A Survey of Some Properties of Even–Even Nuclei

EUGENE EICHLER

Chemistry Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee*

I. INTRODUCTION

The years have passed since the last comprehensive surveys of the properties of even–even nuclei.[1] These explorations proved most valuable in establishing two regions of homogeneous behavior: the rotational region and the vibrational region. Properties of the nuclei in the former region are now quite accurately described by the collective model of Bohr and Mottelson[2] with its numerous extensions. However, as yet no single model describes completely the vibrational nuclei. A number of different approaches have achieved some measure of success.[3–10]

The vibrational nuclei are found in the medium-weight region ($50>A>150$). This designation arises from several systematic characteristics of these nuclei: (1) They all possess low-lying first-excited states with spin and parity 2+ (except for ^{90}Zr and ^{72}Ge, which have 0+ first excited states). (2) The second-excited states appear at approximately twice the energy of the first and have the character 2+, 4+, or 0+. (3) These states are strongly excited in electric quadrupole processes such as Coulomb excitation; conversely, their decay rates are far in excess of the single-particle estimates. Such level spectra are suggestive of a simple quadrupole vibrator as schematically indicated in Fig. 1(a). The 2+ character of the first-excited state and the energy ratio of the second to first state of about two are consistent with experimental observations. The phonon number gives the quanta of vibrational excitation. In the pure vibrational picture, the two- and three-phonon states are degenerate multiplets. However, with real nuclei [Fig. 1(b)], the degeneracy of the upper phonon levels is lifted, and the second excited states have unique spins and parities. The most frequently observed character for the second state is 2+, with 4+ much less usual, and 0+ only rarely observed. A number of 2+, 4+ doublets have been seen, but in only a few instances has the full triplet been identified.[11] As Fig. 1(b) indicates, the energy ratio of the second to first states is somewhat greater than two.

Scharff-Goldhaber and Weneser[3] attempted to explain these properties by considering the weak coupling of four $f_{7/2}$ particles to the vibrating surface. Raz[4] has made a similar calculation utilizing two $f_{7/2}$ particles and considering a definite two-body force between them. Purely collective models have also been tried: Wilets and Jean[5] and, also, Tamura and Komai[6] considered a displaced harmonic oscillator; Davydov and Filippov,[7] as well as Mallman and Kerman[8] attempted to generate the experimental-level scheme by rotations of a non-axially symmetric spheroid.

The newer pairing correlation approach of Kisslinger and Sorenson[9] considers nuclear structure as due to two forces: A short-range two-body force and a long-range collective force. This treatment yields a spectrum of modified shell model states, called quasiparticle states, which are separated from the ground state by an energy gap, cf. Fig. 1(d). The vibrational states are then constructed of contributions from a number of nearby quasiparticle states. (See also Tamura and Udagawa.[10]).

As Fig. 1(c) indicates, there are vibrations of multipole order higher than two. So far only the octupole state[12] (2^3-pole with spin 3−) and the hexadecapole state [13] (2^4-pole with spin 4+) have been observed.

In the hope that the considerable body of new data may aid in further interpretation, some of the nuclear properties of even–even nuclei in the medium-weight region have been surveyed. The energies of the one phonon states is considered first. Then the two-phonon, three-phonon, and octupole states are examined. A similar survey of the one phonon state has been made recently by Ythier and van Lieshout.[14]

In this investigation, the source of most of the data was the Nuclear Data Sheets.[15] In order to gather the recent information, the "Recent References–August,

---

* Operated by the Union Carbide Corporation, Nuclear Division, for the U. S. Atomic Energy Commission.

[1] G. Scharff-Goldhaber, Phys. Rev. **90**, 587 (1953); P. Stahelin and P. Preiswerk, Helv. Phys. Acta **24**, 623 (1952).

[2] A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **26**, No. 16 (1953).

[3] G. Scharff-Goldhaber and J. Weneser, Phys. Rev. **98**, 212 (1955).

[4] B. J. Raz, Phys. Rev. **114**, 1116 (1959); **129**, 2622 (1963).

[5] L. Wilets and M. Jean, Phys. Rev. **102**, 788 (1956).

[6] T. Tamura and L. G. Komai, Phys. Rev. Letters **3**, 344 (1959).

