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SESSION A—REACTION MECHANISMS

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A Survey of Simple Mechanisms of Three-Particle Reactions*

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I. INTRODUCTION

Nuclear reactions with three outgoing particles are very complicated in comparison with reactions where there are only two, albeit in general composite, particles in both the initial and final state. One of the reasons is that while the cross section for a two-particle reaction depends only on two independent kinematical variables, e.g., the center-of-mass energy and angle, there are five such variables in the case of a three-particle reaction. This makes both experimental and theoretical investigations of such reactions much more difficult and time consuming.

It is easy to choose different sets of the five independent kinematical variables. One possible choice would be the total energy in the center-of-mass system, three center-of-mass angles specifying the directions of two outgoing particles with respect to each other and with respect to the beam axis (which is an axis of azimuthal isotropy for the cross section), and the center-of-mass energy of one outgoing particle. The energy and momentum conservation laws enable us to express any other relevant kinematical quantity as a function of these independent variables.[1] However, often it is advantageous to choose as independent kinematical variables the squares of velocity differences $(\mathbf{v}_i-\mathbf{v}_j)^2$, which are both scalars and Galilean invariants and may thus be evaluated in any inertial coordinate system. Here, $\mathbf{v}_i$ and $\mathbf{v}_j$ may be velocities of any participating particle, either incoming or outgoing. The quantities $(\mathbf{v}_i-\mathbf{v}_j)^2$ are nonrelativistic analogs of scalar products of particle four-momenta, which are the basic Lorentz invariant variables in relativistic kinematics. They have a simple physical meaning. When both i and j are outgoing particles $[m_im_j/2(m_i+m_j)](\mathbf{v}_i-\mathbf{v}_j)^2$ is the internal energy of the two-body system formed by particles i and j. When i is an incoming particle and j an outgoing one, $m_im_j\times(\mathbf{v}_i-\mathbf{v}_j)^2$ is up to a constant mass term equal to the square of the four-momentum transfer between particles i and j. In this latter case the term particle is meant to include constituents of composite particles.

It is perhaps not surprising that the most outstanding features of three-particle reactions, as they have been revealed by experiments, are intensity maxima at particular values of either an internal energy variable or a momentum transfer variable. The former arise from sequential processes, where the reaction proceeds via a long-lived intermediate two-body system. The latter occur when one of the incoming particles interacts only with one constituent of the other incoming particle. This is most likely to happen when the constituent is light and weakly bound so that it is often encountered far out in the periphery of the nucleus or some nuclear core. Therefore, such processes are called quasifree or peripheral processes.

In the following, the discussion is simplified by a choice of labels for the particles. We label the incoming particles a and b, and the outgoing ones c, d, and f.

II. SEQUENTIAL PROCESSES

Intuitively one expects intensity maxima of a three-particle cross section at values of

$$E_{\mathrm{rel}}=[m_im_j/2(m_i+m_j)](\mathbf{v}_i-\mathbf{v}_j)^2,$$

where the outgoing particles i and j would have a resonance in a two-body collision. The question is what is the relationship between the dependence of the three-particle cross section on E_{rel} and the corresponding dependence of the two-particle cross section. This question has a clear answer when the resonance is sufficiently narrow, i.e., when the lifetime of the two-body system is so long that the reaction proceeds in

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[1] The kinematics of multiparticle reactions is discussed in greater detail in Č. Zupančič, Hercegnovi Lectures, 1964, Appendix I (to be published).

two steps. The first step is the formation of the intermediate two-body system accompanied by the third particle, and the second step is the decay of the intermediate system occurring predominantly at large distances from the third particle and therefore unaffected by it.

Since we are here considering low- to medium-energy nuclear reactions, the time scale is of the order 10^{-22} sec which is the time it takes a 10-MeV proton to traverse a distance equal to the radius of an O^{16} nucleus. When the participating particles are charged, the effect of the third particle may extend to larger distances and correspondingly longer times, due to the long-range Coulomb field. However, the parameter of interest here is the variation of the Coulomb potential, due to the third particle, over the dimensions of the two-body intermediate system, as compared to the nuclear potential between the two bodies. Because of the comparative weakness of the Coulomb forces, this parameter is small in most cases; moreover, Coulomb effects should, in such cases, be calculable by perturbation methods, thus offering an insight into the complicated problem of the influence of the third particle under particularly simple conditions.

