q^2+(\alpha_l+\mu)^2}\cdots\frac{|\alpha_l+(l+1)\mu|^2}{q^2+|\alpha_l+(l+1)\mu|^2} \quad (5)$$

this gives the asymptotic expression (3) for $\mu \to \infty$ and the expression (4) with the potential in $1/r$ for $\mu \to 0$. Besides the form for an *s* state is the same as the exact solution for the Hulthen potential,

$$-V_0 \frac{\exp(-\mu r)}{1-\exp(-\mu r)}$$

so that by analogy, we can think that $1/\mu$ represents the range of the potential. With range values equal to nuclear radii R or somewhat larger, one obtains Q_c/α_B values (0.62 for *s* state and 0.54 for *p* state) which are in good agreement with experimental ones. The variation of Q_c/α_B with μ is too slow and the experimental uncertainties are too large for allowing an accurate determination of μ. But the expression (5) is helpful for predicting the momentum distribution width for a given l, knowing the corresponding binding energy and by consequence α_l, the only parameter to adjust being μ, which can be chosen as $\sim 1/R$ in first approximation [it must be remarked that the experimental angular correlations measured for the 2*s* state in Ca^{40} [18,21] and Ni^{58} [21] are in good agreement with the $\psi_{2s}(Q)$ expression deduced from Hulthen potential].

One can use also an unique potential (for all states) but in this case, many parameters are involved. Recently, Elton *et al.*[30] have used a four-parameter potential including a finite H.O. potential with an exponential tail for fitting both the binding energies from $(p, 2p)$ and elastic electron scattering data in Li^6 (see also Ref. 27). The fit is obtained with a rather long range of the potential ($1/\mu = 2.5$ F) and it would be interesting to compare these calculations with the $(p, 2p)$ momentum distributions. The same fit was made for S^{32} and Ca^{40} using a Saxon–Woods central potential (3 parameters) with a spin-orbit Thomas term (1 parameter. Good agreement is obtained except for the $1s_{\frac{1}{2}}$ binding energy (33 MeV) whereas, the extrapolation of experimental values suggests a higher value.

Thus $(p, 2p)$ results emphasize the necessity of using realistic wave functions, closely related to binding energies, and in the case of 1*p* nuclides, the necessity of choosing very different potentials for 1*s* and 1*p* states (or what may be equivalent and unique potential with a long tail affecting mainly 1*p* nucleon), involving long range correlations of nucleon in the same state.

A special case to discuss is O^{16}. Tyren *et al.* have observed that the angular correlation distributions are rather different for $1p_{\frac{1}{2}}$ and $1p_{\frac{3}{2}}$ states (Fig. 5) and this difference seems too large to be explained only by binding energies (12 and 18 MeV). The W.K.B. distorted wave calculations of Berggren and Jacob[6] with exponential wave functions do not show such a difference. Radvanyi *et al.*[10] observed also a difference between angular distributions of deuterons corresponding to pick-up of $1p_{\frac{1}{2}}$ and $1p_{\frac{3}{2}}$ neutrons in O^{16}, the ratio $\sigma p\frac{3}{2}/\sigma p_{\frac{1}{2}}$ growing from 1.6 at $\theta_d = 25°$ to 2.7 at 33°. These facts can be compared with analogous observations at lower energies[31] showing that pick-up angular distributions are functions not only of l but also of j, suggesting the existence of a spin-dependent term in the interaction particle nuclei.

