Apparent, but Energy-Dependent Pseudo-Resonances Due to Rescattering in Sequential Processes

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Let us consider the typical process

$$He^3+Li^6\rightarrow\alpha_1+\alpha_2+p \quad (1)$$

with three bodies in the final state. Throughout this paper we restrict ourselves to a consideration of *final-state interactions*.

The total matrix element for this process will consist of a sum of terms as follows:

(i) the three final particles come off independently of each other, there being no two-body correlations;

(ii) one pair of final particles interact strongly, coming off as a two-body state, e.g.,

$$He^3+Li^6\rightarrow\alpha_1+Li^{5*}\rightarrow\alpha_1+p+\alpha_2, \quad (2)$$

or

$$He^3+Li^6\rightarrow p+Be^{8*}\rightarrow p+\alpha_1+\alpha_2. \quad (3)$$

In both (2) and (3) the whole spectrum of accessible resonant intermediate states can be excited. As discussed elsewhere in this conference, the study of such sequential processes can most readily be performed by use of on-line computers which evaluate the invariant quantities

$$s_1=(p+\alpha_2)^2, \qquad s_2=(p+\alpha_1)^2, \qquad s_3=(\alpha_1+\alpha_2)^2.$$

Here p, α_1, and α_2 denote the four-momenta of the proton, and the two alpha particles, respectively. Thus, e.g.,

$$s_1=(E_p+E_{\alpha 1})^2-(\mathbf{P}_p+\mathbf{P}_{\alpha 1})^2$$
$$=(M_p+M_\alpha)^2+M_pM_\alpha(\mathbf{V}_p-\mathbf{V}_{\alpha 1})^2,$$

and is in fact the square of the invariant mass of the (proton+alpha 1) combination. One finds that

$$s_1+s_2+s_3=[\text{mass (He}^3)+\text{mass (Li}^6)]^2$$
$$+2T[\text{mass (Li}^6)]+2[\text{mass }\alpha]^2+[\text{mass }p]^2,$$

where T is the *kinetic* energy of the He^3 in the accelerator beam. Hence a knowledge of two of the three quantities s_1, s_2, and s_3 determines the final state completely, *if* we disregard the spin of the proton. (Of course, the angular correlations between the orientations of the final-state momenta with the *initial*-state momenta and spins are *not* specified in this description.) From the (s_1, s_2, s_3) for each event, a phase-space or Dalitz plot can be built up. Reaction (2) will show itself by bands of constant s_1 at $s_1=[\text{mass (Li}^{5*})]^2$ (Of course by $\alpha_1\rightleftharpoons\alpha_2$ symmetry there will be bands of constant s_2 at the same values.) Similarly reaction (3) will show itself as a band of constant s_3 at $s_3=[\text{mass (Be}^{8*})]^2$.

We emphasize that these bands are located at values of the appropriate s which are totally independent of the incident beam energy T (though the relative excitation function for the different levels will depend somewhat on T). (This is one of the important features of the description in terms of s_1, s_2, and s_3.) Even for nonresonant pair energies, there will be some two-body interaction between pairs (e.g., an attractive zero-energy scattering length). This will show itself by variations in the density of the Dalitz plot, in the appropriate s_1, s_2, or s_3 direction.

(iii) Besides the above two types of matrix-element dependence, (i) and (ii), there is a further type with which we primarily concern ourselves in this paper. This corresponds to higher rescattering corrections. As an example, for the same basic process (1), suppose the intermediate state is Li^{5*}, as in (2). After a time related to the width of the Li^{5*} level, this breaks up into $p+\alpha_2$. It is now possible for the α_2 from the break-up to catch up with and interact with the α_1, provided only that the kinematics is such that α_2 heads in the same direction as α_1, and with a speed which is *greater* that that of α_1. (It is most straightforward to express the conditions in the over-all c.m. system; the final relations are Lorentz-invariant.) One readily finds that these conditions correspond to the following prescription (cf. Figs. 1 a–c).

For the given beam energy T, draw the boundary of the allowed phase-space Dalitz region in the s_1, s_3 plane. Draw the line (or band) corresponding to the Li^{5*} intermediate state $s_1=[\text{mass (Li}^{5*})]^2$. Let this intersect the Dalitz boundary at s_{3+} and s_{3-} ($s_{3+}>s_{3-}$).