The problem of how a long-lived intermediate system is formed in a collision and how it subsequently decays, is well-known from atomic physics and may be directly translated into nuclear physics. A simple example is the excitation of an atomic or molecular state by electron impact and its subsequent decay by the emission of a photon.[2] We are first interested in the dependence of the three-particle cross section on E_{rel}, as revealed, e.g., by the energy spectrum of the third particle, i.e., the outgoing electron. Integrating over the angles of the photon we obtain for the cross section $d^2\sigma$ for the outgoing electron ending in an infinitesimal angular and energy interval

$$d^2\sigma=\frac{\Gamma_{f,x}}{\Gamma}\frac{\Gamma}{2\pi}\frac{dE_{\mathrm{ph}}}{(E_{\mathrm{ph}}-E_c)^2+\Gamma^2/4}d\sigma_{\mathrm{sc}}. \tag{1}$$

Here $d\sigma_{\mathrm{sc}}$ is the two-body cross section for exciting the atomic state, ignoring radiation. It depends on the angle of the outgoing electron but is essentially independent of E_{ph}, which is the photon energy. E_c is the energy difference between the center of the excited state (x) and the final state (f). Γ is the total width of the resonance and $\Gamma_{f,x}/\Gamma$ is the relative probability for the decay of the excited state (x) into the particular final state (f). This expression may be compared to the cross section for (in general inelastic) resonance scattering of photons involving the same excited state. The resonance cross section varies as a function of the incident photon energy E_{iph} as,

$$\sigma_{\mathrm{res}}\sim\lambda_{\mathrm{iph}}^2\frac{\Gamma_{i,x}\Gamma_{f,x}}{(E_{\mathrm{iph}}-E_{\mathrm{res}})^2+\Gamma^2/4}. \tag{2}$$

[2] M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), p. 479.

Here λ_{iph} is the wavelength of the incident photon, E_{res} is the energy difference between the center of the excited state and the ground state, and $\Gamma_{i,x}/\Gamma$ is the relative probability for the decay of the excited state back into the ground state.

These expressions answer our question. They may be carried over into nuclear physics, and are, of course, very familiar there. As long as the intermediate state is long lived it also does not matter whether it decays by photon (i.e., gamma) emission or particle emission, and all photon energies may be replaced by appropriate relative particle energies E_{rel} in the above expressions. However, particle decaying states are usually not quite as narrow as gamma decaying states, which may give rise to important differences in the shape and position of the resonance as observed under different conditions. This is due to the energy dependence of the widths Γ, $\Gamma_{i,x}$, and $\Gamma_{f,x}$ which is in general not negligible in the case of nuclear reactions. Since a width Γ is a product of an energy-independent reduced width and a penetration factor, such effects are particularly noticeable when penetration factors are fast varying. The prime example is the effect of Coulomb penetration factors at low energies. Penetration factors being, in general, increasing functions of energy, a given two-body resonance may thus produce a peak in the three-particle cross section at a value of E_{rel} which is somewhat lower than the corresponding peak energy of the two-body cross section. Another important point to keep in mind is that for the two-body elastic scattering the potential scattering amplitude must be added to the resonance scattering amplitude. Especially when Coulomb scattering is important it is well known that the interference terms may violently change the shape of the two-body elastic scattering cross section in the region of the resonance. In general, the three-particle cross section will thus depend on E_{rel} more like the cross section for an endothermic two-body reaction from an entrance channel where one of the particles is a neutron, rather than matching the E_{rel} dependence of the elastic two-body cross section. Effects of this kind have been discussed in connection with the excited states of He^4.[3]

It is interesting to investigate not only the position and shape of the resonance but also its angular decay pattern. Its dependence on the direction of the third particle, as revealed by angular correlation measurements, gives us important information not only on the spin and parity of the long lived intermediate system, but also on the polarization of the intermediate system which is a sensitive indicator of the excitation mechanism. Such particle decay patterns are, in principle, very similar to the angular patterns of deexcitation gamma rays. Alpha particles yield especially simple patterns due to their zero spin. The mathematical techniques for calculating angular correlations in sequential processes are well developed. For a recent

[3] C. Werntz, Phys. Rev. **128**, 1336 (1962).

application to particle–particle and particle–gamma correlations see Ref. 4, where further references may be found.