[7] A. S. Davydov and G. F. Filippov, Nucl. Phys. **8**, 237 (1958).

[8] C. A. Mallman and A. K. Kerman, Nucl. Phys. **16**, 105 (1960).

[9] L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat.-Fys, Medd. **32**, No. 9 (1960); Rev. Mod. Phys. **35**, 853 (1963).

[10] T. Tamura and T. Udagawa, Prog. Theoret. Phys. (Kyoto) **26**, 947 (1961).

[11] J. K. Dickens, F. G. Perey, R. J. Silva, and T. Tamura, Phys. Letters **6**, 53 (1963).

[12] A. M. Lane and E. D. Pendlebury, Nucl. Phys. **15**, 39 (1960).

[13] P. H. Stelson and R. L. Robinson, in *International Symposium on Direct Interaction and Nuclear Reaction Mechanisms*, Padua University (1962), edited by E. Clementel and C. Villi (Gordon and Breach, New York, 1963), p. 852; H. Crannel, R. Helm, H. Kendall, J. Oeser, and M. Yearian, Phys. Rev. **123**, 923 (1961).

[14] C. Ythier and R. van Lieshout, J. Phys. Radium **22**, 23 (1961).

[15] Nuclear Data Sheets, National Academy of Sciences, National Research Council (Washington, D. C.).

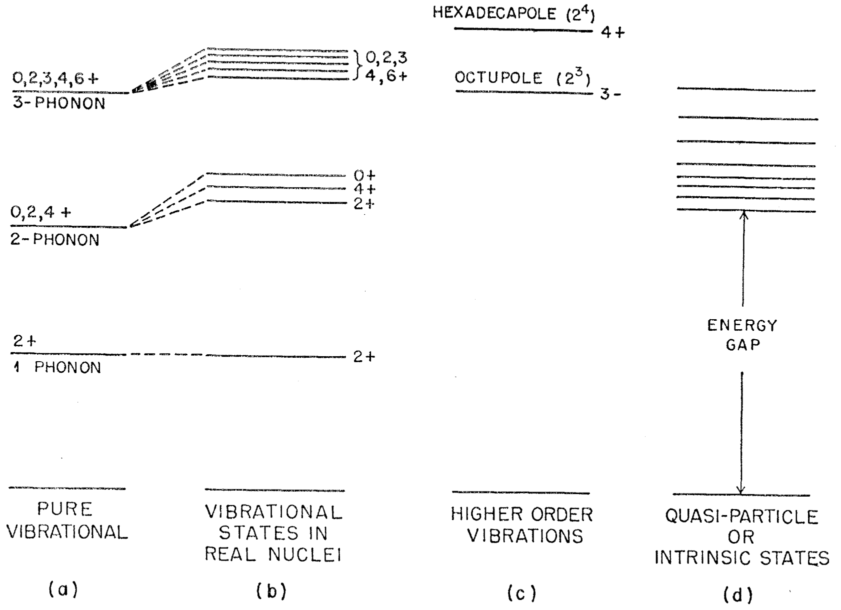

Fig. 1. Typical energy levels of medium-weight even–even nuclei.

1962" from the Nuclear Data Group and the indexes of Nuclear Science Abstracts were used as guides. ("Recent References–July, 1963" was not available at the time this survey was completed.) Unfortunately, space limitations preclude referencing each entry in the illustrations shown below.

II. THE ONE-PHONON STATE

Figures 2 and 3 show the first 2+ excited state energies versus both N, neutron number and Z, proton number. A similar plot was prepared by Nakasima and issued with the Nuclear Data Sheets.[16] Looking first at the neutron number plot, Fig. 2, we are struck by the rapid increase at the major shell closures with a decrease in between to a rather low valley. Beyond the 82 shell, there is a precipitous drop to the rare-earth deformed region. The valley between the shells also descends to a new deformed region as we know from the work of Sheline, Sikkeland, and Chanda[17] and also Lark, Morinaga, and Wapstra.[18]

The valleys between the shells are peppered with fine structure. Starting at the high end of the Z plot (Fig. 3), we see a small peak at 64 protons in the 84 and 88 neutron chains.[18a] This corresponds to the filling of both the $1g_{7/2}$ and $2d_{5/2}$ orbitals. This "64" effect is seen also for neutrons in the 50 proton nuclei. At lower Z values, the $g_{7/2}$ filling at 58 protons produces a peaking which is seen not only in the 78-, 80-, and 84- neutron chains, but also in magic series of $N=82$. There is no 58 neutron peak, but there is one at 56 in the 40 and 42 proton families. This is to be expected since the $g_{7/2}$, $d_{5/2}$ order is inverted for neutrons.