VI. OTHER KNOCK-OUT REACTIONS (p, pd), $(p, p\alpha)$, (etc...)

A large number of experiments, mainly performed by ionographic or radiochemical methods, have shown that light nuclei, such as D, T, He^3, He^4 are emitted with high probability in nuclear reactions induced by protons of medium or high energy. The energy spectra of these nuclei have generally a tail toward high energy which is difficult to explain by the statistical model concerning evaporation in the residual nucleus after nuclear cascade by direct interaction. A possible explanation is the knock-out by the incoming proton (or a cascade nucleon) of a cluster existing inside or at the surface of the target nucleus. On the other hand, reactions induced by heavy ions, such as Li^6, suggest also clustering of the corresponding nucleus. The existence of such clusters has been widely discussed these last years. Therefore, it can be useful to investigate knock-out reactions such as $(p, pd)-(p, p\alpha)$ in a well-defined kinematic analog to the one used in $(p, 2p)$ and see

[29] M. Jean (private communication); and Colloque I.I.S.N. de Bruxelles, 148 (1962).

[30] L. R. B. Elton, R. R. Shaw, A. Swift, and I. S. Towner (private communication).

[31] R. Sherr, E. Rost, and M. E. Rickey, Phys. Rev. Letters **12**, 420 (1964); and R. H. Fulmer and W. W. Dahnick, Phys. Rev. Letters **12**, 455 (1964).

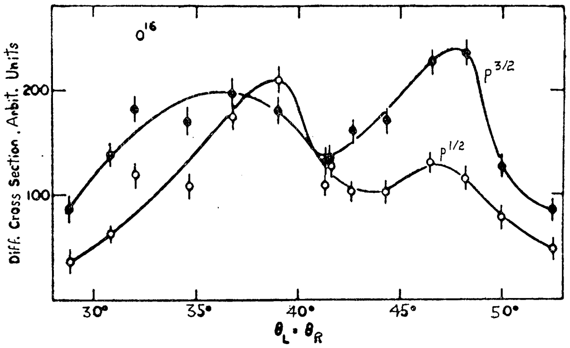

FIG. 5. Angular correlation distribution obtained by Tyren *et al.*, (Ref. 20) at 450 MeV for $1p_{3/2}$ and $1p_{1/2}$ protons in O^{16}.

whether the results can be described by quasifree scattering of the incoming proton on a deuteron or an α particle. In (p, pd) reaction, one must distinguish between the knock-out mechanism and the indirect pickup one (where a nucleon from a $(p, 2p)$ or (p, pn) event picks up another uncorrelated nucleon).

Li^6, Li^7, and C^{12} (p, pd)-Li^6, Be^9, and C^{12} $(p, p\alpha)$ have been recently studied and the results will be summarized here. Zupančič[32] has discussed at this Conference reactions such as $(\alpha, 2\alpha)$. Lefort[33] has recently given a general review on α particles and heavier fragments omitted in nuclear reactions.

The cross section for a quasifree scattering $p+a$ can be obtained in a similar way to quasifree scattering $p+p$. With plane-wave Born approximation, nonrelativistic kinematic and in the case where $Q \ll k_0$ one obtains for a given peak in E_p+E_a spectrum:

$$\frac{d^3\sigma}{d\Omega_p\, d\Omega_a\, dE_p} = \left(\frac{a+1}{a}\right)^2 \frac{aM_p}{\hbar^2} \frac{k_a k_p}{k_0} \left(\frac{d\sigma}{d\bar{\Omega}}\right)_{pa} \rho(q, Q), \quad (6)$$

where a is the mass number of the particle a, $\mathbf{q}=\mathbf{k}_0-\mathbf{k}_p$, the momentum transfer to a, $\mathbf{Q}=-\mathbf{k}_R=\mathbf{k}_a+\mathbf{k}_p-\mathbf{k}_0$, the momentum of "$a$" in the target nucleus if the impulse approximation is valid, $d\sigma/d\bar{\Omega}_{p,a}$ is the free elastic scattering $p+a$ in their center of mass for the momentum transfer q, $\rho(q, Q)$ is generally a complicated function of q.