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as in Fig. 1b, the kinematic conditions are fully satisfied at $s_3 = s_{3-}$, and nowhere else; if s_{3-} is not on this arc, they can never be simultaneously satisfied.

Suppose the beam energy T is such that Fig. 1b is appropriate. Then when the Li^{5*} breaks up, some of the α_2' will in fact satisfy the kinematic conditions and hence interact with the α_1', at the unique s_3 value s_{3-} [notice that $s_3 = (\alpha_1 + \alpha_2)^2 = (\alpha_1' + \alpha_2')^2$, another useful feature of the invariant variables]. This interaction will occur *regardless* of whether s_{3-} corresponds to a level of Be^{8*}. Provided only that *some* attractive $\alpha\alpha$ interaction occurs at this energy, there will be an overall enhancement in the process (1), for this value of s_3 (and *independent* of the value of s_1), i.e., an enhancement in the shaded *band* in Fig. 1b.

Then if, and only if, s_{3-} is on the lower right-hand arc ab,

Certain remarks are in order:

(a) If the α_2 heads in the same direction as p, this just leads to an extension of the Li^{5*} lifetime, so this effect is already included in the direct process (2), with the experimental width of Li^{5*}.

(b) If the p from the breakup attempts to catch up and interact with the α_1, it turns out that its speed is *never* sufficient to reform the *same* Li^{5*} level; however it can catch up and interact with the α_1 for certain T. This will lead to s_2 bands, as found on the $(s_1 s_2)$ plot.

(c) Of course since there are many possible levels of Li^{5*}, all these must really be considered.

(d) Finally, if s_{3-} turns out to be near or at a Be^{8*} level, the enhancement is the product of the kinematical effect times the appropriate s_3 Breit–Wigner form for the Be^{8*} level.

We see from Fig. 1b that the rescattering s_3 enhancement occurs in an s_3 band, precisely similar to a direct process (3) through a Be^{8*} level. Thus for a *fixed* beam energy T, such a rescattering enhancement will simulate a (previously unknown) Be^{8*} level. It can be distinguished from such an effect by observing that s_{3-} is a strong function of T (cf. Figs. 1a–c); in fact the effect is only present for a certain region of T.

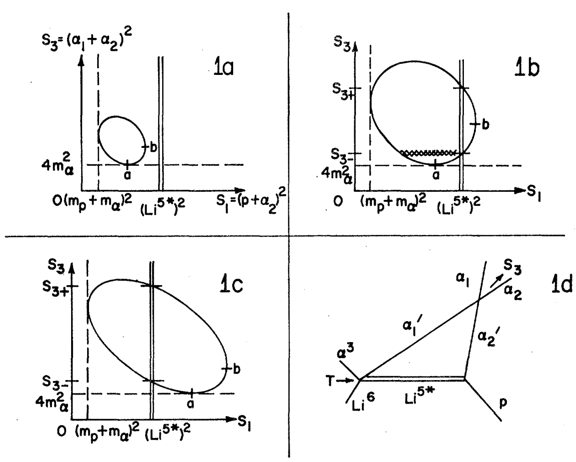

FIG. 1. (a–c) The location of the rescattering band, for various T.—In each case the relevant arc is ab—in (a) T is below the Li^{5*} threshold so there is no effect; in (b) T is satisfactory, leading to the shaded s_3 band; in (c) T is too large so that the kinematic conditions cannot be met. (d) The triangle graph which determines Figs. a–c.

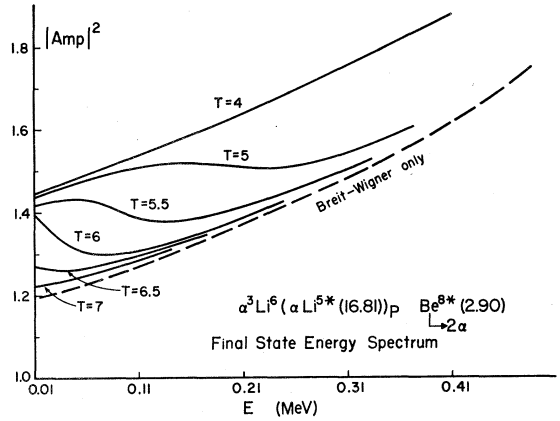

FIG. 2. The probability distribution of relative $\alpha_1\alpha_2$ energy E for various fixed beam energies T for the process $He^3 + Li^6 \rightarrow \alpha + Li^{5*}(16.81) \rightarrow p + Be^{8*}(2.90) \rightarrow p + 2\alpha$.