The effect of the 38 and 40 subshells is interesting. First of all, there is a distinct peak at $N=38$ seen in the zinc ($Z=30$) and germanium ($Z=32$) families. However, there seems to be none for $N=40$ unless the 34 proton upswing can be called a peak. On the other hand, at $Z=40$, there is a distinct peaking. In fact, ^{90}Zr with 40 protons and 50 neutrons behaves like a doubly magic nuclide having a 0+ first-excited state and first-2+ excited state of 2.18 MeV, the highest above the light element region. There is also a peak in the $N=52$, $N=54$, and $N=56$ chains at 40 protons. But there is no $f_{5/2}$ filling effect at 38 protons.

To recapitulate briefly, there is definite evidence of shell-model fine structure at certain points between the major shell closures for many isotopic and isotonic families. The presence of these effects in some chains and their absence in others suggests several thoughts about even–even nuclei and their vibrational motion.

First, the great sensitivity of the phonon energy to subshell closure supports the view that vibrational motion involves principally the particles beyond the major closed shells. Further, the normal shell-model ground-state filling scheme must be altered in even–even nuclei by the pairing force. Some subshell splittings become large, resulting in the discontinuities cited above.

---

[16] R. Nakasima, Nuclear Data Sheets, NRC 61-4-127.

[17] R. K. Sheline, T. Sikkeland, and R. N. Chanda, Phys. Rev. Letters **7**, 446 (1961).

[18] N. Lark, H. Morinaga, and A. Wapstra (private communication).

[18a] *Note added in proof.* There is also a peak in the 86-neutron chain since the first excited state of ^{150}Gd occurs at 0.639 MeV according to K. Toth *et al.*, Phys. Rev. **116**, 118 (1959); and A. T. Strigachev *et al.*, Izvest. Akad. Nauk SSSR Ser. Fiz. **25**, 813 (1961) [English transl.: Bull. Acad. Sci. USSR Phys. Ser. **25**, 824 (1962)].

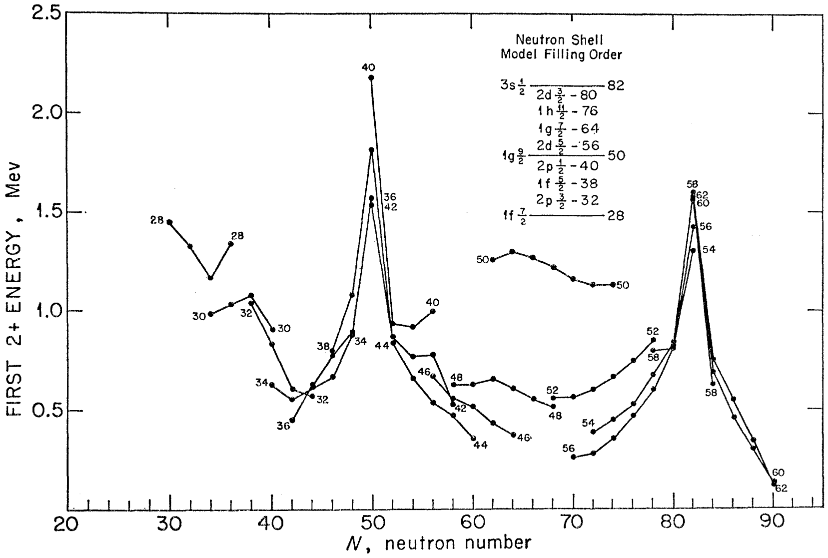

FIG. 2. Plot of the energies of the first 2+ excited states of even–even nuclei vs neutron number. Lines join isotopic chains and are marked with proton numbers.

However, note that these peaks in the 2+ energy values occur for those isotopes whose proton numbers are not too far removed from closed shell numbers (or conversely for those isotones whose neutron numbers are near magic values). For example, subshell effects are present in the isotopic chains with proton number 50 and 48 but not 44 and 46. Thus, it appears that for nuclei fairly distant from closed shells the fine structure is washed out.