The interpretation is particularly simple when one choose an explicit cluster model where the relative motion of the clusters "a" and "$A-a$" is described by an orbital wave function $\chi(R)$. In this case $\rho(q, Q)$ can be factorized as $\rho(Q)\cdot P(q)$ where

$$\rho(Q) \alpha \left| \int \exp(iQR)\chi(R)\, d^3R \right|^2$$

is simply the momentum density distribution of "a."

$$P = \int \psi_a^*(r_i) \cdot \psi_{``a"}(r_i)\, d^3r_i$$

(where ψ_a and $\psi_{``a"}$ are the internal wave functions of the free a particle and of the cluster "a") can be considered as the probability for finding the cluster "a".[29] According to the treatment of the problem, P may or may not be a function of q. In the experiments where q has a little variation $P(q)$ can be considered as constant and the $\rho(Q)$ distribution can be easily deduced from the experimental ones. But it must be remembered that this interpretation is only valid when the cluster model is valid, i.e., when $P(q)$ is near the unity. In the other cases, the factorization above is not obtained and the interpretation of the experimental data is more complicated.

(1) The Li^6, $Li^7(p, pd)$ and $Li^6(p, p\alpha)$ reactions were studied at 155 MeV by Ruhla *et al.*[34] They obtained angular correlation distributions for (p, pd) with symmetric kinematics $k_p=k_d$, and $\Theta_p=\Theta_d$ being varied around the value for elastic p–d scattering (51°). They also obtained the E_p distribution for $(p, p\alpha)$ with $\Theta_p=\Theta_\alpha=55°$, value for p–α scattering. The energies were measured by a magnetic analyzer (E_p) and a total energy plastic scintillator (E_d or E_α). The time of flight on the d or α path selected the mass of the particles detected. The summed energy spectrum E_p+E_d (or E_α) for Li^6 shows the peak at E_0–E_B–E_R expected from quasifree scattering. The angular correlation distribution for (p, pd) and the E_p distribution for $(p, p\alpha)$ have both a pronounced maximum at $Q=0$ as it is expected if the "α" and "d" in Li^6 are in a relative S state. (Fig. 6.)

These distributions are very narrow and have the same width (the half-width at half-height is $\sim$30 MeV/c). The important fact that the width is the same for the two distributions shows that the knock-out mechanism is valid and also the impulse approximation.

Devins *et al.*[35] have obtained similar results on the $Li^6(p, pd)$ reaction at 30 MeV by studying the angular distribution $d^2\sigma/dr_p dr_a$.

The $Li^7(p, pd)$ summed energy spectrum is broad, and also the corresponding angular correlation distribution, the differential absolute cross section is significantly lower than for Li^6 so that the interpretation is not clear (knock-out or indirect pickup).

The two Li^6 distributions are in good agreement with the momentum distribution $\rho(Q)$ obtained in the cluster model with a motion of "α" and "d" in a 2s state. Describing this motion by a Hulthen's potential of range 3.3 F or a rectangular well ($R=3.4$ F, $V=35$ MeV as calculated by Gammel *et al.*[36] for the cluster model of Li^6) and using the binding energy of the d or

[32] Č. Zupančič, Rev. Mod. Phys. **37**, 330 (1965).
[33] M. Lefort, Ann. Phys. (Paris) **9**, 249 (1964).
[34] C. Ruhla, M. Riou, J. P. Garron, J. C. Jacmart, and L. Massonnet, Phys. Letters **2**, 44 (1962); and C. Ruhla, M. Riou, M. Gusakow, J. C. Jacmart, M. Liu, and L. Valentin, Phys. Letters **6**, 282 (1963).
[35] D. W. Devins, H. H. Forster, S. M. Bunch, and C. C. Kim, Phys. Letters **9**, 35 (1964); and D. W. Devins, H. H. Forster, and B. L. Scott, Rev. Mod. Phys. **37**, 396 (1965).
[36] J. L. Gammel, B. J. Hill, and R. M. Thaler, Phys. Rev. **119**, 267 (1960).