An exceptional case, where the two-body cross section does not exhibit a proper resonance and yet the three-particle reaction may be considered to be approximately sequential, may occur when the intermediate system is in a relative s state at very low energy. The prime examples are two-nucleon systems pn, nn, and pp. Their decay is certainly not exponential, thus they do not possess a uniquely defined lifetime. However, one may well consider the *average* time they spend together, as a function of their relative energy.[5] Since their relative position is uncertain within their relative wavelength, this time may be defined as

$$\tau = v^{-1}(r+1/k+d\delta/dk), \tag{3}$$

where v and k are the relative velocity and wave number, respectively, r is the range of nuclear interaction, and δ is the nuclear phase shift.[6] In the effective range approximation we have $k \cot \delta = a^{-1}+\frac{1}{2}r_0k^2$ for the (n, n) and (p, n) scattering, and $C^2k \cot \delta = a^{-1}+\frac{1}{2}r_0k^2-\rho^{-1}h(\eta)$ for the (p, p) scattering. Here, a is the scattering length (defined so that it is positive for the virtual singlet s state of a two-nucleon system), r_0 is the effective range (which is of the order of r), $\rho = \hbar^2/me^2$ (proton Bohr radius), $\eta = e^2/\hbar v$, and

$$C^2 = 2\pi\eta/[\exp(2\pi\eta)-1].$$

$h(\eta)$ is a function evaluated in Jackson and Blatt.[7] Using these expressions one may compute the average lifetimes of two-nucleon systems; one finds that in the interesting low-energy region they are indeed long compared to 10^{-22} sec, and in general also long compared to the time spent by the third particle in traversing the distance $r+1/k$. In this sense one may consider reactions, involving the formation of two-nucleon systems at low relative energy, as quasisequential.

In this case it is nevertheless customary to speak of final-state interactions rather than intermediate systems. An approximate treatment of the final-state interactions due to Watson,[8] yields as the enhancement factor above phase-space intensity an expression, known from Coulomb corrections in atomic and nuclear processes such as photoeffect, bremmstrahlung, and β decay, namely, $B = |\langle u/u_0\rangle|^2$, u being the exact wave function describing the relative motion in the two-body system and u_0 being the corresponding wave function in the absence of interaction between the two bodies. The angular brackets indicate a suitable average over the three-particle interaction volume. Since this average depends somewhat on the details of the three-particle reaction mechanism, the usual procedure is to approximate it further to

$$B = |u/u_0|^2 \tag{4}$$

calculated either at zero separation distance or some small separation distance like the interaction radius. This expression is to a fair approximation

$$B = \frac{(r_0^{-1}+a^{-1}+\frac{1}{2}r_0k^2)^2}{k^2+(a^{-1}+\frac{1}{2}r_0k^2)^2}. \tag{5}$$

The proton–proton interaction gives rise to

$$B = \frac{[r_0^{-1}+a^{-1}+\frac{1}{2}r_0k^2-\rho^{-1}h(\eta)]^2}{C^2k^2+C^{-2}[a^{-1}+\frac{1}{2}r_0k^2-\rho^{-1}h(\eta)]^2}. \tag{6}$$

If one evaluates these expressions, one sees that while the neutron–neutron and singlet neutron–proton systems are strongly peaked at zero relative energy, the proton–proton interaction yields a much broader and lower peak with a maximum at about 1 MeV in relative energy.[9] This effect is caused by the Coulomb field and is mainly due to the smaller proton–proton scattering length and the occurrence of C^2 in expression (6). Since the average lifetimes computed from expression (3) are decreasing functions of relative energy, the pp system has thus, if anything, less right to be considered quasistationary than the nn and singlet np systems.

Any kind of sequential process, be it of the resonance or final-state interaction type, corresponds to at least one pole of the three-particle reaction cross section in the unphysical region of the complex E_{rel} plane. The long lifetime of the intermediate system reflects in the nearness of the pole to the (physical) positive real axis. Isolated poles being the simplest singularities of an analytic function, this connection between them and narrow long-lived states is another indication that one is dealing with a process of especially simple nature.