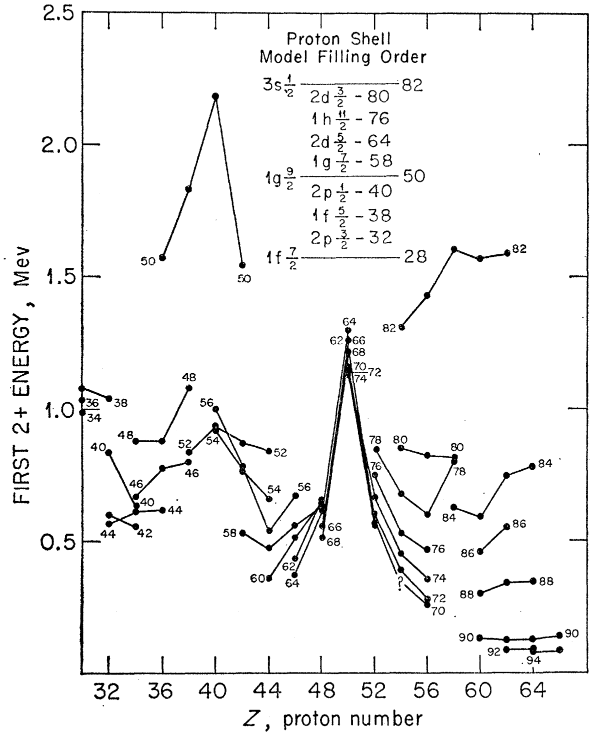

FIG. 3. Plot of the energies of the first 2+ excited states of even–even nuclei vs proton number. Lines join isotonic chains and are marked with neutron numbers.

Are there other more subtle manifestations of the shell-model orbitals? To look for such details, it was necessary to prepare a different correlation of the first-excited state. In Fig. 4, the 2+ energies are plotted versus the logarithm of the number of particles or holes (whichever is smaller) in the major unfilled shell for a number of the chains which do not exhibit subshell effects. These data for each mass chain fall on a straight line with an almost common slope for all the mass chains. The effect of proton number is also apparent in this plot. Note that the Ru values ($Z=44$; 4 protons beyond the strong 40 subshell) and the Xe values ($Z=54$; 4 protons beyond the 50 shell) lie on a common line. An attempt was made to correlate the 2+ energy with the logarithm of the mass number, A; this was not very instructive as the inset indicates.

This effect of proton pairs was also examined by making a similar plot of the data for the isotonic families $N=60$, 72, 74, and 76; they all fall roughly on lines parallel to the isotopic lines. The proton situation is less satisfactory, since shell and subshell perturbations permit, at most, a smooth three-member chain. Thus, it would appear that (1) the variation of phonon energy with addition of pairs is independent of the orbital character of the added particles; and (2) addition of pairs of neutrons or protons (or holes) has roughly the same effect on the phonon energy.

Another perhaps trivial but amusing effect can be seen in the systematics of the first 2+ states. Looking at the neutron plot, we note that although the Sn nuclei ($Z=50$) have their maximum energy at $N=64$ (^{114}Sn), the neighboring Cd nuclei ($Z=48$) have theirs at 62. Notice that this nucleus, ${}^{110}_{48}\text{Cd}_{62}$, is ${}^{114}_{50}\text{Sn}_{64}$ minus 2

