Seminar on the Embedding Problem*

Introduction

In the 1925 preface to his textbook on "Riemannian Geometry," L. P. Eisenhart was noting that, "The recent physical interpretation of intrinsic differential geometry of spaces has stimulated the study of this subject." In such a context, he devoted a large part of the book to the description of Riemann spaces embedded in flat spaces with added dimensionality. It turned out, however, that general relativity was handy enough when treated in terms of the curvilinear coordinates of the Riemann space itself; the embedding seemed to add extraneous spatial extensions which added little to the understanding of gravitation and the cosmology. From time to time, some interesting results would be derived in this way, but they would also be directly derivable from the Riemannian metric, interest in the embedding thus subsiding again. An example of this sort was provided by C. Fronsdal's study of the complete Schwarzschild solution via its embedding [Phys. Rev. **116**, 778 (1958)]; the remarkable insight into that problem did not give rise to an interest in the embedding method, since the same result was achieved shortly afterward by M. D. Kruskal [Phys. Rev. **119**, 1743 (1960)] without using it.

The theory of elementary-particle physics was developed in terms of a flat space and its symmetry properties—the Lorentz group. It is not surprising, therefore, that attempts to describe properties of particles in curved space–time should lead to a renewed interest in the embedding method. The applications vary, from the development of additional techniques in handling the Lorentz group, to attempts to identify the "internal" symmetries of particle physics with the symmetries of the normal, "extraneous," piece of the embedding space.

We have deemed it worthwhile to convene some physicists and mathematicians interested in this subject to a discussion of the method and its application. The following papers formed the nucleus of a seminar, which, we hope, will prove fruitful to both relativity and particle physicists, and may provide some mathematicians with the motivation to complete the study of those differential geometry aspects which are extremely mysterious at this stage.

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Isometric Embedding of Riemannian Manifolds into Euclidean Spaces

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This is a review of differential geometry results pertaining to the problem of embedding curved space–time in a pseudo-Euclidean space.

I. INTRODUCTION

Recent work in elementary-particle physics ties up internal symmetries of elementary particles, under nonstrong interaction, with symmetries of generalized space–time curvature. The mathematical aspect of the problem is then to obtain information on the embedding class of various four-dimensional relativistic metrics; the embedding considered is isometric and smooth, and is either local or global.

In the mathematical literature there are numerous classical results on local (isometric) embeddings, but only a few which may be of help in determining the embedding class for the present metrics. As for global embedding, there are several fairly recent results, but they only apply to positive definite metrics.

In this talk I shall survey most of the known results concerning global embeddings, as well as some results concerning local embeddings which might be useful for relativistic metrics.

II. GLOBAL ISOMETRIC EMBEDDING

In this section we consider isometric embedding of a Riemannian manifold $|V_n$ of dimension n into a Euclid-