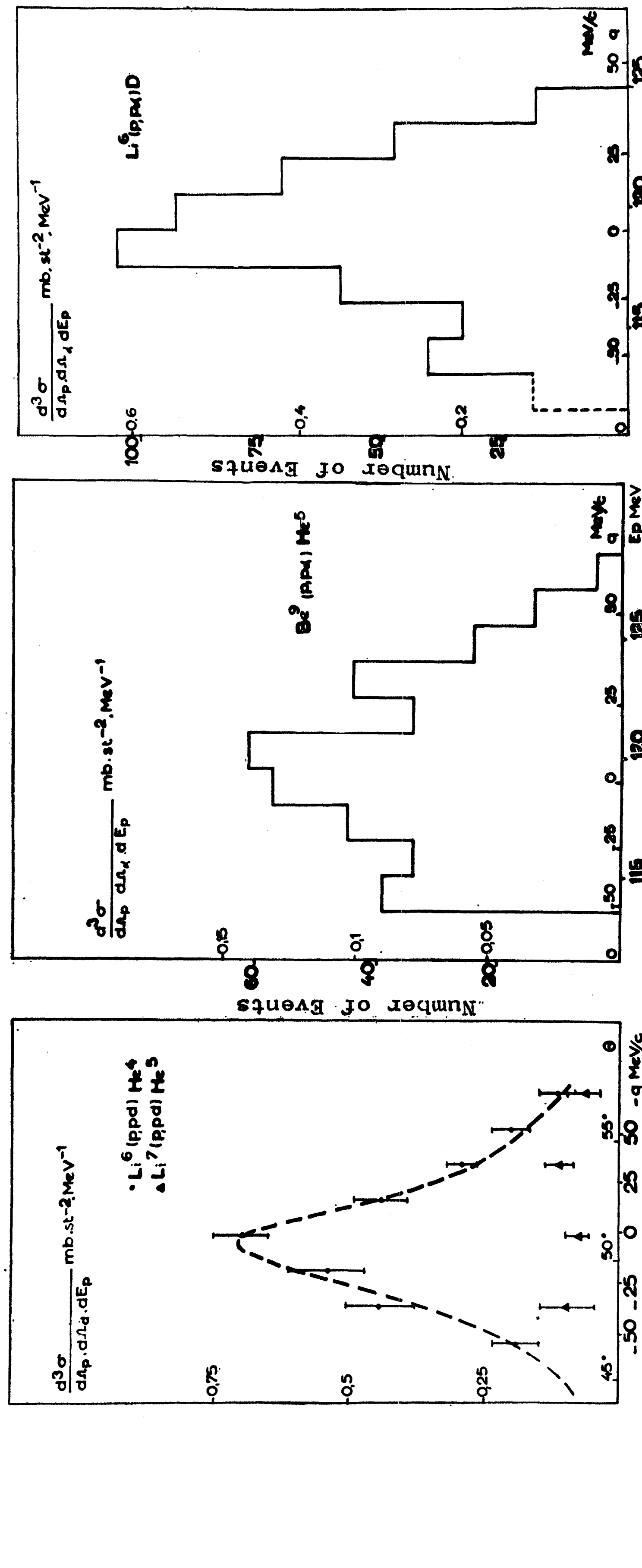

FIG. 6. Angular correlation distribution for Li^6, Li^7 (p, pd) reactions at 155 MeV ($|k_p| = |k_d|$ and $|\theta_p| = |\theta_d| = \theta$) and E_p distributions for Be^9, Li^6 $(p, p\alpha)$ reactions ($|\theta_p| = |\theta_\alpha| = 55°$) N: Number of events (Ruhla *et al.*, Ref. 34).

α in Li^6 (1.47 MeV) one obtains a good agreement between the calculated and observed distributions.[34] The low value of energy is thus related to the small width of the momentum distribution. The values of the probability, P, are high: $\sim$0.30 for $Li^6(p, pd)$ where $q=2.2\pm0.2\ F^{-1}$ and $\sim$0.20 for $Li^6(p, p\alpha)$ where $q=2.46\pm0.03\ F^{-1}$. These values must be taken as lower limits for the probability of clustering "α"+"d" in Li^6, as the formula (6) does not take into account the absorption of the particles involved.