So far our discussion has been in direct physical terms. Exactly the same conclusions are reached if we consider the process (1) by successive orders in perturbation theory. Processes (2) and (3) correspond to a full treatment of each two-body channel, taken one at a time. Our rescattering effect corresponds to the triangle graph shown in Fig. 1d. Such graphs have been the subject of much theoretical work, particularly with application to higher resonances in elementary particle physics. A recent survey of the subject, with full references, is given by one of the present authors.[1] The conclusions are as above.

The triangle graph also enables us to perform explicit calculations to estimate the size of such effects. Such calculations have been performed for various elementary particle high energy processes.[2] It turns out that, in general, the rescattering enhancement is at most only 25% of background, and hence (with the limited statistics presently available in high-energy physics) no such effect has been identified *un*ambiguously (but see the suggestive work of Anisovich and Dakhno[3]). Thus at the present time there is no real

[1] C. Kacser, "Theoretical and Experimental Relationship between Triangle Singularities, Peierls Mechanism and Resonance Poles," Phys. Letters **12**, 269 (1964); see also I. J. R. Aitchison and C. Kacser, Phys. Rev. **133**, B1239 (1964).

[2] See, for instance, I. J. R. Aitchison, Phys. Rev. **133**, B1257 (1964), for details of the calculational method. A convenient analytic approximate form is given by I. J. R. Aitchison, Nuovo Cimento (to be published).

[3] V. V. Anisovich and L. G. Dakhno, Phys. Letters **10**, 221 (1964).

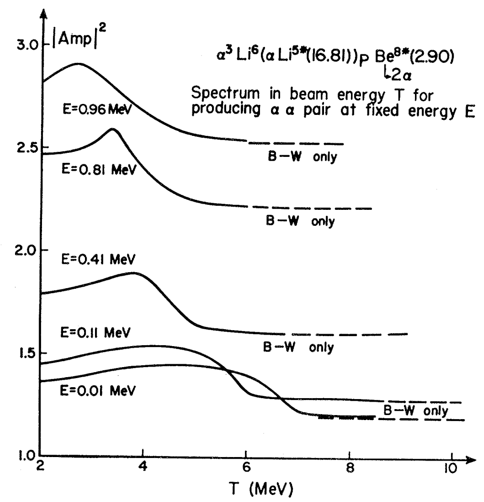

FIG. 3. The spectrum versus beam energy T for producing an $\alpha\alpha$ pair at a fixed (selected) relative energy E, in the process $He^3+Li^6\rightarrow\alpha+Li^{5*}(16.81)\rightarrow p+Be^{8*}(2.90)\rightarrow p+2\alpha$.

experimental verification of the whole complex of ideas involved in the study of analyticity of high-order perturbation theory graphs.

We have performed calculations for various nuclear reactions. Again the largest effects are 25% peaks, but with the better statistics available, such effects should be observable. Their observation and study is of great importance since (i) unless these rescattering enhancements are fully understood, they can readily be confused with "new" resonance levels—the crucial test here is whether their positions depend on the incident beam energy; and (ii) such rescattering enhancements are a necessary consequence of present final-state-interaction theory—if no such effects exist, the theory must be *totally* reformulated.

Finally therefore we come to the actual predictions. In each case we have been careful to add in the background term. Thus for instance in the reaction[4]

$$\text{I:}\quad He^3+Li^6\rightarrow\alpha_1+Li^{5*}(16.81)\rightarrow p+\alpha_1+\alpha_2$$

with rescattering between α_1 and α_2, s_{3-} is fairly near the Be^{8*} level at 2.90 MeV. We have hence used this to describe the $\alpha_1\alpha_2$ interaction in this region when calculating (I). However, we have added coherently with (I) both the $\alpha_1\rightleftharpoons\alpha_2$ graph, *and* the *direct* two-body reaction

$$\text{I}':\quad He^3+Li^6\rightarrow p+Be^{8*}(2.90)\rightarrow p+\alpha_1+\alpha_2.$$

I′ is expressed by means of the Breit–Wigner form in s_3 for $Be^{8*}(2.90)$, and in the figure is referred to as "B–W only." The relative scale between I and I′ is reasonably well determined in terms of the energy and width of the Li^{5*} (16.81) level. The absolute scale of each depends on the initial state interaction, which we do *not* treat here. This means that all our graphs are to be multiplied by a (hopefully) *smooth* function.