It is clear that not only s-wave type of final-state interactions but also narrow resonances occur mainly at low relative energies. As the relative energy becomes higher, penetration factors increase and more channels open, thus diminishing the probability of operation of selection rules. Resonances start to overlap, the intermediate systems live shorter, and the decay occurs often while the third particle is still in the vicinity. Very complicated phenomena then arise, some of which will be briefly mentioned in Sec. IV, but on the whole, little is understood about them, even qualitatively.

On the other hand, there seems to be no reason apart from experimental difficulties, why narrow resonances and final-state interactions at low relative energy should not be studied at higher total energies, in the region of 100 MeV and above. Actually, since this energy would

[4] J. G. Cramer and W. W. Eidson, Nucl. Phys. **55**, 593 (1964). For a short discussion c.f. also Appendix II of Ref. 1.

[5] This does not contradict the uncertainty relation. In the same sense one may assert, e.g., that the *average* space coordinates of a wave packet may be zero, while also its *average* momentum is zero.

[6] E. P. Wigner, Phys. Rev. **98**, 145 (1955); and Chap. 8 of Ref. 2.

[7] J. D. Jackson and J. M. Blatt, Rev. Mod. Phys. **22**, 77 (1950).

[8] K. M. Watson, Phys. Rev. **88**, 1163 (1952); and Ref. 2, p. 540.

[9] The p–p final-state interaction is discussed in more detail in R. J. N. Phillips, Nucl. Phys. **53**, 650 (1964).

mainly appear as the energy of relative motion of the third particle with respect to the intermediate system, the use of higher energies would cause the third particle to move away faster and thus increase the range of validity of the simple expressions (1) and (4). The problem of accurate energy determination is not as bad as it appears at first sight. In coincidence measurements one can find kinematical configurations where the *relative* accuracy of the energy of the outgoing particles determines the *relative* accuracy of the internal energy of the two-body system, thus making such studies feasible.

At present the higher energy reactions are mainly exploited for the study of a different kind of process, namely, peripheral or quasifree interactions.

III. QUASIFREE PROCESSES

When the initial relative kinetic energy of projectile and target is large compared to the binding energy of some of their constituents, one may, in a sense, neglect the binding energy and treat the collision as quasifree. For simplicity of discussion we shall, in the following, assume that the projectile a is tightly bound while the target b consists of a core b' and a particle b'' weakly bound to the core. One should, of course, keep in mind that in practical cases the situation may be reversed or that both target and projectile may have a loose structure. We call core the heavier of the two constituents of the target; if they are of the same mass the choice is arbitrary.

The most general peripheral process giving rise to three particles in the final state is a two-body collision of the projectile a with either the particle b'' or the core b', such that the target is broken up. The struck constituent of the target interacts with the projectile to give the final (in general different) particles c and d, while the other constituent of the target is only little affected and thus emerges with small laboratory momentum as particle f.

It helps intuition to draw a diagram of such a process. We only draw one of them corresponding to the interaction of the projectile with the lighter constituent. It is also instructive to compute this process in the simplest plane-wave Born approximation, where all particles are treated as free, except for the initial bound state b, which is described by a bound state wave function $\psi(\mathbf{r}_{b'}-\mathbf{r}_{b''})$. With very little algebra one obtains the result that the matrix element for the process in Fig. 1 is a product of two factors, one for each vertex. The lower vertex yields a form factor which is the Fourier transform of the initial bound state wave function, the momentum variable of this transform being the momentum transferred during the collision to b'. In the laboratory system where the initial velocity of b is zero, this momentum transfer is just $\mathbf{p}_f$ and the Fourier transform is $\psi(\mathbf{p}_f)=\int \exp(i\mathbf{p}_f\mathbf{x})\psi(\mathbf{x})\,d\mathbf{x}$. The upper vertex yields the Born-approximation amplitude for the reaction $a+b''\rightarrow c+d$. One may improve on

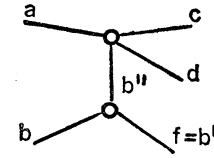

FIG. 1. Diagram of a quasifree process.