The question now is why the probability of clustering "α"+"d" in Li^6 is high at least in the momentum range explored and how this can be related with the shell model. As we have seen before (Sec. V) the $(p, 2p)$ results suggest, *particularly in the case of Li^6*, very different well parameters and wave functions for $1p$ and $1s$ states. Consequently, special correlations between nucleons in the same state are expected. In the cluster model the wave function is

$$\Psi_{Li^6} = \mathbf{A}\Psi_{``\alpha"}\Psi_{``d"}\chi_{\alpha-d},$$

where $\mathbf{A}$ is the operator for complete antisymmetrization. Wildermuth and Kanellopoulos[37] have shown that for equal parameters of Gaussian functions in Ψ_α, Ψ_d, $\chi_{\alpha-d}$ the cluster wave function is identical with the H.O. shell model wave function with one parameter. Elton and Jackson[38] have emphasized the necessity of *complete antisymmetrization* in the wave function above, the clustering being more or less destroyed by this operation, according to the choice of the functions. However they show that a two parameter H.O. wave function in L–S coupling shell model for Li^6 is equivalent to a cluster model wave function with different parameters (whose values are reasonable for describing the internal motion in "α" and "d" and their relative motion). The degree of clustering is larger when H.O. shell model parameters are more different, as it is the case in Li^6.

On the other hand, Sakamoto[39] has shown that the shell model with one parameter H.O. wave function cannot reproduce the experimental data, both in the absolute values of the cross section and the momentum distribution of $Li^6(p, pd)$ reaction. Jackson has discussed the theoretical interpretation at this conference.

It can be hoped that theoretical treatment including both distortion, antisymmetrization, and shell-model realistic wave functions will progress, although high calculation difficulties, and will help to understand why the cluster model stands and where. The semi-empirical relation between $(p, 2p)$ and (p, pd) results shows already that clustering is possible in $1p$ nuclides, with higher probability in the case of Li^6.

[37] K. Wildermuth and Th. Kanellopoulos, CERN **59,** 23 (1959); K. Wildermuth, Nucl. Phys. **31,** 478 (1962).
[38] L. R. B. Elton and D. F. Jackson (private communication).
[39] Y. Sakamoto, Nuovo Cimento **28,** 206 (1963); and Phys. Rev. **134,** B1211 (1964).

(2) The $Be^9(p, p\alpha)$ reaction was also studied by Ruhla *et al.*[34] who obtained a distribution similar to the Li^6 one: maximum at $Q=0$, characteristic of an s state, distribution rather narrow (40 MeV/c at half-height) which can be related to the binding energy of an α particle in Be^9, 2.53 MeV. But the probability obtained is much lower $P=0.065$, meaning that the validity of the cluster model is certainly not so large as in Li^6. (Fig. 7.)

(3) The $C^{12}(p, p\alpha)$ reaction has been studied by many authors Cuer *et al.*[40] at 180 and 340 MeV observed with nuclear emulsions, one proton and α particle due to $p+\alpha$ scattering and two other low energy α particles due to the disintegration of the residual nucleus Be^8 left in its ground state or 2.9 ($J=2$) and 12-MeV ($J=4$) excited states. They pointed out the existence of α clusters in C^{12}. Gauvin *et al.*[41] confirmed these results by the same method at 90 MeV and pointed out the possibility of obtaining directly the $\rho(Q)$ distribution by direct measurement of the recoil momentum of Be^8.

James and Pugh[42] studied angular correlation distribution at 150 MeV with telescopes ($\Theta_p=45°$, $E_\alpha=$ 22.5 4 MeV, Θ_α varied between 25° and 100°) and obtained a rather broad distribution (130 MeV/c at half-height for all events).

Yuasa *et al.*[43] studied coplanarity and angular distribution at 123 MeV by propane bubble chamber, tested the validity of the knock-out mechanism and estimated a total cross section (5 to 7 mb).