In Fig. 2, we show, versus E (the relative $\alpha_1\alpha_2$ kinetic energy), the squared modulus of the matrix element for I+I′, for various fixed-beam kinetic energies T. When multiplied by a smooth function of E to account for the initial-state interactions, these curves are closely related to the density of events in the Dalitz plot as a function of $s_3=(2m_\alpha+E)^2$. For beam energies in the region 5–6 MeV, we see a 25% peaking *which moves with beam energy*, but which for a fixed-beam energy (e.g., $T=5.5$ MeV) might well be mistakenly analyzed as a weakly excited Be^{8*} level with $E\approx 50$ keV.

In Fig. 3, we show the *same* process I+I′, this time showing the excitation function versus beam energy T for producing an $\alpha\alpha$ pair at a fixed (selected) relative energy E. Particularly for $E=0.81$ MeV, the expected effect is rather striking.

In both figures, process I by itself gives, in general, very peaked effects, but these get less peaked when the background is included from I′ (called B–W only). Many other competing processes must be added in; in particular all the possible intermediate resonance levels of Li^{5*} should be treated. Since their individual excitation functions are not known, we do not know the appropriate weighting, and hence have not considered this here.

We have performed similar calculations for

$$\text{II:}\quad He^3+Be^9\rightarrow\alpha\, Be^{8*}(16.92)\rightarrow Be^{8*}(2.90)+\alpha\rightarrow 3\alpha$$

$$\text{II}':\quad He^3+Be^9\rightarrow Be^{8*}(2.90)+\alpha\rightarrow 3\alpha$$

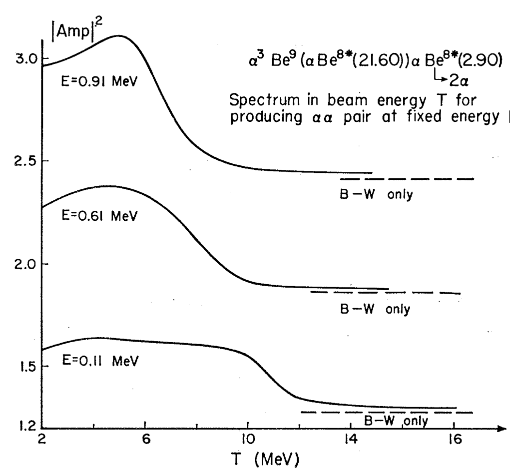

FIG. 4. The spectrum versus beam energy T for producing an $\alpha_1\alpha_3$ pair at a fixed (selected) relative energy E, in the process $He^3+Be^9\rightarrow\alpha_1+Be^{8*}(21.60)\rightarrow\alpha_2+Be^{8*}(2.90)\rightarrow\alpha_2+\alpha_1+\alpha_3$.

[4] All nuclear data are taken from F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959).

and for

$$\text{III:}\quad He^3+Be^9\to\alpha\ Be^{8*}(21.6)\to Be^{8*}(2.90)+\alpha\to 3\alpha$$

$$\text{III}':\quad He^3+Be^9\to Be^{8*}(2.90)+\alpha\to 3\alpha.$$

In each case the effects have the same general form. The most striking effect is for process III, as a function of T. This is shown in Fig. 4.

The choice of examples is governed by a consideration of the width of the *intermediate* resonance level. If the width is very large, this level only exists for a very short time, so that all particles at breakup are within each other's range of interaction. Hence the kinematic condition can be violated to some extent, yet rescattering still occur—i.e., the effects get broadened and decreased in size. On the other hand, if the width is very small, the rescattering has very small probability. Thus the width must be neither too small nor too large. (We remark that these conclusions are verified on detailed theoretical analysis.)

In summary: We have argued that rescattering effects always occur in three-body final-state interactions. Such effects can be confused with new resonance levels, but can readily be distinguished from the latter by their dependence on incident beam energy. The importance of the study of such effects is twofold:

(i) they must not be confused with ordinary resonances;

(ii) owing to experimental difficulties, such effects have not yet been unambiguously identified. Since they are a necessary consequence of present day theoretical ideas, their verification (or lack of same) is of basic importance.