this calculation by inserting at this place instead of the Born amplitude the exact two-body collision amplitude. For practical calculations it is, at this stage, the simplest to forget about the binding of the struck particle altogether and assume the two-body collision amplitude to be the same as in a free two-body collision. This procedure is known as the peripheral model or the impulse approximation.[10,11] With a little care in performing the kinematical transformations one easily obtains the result for the three-particle cross section in the form

$$d^4\sigma=K\left(\frac{T_i^{\mathrm{cm}}}{T_f^{\mathrm{cm}}}\right)^{\frac{1}{2}}\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{cm}} |\psi(\mathbf{p}_f)|^2\times\text{phase-space factors.} \tag{7}$$

Here K is a constant, $d^4\sigma$ is the three-particle cross section for the outgoing particles ending in an infinitesimal region of final momentum space, and the phase-space factors are the corresponding phase-space factors of the three-particle reaction.[1] $(d\sigma/d\Omega)_{\mathrm{cm}}=(d\sigma/d\Omega)(\theta_{\mathrm{cm}})$ is the two-body differential cross section for the reaction $a+b''\rightarrow c+d$ as measured in the two-body center-of-mass system at a relative final energy $T_f^{\mathrm{cm}}=[m_cm_d/2(m_c+m_d)](\mathbf{v}_c-\mathbf{v}_d)^2$ and at the angle

$$\theta_{\mathrm{cm}}=\arccos\frac{(\mathbf{v}_a-\mathbf{v}_{b''})\cdot(\mathbf{v}_c-\mathbf{v}_d)}{|\mathbf{v}_a-\mathbf{v}_{b''}||\mathbf{v}_c-\mathbf{v}_d|}.$$

T_i^{cm} may be taken as $T_i^{\mathrm{cm}}=[m_am_{b''}/2(m_a+m_{b''})]\times(\mathbf{v}_a-\mathbf{v}_{b''})^2$. The velocity $\mathbf{v}_{b''}$ may be calculated from momentum conservation in the lower vertex. The precise choice of these parameters is somewhat arbitrary due to the approximate replacement of the amplitude for collision with a bound particle by the free-particle amplitude. If the precise choice of the above parameters would greatly influence the calculated cross section, this would indicate that the approximation is badly justified.

The gross behavior of the three-particle cross section for a peripheral process is determined by the term $|\psi(\mathbf{p}_f)|^2$ in expression (7). For the purpose of illustration let us assume that the particles b' and b'' are bound in a relative s state, and that we are only interested in low values of p_f so that we may approximate $\psi(\mathbf{x})$ by its asymptotic value $e^{-\alpha x}/x$. Then $\psi(\mathbf{p}_f)$ is proportional to $(p_f^2+\alpha^2)^{-1}\sim[E_f+E_{\mathrm{R.B.}}]^{-1}$. Here $E_f=$

[10] E. Ferrari and F. Selleri, Nuovo Cimento, Suppl. **24**, 453 (1962).
[11] Chapter 11 of Ref. 2.

$p_f^2/2m_{b'}$, while $E_{\text{R.B.}}$ is the reduced binding energy equal to the binding energy of the target b divided by $1+m_{b'}/m_{b''}$. The value of α, if desired, may be deduced from these relations. In this approximation we thus again get a pole-type behavior of the three-particle cross section but this time the pole is in the unphysical region of the complex E_f plane. E_f is actually a momentum transfer type of variable, being equal to the invariant quantity $\frac{1}{2}m_{b'}(\mathbf{v}_f-\mathbf{v}_b)^2$. A complimentary but equivalent way of looking at a peripheral process is indicated by the fact that $|\psi(\mathbf{p}_f)|^2$ is the probability distribution for the momentum of particle b'' in the target. In the peripheral model the reaction proceeds as if it were a free two-body reaction but with an initial velocity spread of the struck particles b''.

There are, of course, serious limitations for the applicability of the peripheral model to three-particle reactions. The main one is the requirement that the incident energy be large compared to the reduced binding energy of particle b'' in the target. If the model is to make sense, it is also necessary that the final velocities of particles c and d relative to b' be sufficiently large, so that they escape quickly from the sphere of influence of b'. Finally, the incident particle a should not interact strongly with the core b' while striking the particle b''.