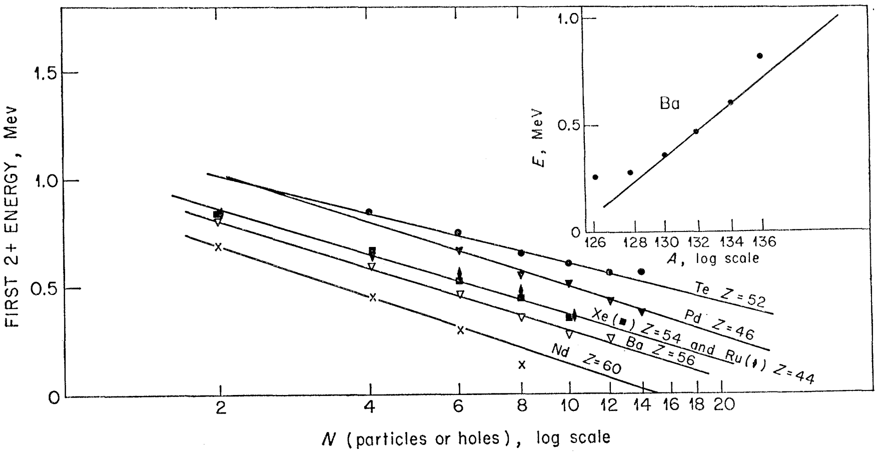

Fig. 4. Plot of the energies of the first 2+ excited states vs the number of neutrons or holes in the major unfilled shell (log scale) for several isotopic families. The straight lines are arbitrary fits to the points. Inset: plot of the energies of the first 2+ excited states of the Ba nuclei vs the logarithm of A, the mass number.

protons and 2 neutrons, i.e., an α-particle. There is another example of this situation: ${}^{86}_{38}\mathrm{Sr}_{48}$ is ${}^{90}_{40}\mathrm{Zr}_{50}$ minus an α-particle and has an anomalously high first 2+ energy of 1.08 MeV.

III. THE TWO-PHONON STATE

The two-phonon state is a triplet of levels with spins 2+, 4+, and 0+ (in order of frequency of experimental observation). This triplet occurs at slightly more than twice the energy of the first 2+ state.

Since the 2+ is the most commonly observed second state, Fig. 5 includes the energy ratios of the second to first 2+ states versus neutron number. No strong pattern is obvious. There is a tendency for the ratio to drop just before a major shell closure; these occur at $Z=38$, $N=48$; $Z=54$; $N=80$; and $Z=58$, $N=80$. There are only three second 2+ states in single closed shell nuclides on this plot: ^{116}Sn with a very low ratio of 1.64, ^{140}Ce with a ratio of 2.5, and ^{136}Xe with a ratio of 2.0.

According to the simple vibrational approach,[3] the splitting of the degeneracy of the second phonon triplet is due to the coupling of particles beyond closed shells to the spherical "core." At or near closed shells, the coupling would be weak, hence the splitting small. Moving away from the shell, the coupling should grow stronger and the splitting larger. Although this is a vastly oversimplified description, let us examine the experimental situation.

Between Se and Nd ($Z=34$ to 60), there are about 11 nuclei with second phonon multiplets with splittings less than about 50 keV. The other nuclei either have as yet unidentified multiplets or splittings of 100 keV or more. The 11 with <50-keV splitting lie both near to and far away from closed shells. However, the smallest splittings, by far, are for five Ru and Pd isotopes with splittings less than 20 keV. The record goes to ^{102}Ru which has a 2+ and a 4+ state both exactly at 1.105 MeV. These nuclei have 4 or 6 protons or proton holes and 4 to 12 neutrons beyond closed shells. It will be interesting to see if similar small splittings occur in the "almost deformed" neutron deficient Ba and Xe isotopes.

About 20 $\delta(2+, 2+, 0+)$ values (where δ is the E2/M1 mixing ratio) have been measured for nuclei in the region under discussion. Several attempts[4,19] have been made to theoretically or semiempirically explain the fluctuations in the δ values. None of these approaches has been completely successful. The δ values are remarkably consistent considering the experimental difficulty in obtaining highly accurate correlation parameters. One can generalize that the 2+→2+ transition from every second 2+ in this region has a $\delta > 3$; i.e., the transition is >90% E2. There are only three exceptions to this rule: The radiations from second 2+ levels in ^{92}Zr and ^{140}Ce are essentially 100% M1. These results suggest, of course, that these latter states are not collective in nature. The other exception is the value of

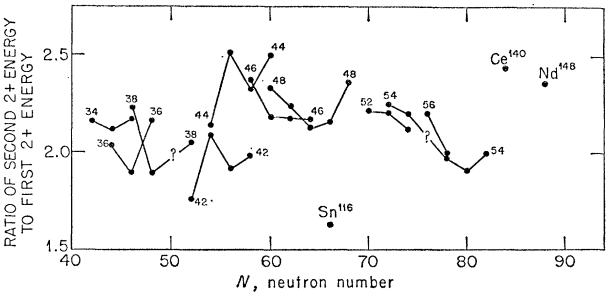

Fig. 5. Plot of the ratios of the energies of the second 2+ to the first 2+ excited states vs neutron number. Lines join isotopic families and are marked with proton numbers.