The knock-out seems well confirmed in this case, but due to the broad distribution and the presence of many states of Be^8 the interpretation in terms of an α-cluster momentum distribution seems not quite simple. It must be noted that the presence of high momentum α particles in C^{12} requires high-energy incoming protons for an easier interpretation of experiments.

(4) The $C^{12}(p, pd)$ reaction was studied at 155 MeV by Radvanyi *et al.*[44] for testing the mechanism. They obtained a broad angular correlation distribution (with Θ_d fixed at 38° or 28°, E_d fixed at 81 or 99 MeV and Θ_p variable) with a maximum at the angle expected for knock-out.

VII. $(\Pi^+, 2p)$ AND (Π^-, pn) REACTIONS

These reactions are not knock-out reactions but absorption of one pion by interaction with two nucleons.

[40] P. Cuer, Phys. Rev. **80,** 906 (1950); P. Cuer, J. Combe, and A. Samman, Compt. Rend. **240,** 25 (1955); A. Samman and P. Cuer, J. Physique **19,** 13 (1958).
[41] H. Gauvin, R. Chastel, and L. Vigneron, Compt. Rend. **253,** 257 (1961); and H. Gauvin and R. Chastel (private communication).
[42] A. N. James and H. G. Pugh, Nucl. Phys. **42,** 441 (1963).
[43] T. Yuasa and E. Hourani, Rev. Mod. Phys. **37,** 399 (1965).
[44] P. Radvanyi *et al.* (private communication).
[45] M. Jean, Proc. of the Conf. on Direct Interaction, Padua (1962); and Nuovo Cimento (to be published).

The (Π^+, $2p$) reaction with Π^+ in flight was suggested by Jean[45] for looking at correlations between a neutron and a proton inside the target nucleus. This method can give the same information as (p, pd) but in a more general manner: the two nucleons can be correlated in an other way that in a deuteron and according to the geometry chosen it is possible to explore the momentum of the ($p+n$) pair with respect to the residual nucleus or the momentum of one nucleon of the pair with respect to the other. These reactions are being studied at Cern by Charpak *et al.*[46] with Π^+ of 100 MeV on D, Li^6, Li^7, C^{12}, and N^{14}, the localization and the range of the outgoing protons are measured by spark chambers so that summed energy spectra and angular correlation distribution can be obtained.

The (Π^-, pn) reaction with Π^- at rest was suggested by Ericson[47] for giving information on correlations between two protons.

Reactions by protons, electrons, pions . . . at medium energy seem now a good tool for exploring nuclear matter, individual states, and correlations, and great progress can be hoped for in the near future.

[46] G. Charpak, J. Favier, M. Gusakow, and L. Massonnet (private communication).

[47] T. Ericson, Proc. of the Conf. on Direct Interaction, Padua (1962), p. 39.

Discussion

Jackson: I would like to ask about the interpretation of the Li^7 (p, pd) reaction. If you use a cluster model to interpret this reaction, you do, in fact, predict a minimum. I would like to know if you have any other evidence besides the magnitude of the cross section which would suggest that this is not a knock-out process?

Riou: There is just an indication for the possibility of minimum in Li^7 (p, pd) reaction, but the results of the experiments are not good enough to say you have really a minimum.

Pugh: Do you see any prospect of improving energy resolution? This would be very valuable, and many people have worked a long time to get 2 MeV. Can we do better?

Riou: If you use magnets in (p, $2p$) studies, as was done at Chicago and Orsay, the intrinsic energy resolution can be good enough, but in this case, you have some difficulties with target effects. With the actual synchrocyclotron having an internal beam of 1 μA, we are obliged to use prismatic targets of about 10 MeV for performing an experiment with good angular resolution and sufficient statistics. In this case, the energy width coming from target effects is 1 MeV or more. Progress can be hoped for with thinner targets and more intense beams.