Serious as these requirements are, quasifree processes play a surprisingly important role in many atomic and nuclear collisions. The reason is that for any given composite target there are usually projectiles available with energies much larger than the binding energy. Then there are always large parts of the physical region where the other requirements are also well satisfied. Well known examples of three-particle quasifree processes occur in the scattering of epithermal neutrons from atoms in molecules involving the breakup of the molecules and in the Compton scattering of x rays from bound electrons.

In nuclear physics the model is well applicable to knockout reactions at energies beyond 100 MeV. One difficulty in the nuclear case is the presence of several loosely bound nucleons in the nucleus. To projectiles of sufficiently high energy, the whole nucleus actually looks somewhat like a huge tightly packed macromolecule would look to an epithermal neutron. In such cases knockout reactions may hardly be called "peripheral" but they are still quasifree and may be treated by an extension of the model which takes into account the collisions of the projectile with several loosely bound particles in the target.[11]

An extrapolation procedure, suggested by Chew and Low,[12] in principle overcomes the limitations on the validity of the peripheral model. At the pole $E_f=-E_{\text{R.B.}}$ the peripheral model should be exactly valid and thus the experimental three-particle cross section, extrapolated to this pole, should exactly conform to Eq. (7). In practice, the extrapolation is unreliable except in cases when the peripheral model is rather well justified in the first place, and thus the Chew–Low method offers only an improvement on the quantitative application of the model.

In situations when the peripheral model is applicable, expression (7) enables us to extract important two-body parameters, often otherwise inaccessible, from three-particle reaction cross sections. One quantity of interest is $d\sigma/d\Omega$ for a two-body collision. In cases when the particle b'' is unstable this may be the only practical method of measuring its two-body cross sections. The still rather uncertain cross section for elastic scattering of neutrons on neutrons is the most important example in low-energy nuclear physics, while in elementary-particle physics examples are copious. The other quantity of interest is the form factor $|\psi(\mathbf{p}_f)|^2$. In complicated composite systems, such as most nuclei but the lightest ones, $\psi(\mathbf{x})$ represents only that component of the total wave function which describes the virtual disintegration of the target b into particle b'' and core b'. When the target does not like to appear as a loosely bound system of b' and b'', $|\psi(\mathbf{p}_f)|^2$ and the corresponding enhancement of the three-particle cross section are small. The form factors are thus important clues to our understanding of the nuclear wave function. Finally, the studies of the binding energies of nucleons on different shells in nuclei are made possible by the quasifree scattering process which knocks a nucleon out of the interior of the nucleus without giving the latter time to rearrange itself.

IV. MORE COMPLICATED PROCESSES

The study of three-particle reactions, like many other branches of physics, is proceeding in two more or less opposite directions. On one hand, one tries to exploit simple and relatively well-understood limiting situations to extract quantities of interest for related fields. The previous two sections indicated how simple three-particle reactions may be used to obtain nuclear spectroscopic data. On the other hand, one tends to penetrate into the unknown land in between or beyond these simple limiting situations.

There are some features of three-body reactions which have not been mentioned till now, but which may be understood by a proper extension of the simple ideas presented in the previous two sections. Some of them arise from the fact that, in general, in quantum mechanics one has to add amplitudes for simultaneously occurring processes and properly take into account the principle of indistinguishability of identical particles. In situations when the process giving rise to a single amplitude is well-understood, their superposition should be relatively straightforward. Another area which should not present great conceptual difficulties, though it might be quite onerous for computations, is the transition between the two limiting cases of sequential

[12] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).

and peripheral processes, under conditions when three-body effects are not important. As an example of what I mean, consider the scattering of x rays from electrons bound in the atom. In one limit, the inelastic scattering of the photon is accompanied by the excitation of the atom and its subsequent radiative decay at a much later time. This is the sequential process. In the other limit, a weakly bound electron is ejected by a high-energy photon into a practically free state. This is the peripheral process. Clearly any intermediate situation may be mastered by taking into account the exact Coulomb wave functions of the electron in the continuum. This corresponds to a distorted wave calculation in nuclear physics and is also in principle straightforward though sometimes hard to carry through.