---

[19] T. Tamura and H. Yoshida, Nucl. Phys. **30**, 579 (1962); D. P. Grechukhin, Nucl. Phys. **40**, 422 (1963); M. Sakai, Institute for Nuclear Study, University of Tokyo, Institute Report 37; V. R. Potnis and G. N. Rao, Nucl. Phys. **42**, 620 (1963); I. Asplund, L. G. Stromberg, and T. Wiedling, Arkiv Fysik **18**, 65 (1960).

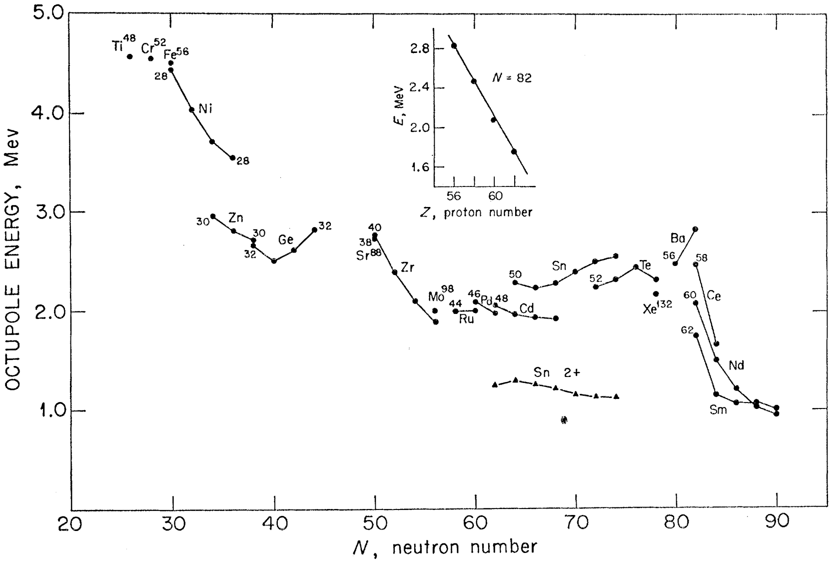

FIG. 6. Plot of the energies of the lowest octupole states (3−) in medium weight even–even nuclei vs neutron number. Inset: Octupole energies vs proton number for the isotonic chain $N=82$. Most of the data is from Hansen and Nathan, Ref. 25. Additional references are: ^{48}Ti, ^{52}Cr, ^{56}Fe, ^{58,60}Ni, ^{64}Zn: K. Matsuda, Nucl. Phys. **33**, 536 (1962). ^{58,60}Ni, ^{64,66,68}Zn: H. W. Broek, T. H. Braid, J. L. Yntema, and B. Zeidman, Nucl. Phys. **38**, 305 (1962); Phys. Rev. **126**, 1514 (1962). ^{70,72,74,76}Ge and ^{62,64}Ni: J. K. Dickens, F. G. Perey, and R. J. Silva, Neutron Physics Division Annual Report, Oak Ridge National Laboratory, ORNL-3499; Phys. Rev. **132**, 1190 (1963).

$-1.0 \geq \delta \geq -0.4$ observed by Sen Gupta and Van Patter[20] in ^{60}Ni.

IV. THE THREE-PHONON STATE

As indicated earlier, the next stage of quadrupole excitation, namely 3 phonons, excites a quintet of states: 0, 2, 3, 4, 6+. In an ideal vibrator, this band would lie at 3 times the energy of the first phonon state. Such a quintet of states has been seen by several groups utilizing inelastic scattering.[21] The energies of these states are predicted by the models of Kerman and Shakin,[22] and Raz.[4]

TABLE I. Log *ft* values for allowed beta decay to octupole states.