Notes added in proof.

(i) When writing this paper, we were not aware of the work of R. Fox [Phys. Rev. **125,** 311 (1962)]. Fox considers only the "static" limit, in which the two rescattering particles are much lighter than the intermediate resonance. This leads him to concentrate on effects observable in angular distributions. His formulation cannot be readily extended for the general mass case (e.g., reaction 1).

(ii) Very recently rescattering effects have been observed by J. Lang, R. Müller, W. Wölfli, R. Bösch, and P. Marmier [Phys. Letters **15,** 248 (1965)]. The reaction was $d+C^{12}\to n+N^{13*}\to(n+p)+C^{12}$. Unfortunately the experiment was performed at only one beam energy so that the interpretation has not been fully tested.

(iii) We have since written two detailed papers on the rescattering effect. One contains the general formulation and has been submitted to *The Physical Review,* while the other, of a more practical nature, is to be submitted to *Nuclear Physics.*

Discussion

DONOVAN: The way one actually carries out these experiments is not to measure the whole Dalitz plot at one time, but rather pairs of angles. The way you usually study final-state interactions and find out what nucleus they belong in is to leave the angle of one detector fixed and move the other one, and this effect won't move the right way. It would look like something phony; not really a level in Be^8.

KACSER: That is not the case. For a given beam energy, there is *no* way of distinguishing this effect from a level in Be^8. If you change the lab angles, the effect will be there for all angles. For a given beam energy it looks exactly like an ordinary resonance of Be^8.

DONOVAN: What if you move one detector out of the plane?

KACSER: It doesn't matter. This is one of the useful features of invariant variables. The fact is that the effect is a function only of the S of the two alphas, and hence it gives a straight line on the Dalitz plot, exactly like any $(\alpha_1\alpha_2)$ resonance.

DONOVAN: The point I'm trying to make is that in changing the angles in every direction, it looks like a state in Be^8 but you can't see it except under the special conditions. States in Be^8 would be seen everywhere in the kinematically allowed region except where an accidental cancellations occurs.

ZUPANČIČ: Could you explain why you get a whole band, and not just a point?

KACSER: Yes. Consider Fig. 1d. At the $\alpha\alpha$ vertex you get rescattering, and it is scattering which can certainly lead to momenta out of the plane. All that is conserved is the four-momentum $\alpha_1'+\alpha_2'=\alpha_1+\alpha_2$. Hence we now do have a three-dimensional picture, with the final momenta out of the plane. The relative $\alpha_1\alpha_2$ energy *remains invariant,* but the rescattering by an arbitrary angle gives the extra freedom that leads to a band, i.e., all angles are possible between the final α_1 and the p.

ZUPANČIČ: The model you drew showed one alpha catching the other one.

KACSER: Yes, but after they have caught, they could go off in any direction.

PHILLIPS: I would like to make a comment here. Your arguments, as you gave them there, were entirely classical arguments. We all like to think that way. But we also know that ultimately this quantum mechanics.

KACSER: This is one of the remarkable features. The theoretical high-energy physicists have in fact been able to prove that the classical arguments do give the correct result.

PHILLIPS: Your figure shows an alpha particle catching up another alpha particle and scattering. That particular sort of diagram implies that the last thing that happened physically is that two alpha particles interacted and the proton was emitted.

If one simply picks the two diagrams for (2) and (3), the first with the direct emission of the alpha, and the second with the direct emission of the proton, and one makes a linear combination of these, then one has everything you have there.

KACSER: In a very real sense you are correct. But not fully. You ask in what way does my rescattering effect (Fig. 1*d*) differ from the process $He^3+Li^6\to$"blob"$\to p+(\alpha\alpha)$ where the $\alpha\alpha$ interaction is put in, that is, my process (3). The answer is that the simplest calculation for (3) would use a "blob" with *no* structure (except for angular momenta, etc.) What we are doing is including a very particular type of structure in the "blob." Not all the structure. Not the structure due to the initial-state reaction which we totally ignored. (Our curves have to be multiplied by a smooth function to include the initial-state effects.) But we are putting some specific structure inside the blob. This blob is not a point any more. And that is what makes the rescattering different.