The really hard problem is presented by the presence of three strongly interacting particles in the final state. Somewhere in the intermediate area between the two extreme simple processes, characteristic features of the three-body problem should manifest themselves. One prediction that one can probably safely make, is that the presence of the third particle will tend to widen and distort the resonances in sequential processes, but qualitative estimates of this effect are at present not available. A few years ago we carried out an experiment[13] which probed into this intermediate area. The disquieting experimental indication (from the point of view of a physicist interested in three-body problems) was that there might be a smooth transition between the two limiting processes, much like in inelastic x-ray scattering. The specific three-body features of three-particle nuclear reactions might be hidden in finer details of the cross sections. In view of the recent promising developments in the theoretical investigations of the three-body problem, the experimental studies of such fine details should, in the future, offer an exciting field of research.

Discussion

Henley: In the intermediate region, are there any clear experimental features that allow you to distinguish between which of these two extremes might be applicable?

Zupančič: In the intermediate region, in general, none of them is applicable. However, if you do have a sequential process, then you can make some very definite statements about the correlation between the relative velocities in the first step and the relative velocity of the decay products in the second step.

Let me talk about this simple case as an example:

$$O^{16}+\alpha\rightarrow O^{16*}+\alpha'\rightarrow C_{g.s.}{}^{12}+\alpha'+\alpha''.$$

Suppose this process goes in two steps, the first one being the excitation of O^{16} and the second one being the decay of the O^{16} excited state into the C^{12} ground state and α''. You can now measure the angular correlation between α' and α'', and that was done in an experiment which we performed.

Then you can make a prediction that does not depend on the mechanism of the excitation process. It just depends on the fact that the O^{16} excited state is isolated and long lived—in other words, that you really are in the limit of the sequential process. Now consider the plane defined by the incoming alpha particle and first outgoing alpha particle. The normal to this plane is an axis of two-fold symmetry of the decay pattern, that is, the angular pattern of α'' measured with respect to center-of-mass system of the O^{16} excited nucleus. This angular pattern should repeat itself after 180°; that, as I say, is a very general prediction. It depends only on the assumption that you have a sequential process.

Furthermore, it is very sensitive, because all types of disturbances give interference terms. If you don't have a very narrow state, the interference terms due to tails of other states will come in; then, the symmetry prediction won't be true any more. This is a very sensitive test of the purity of the sequential process. In the particular case of the reaction we measured, it turned out that this symmetry did, indeed not exist exactly. On the other hand, it was still approximately there, which is quite surprising in such a reaction where we were dealing with an intermediate situation.

Phillips: I got the impression from what you said that one would have this repetition of periodicity of π in the correlation. I would point out, though—and I believe you made the statement—that this was a sensitive test, whether or not the reaction was sequential. If the reaction is sequential and more than one state is involved, that is, more than one original compound nuclear state in this case, then your statement is no longer necessarily true, and yet the reaction mechanism can indeed be sequential.

Zupančič: The other state would also have to be of the opposite parity.

Phillips: It is more than that. There can only be one state involved, and you cannot know which alpha particle you are looking at—the one that comes out of oxygen or the first one. Under that circumstance, the statement you made is not true. So I think at this time there is no sort of simple rule-of-thumb way of knowing whether the reaction is sequential.

Zupančič: I am quite aware of what you say and in my talk I have tried to simplify things.

I quite agree that this kind of symmetry prediction is not true as soon as you get another overlapping state of opposite parity; and in general this could happen in the region where the separation of states is quite large compared to their widths, if just by accident two states of opposite parity lie near to each other. That would be an exceptional case, and I talked rather about the typical case where you will get that kind of overlap only when the separation between the states becomes of the same order of magnitude as their width, and those are really the cases where you are in the intermediate situation.

You also get complicated things when you have to deal with a situation where the contribution from two amplitudes are of about equal strength. I was just talking about simple cases, and not such fine complications, which certainly arise.

This kind of prediction is also not true when you have some background; interferences with the background spoil such a symmetry very quickly. But now, what is the background? It may be considered as the tails from other states which contribute fast-decaying components. In other words, if you have considerable background, to that extent you do not have a pure sequential process, and all I wanted to say is that these symmetry predictions are really very sensitive to even small admixtures of more general processes.

I just don't think you can explain all three-body reactions by the sequential process.

[13] P. F. Donovan, J. V. Kane, Č. Zupančič, C. P. Baker, and J. F. Mollenauer, Phys. Rev. **135**, B61 (1964).