Parent nucleus	*S* Spin and parity	Daughter	Type of decay	Octupole state energy	Log *ft*[a]
^{72}Ga	3−	^{72}Ge	β^-	2.55	8.0
^{72}As	2−	^{72}Ge	β^+	2.55	6.2
^{74}Ga	(3−, 4−)	^{74}Ge	β^-	2.6	7.7
^{88}Rb	2−	^{88}Sr	β^-	2.74	6.7
^{88}Y	4−	^{88}Sr	E.C.	2.74	6.6
^{92}Y	2−	^{92}Zr	β^-	2.33	7.1
^{112}Ag	2−	^{112}Cd	β^-	1.97	7.5
^{124}Sb	3−	^{124}Te	β^-	2.25	7.7
^{124}I	2−	^{124}Te	E.C.	2.25	6.6
^{140}La	3−	^{140}Ce	β^-	2.47	7.6

[a] Values are taken from the Nuclear Data Sheets.[15]

Decay scheme studies should help characterize new 3-phonon states. Where a 2+ member of a 3-phonon set is involved in a 2+, 2+, 0+ cascade, measurement of the γ,γ angular correlation yields δ–a criterion of the collective character of the state. Decay scheme workers can also begin searching for high-lying states which preferentially populate 2-phonon states over the one phonon state. Van Lieshout, Ricci, and Girgis[23] and Yoshizawa[24] have found examples of this effect. Of course, this is analogous to the preference of 2+→2+ transition over the 2+→0+ transition from the 2-phonon level.

V. THE OCTOPOLE STATE

The octupole vibrational state is now a well-established feature of even–even nuclei.[12] Very little credit for this achievement is due to radioactivity studies, since the octupole states are not strongly populated by beta decay. A survey of the known octupole states in the region from Ni to Ce was made, searching for cases of β decay from 2−, 3−, or 4− parents to 3− states, i.e., allowed decay. Table I contains the results of this survey. Clearly, all of these decays are

---

[20] A. K. Sen Gupta and C. M. Van Patter, Phys. Letters **3**, 355 (1963).

[21] F. Perey, K. Dickens, and R. J. Silva (private communication); H. W. Broek, Phys. Letters **3**, 132 (1962); C. W. Paris and W. W. Buechner, *International Conference on Nuclear Physics, Paris* (1958), edited by P. Gugenberger (C. Lockwood, London, 1959), p. 515.

[22] A. K. Kerman and C. M. Shakin, Phys. Letters **1**, 151 (1962).

[23] R. van Lieshout, R. A. Ricci, R. K. Girgis, Nuovo Cimento **21**, 379 (1961).

[24] Y. Yoshizawa, Phys. Letters **2**, 261 (1962).

hindered in comparison to a normal allowed decay in this region; allowed log ft values are typically between 5 and 6.

Most of the evidence for this type of vibration has come from experiments involving inelastic *p*, *d*, α, or e^- scattering. Figure 6 shows the lowest octupole states for the region of neutron number 26 to 90. The bulk of the data is from the paper of Hansen and Nathan[25]; other references are given in the figure caption.

The variation in the energy of the 3− state with N is much smoother than that of the 2+ quadrupole state. Superimposed on a general decrease with N (or A) are modest upswings at the major shells of 50 and 82 neutrons. The isotopes of tin (50 protons) have their 3− levels only a bare 200 keV above the neighboring chains (cf. Hansen and Nathan[25]). The first 2+ energies for Sn are included in this figure. It is interesting that the tin 3− states exhibit a trend opposite to that of the 2+ states: A minimum followed by a gradual increase. The only isotonic family of 3− states of any length, $N=82$, is plotted in the inset and shows a linear decrease.

Clearly, the octupole state arises principally from collective motion of the entire nucleus; the effect of the "cloud" of particles in unfilled shells is a secondary one. The most striking change in the 3− energy is the sharp decrease at the threshold of the rare earth deformed region. It is not surprising that the octupole state is drastically affected by the strong deformation of the nucleus. Perhaps this abrupt change could better signal the onset of deformation than the slowly decreasing first 2+ state which reflects mainly the behavior of the "valence" or "cloud" nucleons.

ACKNOWLEDGMENTS

The author is grateful to Dr. Raymond K. Sheline, Dr. G. Davis O'Kelley, Dr. Noah R. Johnson, and Dr. Robert J. Silva for their encouragement of this effort, the stimulation of their ideas; and finally to the latter three for their constructive criticism of the manuscript.

---

[25] O. Hansen and O. Nathan, Nucl. Phys. **42**, 197 (1963).