KANE: I would just like to reiterate Zupančič's question. It seems to me the conservation of energy and momentum gives you a spot on the diagram, Fig. 1b, rather than a band.

KACSER: No. By Dalitz plot I mean a plot which includes all possible angles. You are probably thinking in terms of a differential plot, obtained with two counters set at fixed angles.

Even in the full Dalitz plot it is not immediately obvious one should get a band. The reason is that it is the intermediate α_1', α_2' that must satisfy the "catch-up" condition. They then rescatter, giving the observed α_1 and α_2. By four-momentum conservation $S_3=(\alpha_1+\alpha_2)^2=(\alpha_1'+\alpha_2')^2=S_3'$. But $p+\alpha_1'\neq p+\alpha_2$ so that while $S_1'=(p+\alpha_2')_2=[\text{mass Li}^{5*}]^2$, S_1 is not determined, and can take on all possible values.

ZUPANČIČ: I don't have any objection whatsoever. I believe your calculation. I would just like to understand it physically. Why is it a condition that you should have the vertical band crossing the elipse in order that the horizontal band appears (Fig. 1b)? Why is it discontinous?

KACSER: Why didn't I get an effect in Fig. 1a? Because then, if the excitation of the Li^{5*} means anything at all on the far tail of the Breit-Wigner resonance, this particular Li^5 state would be exceedingly weak. To that extent it would also break up and give some effect. But I am looking for an effect you can observe, and this is best in the case of Fig. 1b.

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The $Be^9(He^3, \alpha\alpha\alpha)$ Reaction

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The Q values for many He^3 and tritium-induced reactions on light nuclei are sufficiently high that it is possible for the systems to decay to three- or four-body final states. $Be^9(He^3, \alpha\alpha\alpha)$ is an interesting example of such a reaction leading to a three-body final state.

Studies of the spectra and angular distribution for the $Be^9(He^3, \alpha)2He^4$ reaction at bombarding energies of 3 and 4 MeV by Erskine and Browne[1] and by Dorenbusch and Browne[2] have shown the existence of a large continuum in the α-particle spectrum, a feature characteristic of the single-particle spectra for multiparticle reactions. Dorenbusch and Browne[3] have proceeded to fit the shape of this continuum under the assumptions that the continuum results from the breakup of states of a compound nucleus, C^{12}, and that the density of final states is uniformly distributed in phase space.

Since all the states of Be^8 are unbound for α decay and since many of these states are rather short-lived, one might expect that, indeed, the phase-space distribution for the final state would not be significantly modulated by the interactions among the components of the final state for the $Be^9(He^3, \alpha\alpha\alpha)$ reaction, even at low bombarding energies. The study of the single-particle spectra, however, is not adequate to determine unambiguously the nature of the process leading to the multibody final state.

The origin of the continuum can be uniquely determined for the 3 α-particle final state by measuring the energies of two of the α particles, E_A and E_B, at angles θ_A and θ_B. Conservation of energy and momentum restrict all such events to a kinematic curve $E_B(E_A)$ in the two-dimensional energy spectrum E_AE_B at fixed θ_A and θ_B. Reactions which proceed by sequential processes through discrete states of the intermediate Be^8 system will appear as points on this curve, or segments of the curve in the case of broad resonances.

If the $Be^9(He^3, \alpha\alpha\alpha)$ reaction proceeds through the 0^+ ground state of Be^8 with the emission of the initial α particle at the angle θ_A, the angle θ_B at which one of the α particles from the subsequent breakup of the Be^8(g.s) is emitted is limited to a narrow cone (half-angle$=7.2°$ for $\theta_A=60°$) about the recoil direction of the Be^8(g.s.) system. Figure 1(a) represent the velocity vector diagram of $Be^9(He^3, \alpha)Be^8\text{(g.s.)}\rightarrow 2\alpha$ reaction where $V_{C.M.}$ denotes the total center-of-mass velocity. V'_A and V'_{Be} are the velocities of the initial α and the Be^8 in the total center of mass system, and

* Supported in part by the U. S. Atomic Energy Commission.

[1] J. R. Erskine and C. P. Browne, Phys. Rev. **123**, 958 (1961).

[2] W. E. Dorenbusch and C. P. Browne, Phys. Rev. **131**, 1212 (1963).

[3] W. E. Dorenbusch and C. P. Browne, Phys. Rev. **132**, 1759 (1963).