Upper Critical Field of Solid Solution Alloys of the Transition Elements

C. K. JONES and J. K. HULM
Westinghouse Research Laboratories, Pittsburgh, Pennsylvania

B. S. CHANDRASEKHAR
Department of Physics, Western Reserve University, Cleveland, Ohio

I. INTRODUCTION

Superconductors may be divided into two classes, according to whether the penetration depth λ is greater or less than the coherence length ξ. Those with $\lambda > \xi$ have been called London superconductors, and those with $\lambda < \xi$ Pippard superconductors.[1] It has become evident, both from the phenomenological development by Abrikosov[2] of the Ginzburg–Landau[3] theory, and from the work of Gor'kov[4] based on the microscopic BCS theory, that the great majority of superconducting alloys and compounds are of the London type. This is in contrast to most elemental superconductors, which are of the Pippard type except in a very narrow temperature range close to T_c where they show the London type of behavior. Perhaps the best-known property of the London superconductor is the mixed state,[2] which occurs between a lower field H_c and an upper field H_{c2}. In this state (not to be confused with the geometrical intermediate state), there is partial flux penetration into the sample, and the transition to the normal state is completed at H_{c2} which is greater than the thermodynamic critical field H_c. Abrikosov[2] showed that if the G–L order parameter κ is greater than $1/\sqrt{2}$, the mixed state appears, and the upper critical field H_{c2} is given by

$$H_{c2} = \sqrt{2}\kappa H_c\,, \tag{1}$$

where κ may be written in terms of the fluxoid quantum ϕ_0, the actual penetration depth λ_e, and H_c as

$$\kappa = \sqrt{2}\phi_0^{-1}\lambda_e^2 H_c\,. \tag{2}$$

Ginzburg[5] suggested that for London superconductors Eq. (2) may be valid for a wide range of values of the reduced temperature t. If one assumes the Gorter–Casimir temperature dependencies for λ_e and H_c (cf. Tinkham[6]), one gets

$$H_{c2} = 2\sqrt{2}\kappa_1 H_c(0)(1 - t^2)/(1 + t^2)\,, \tag{3}$$

where κ_1 is the value of κ at $t = 1$, and $H_c(0)$ denotes the value of H_c at $t = 0$. We note that Eq. (3) predicts that, at $t = 0$,

$$H_{c2}(0) = 2\sqrt{2}\kappa_1 H_c(0)\,. \tag{4}$$

Gor'kov[4] showed from the microscopic theory that, close to $t = 0$,

$$H_{c2}(0) = 1.77\kappa_1 H_c(0) \tag{5}$$

and for the variation with temperature he suggested the following polynomial, which gives the Abrikosov–Ginzburg result at $t = 1$:

$$H_{c2}(t) = \kappa_1 H_c(0)(1.77 - 0.43t^2 + 0.07t^4)(1 - t^2)\,. \tag{6}$$

Shapoval[7] calculated the upper critical field from the microscopic theory for the case $l \ll \xi_0$, where l is the electronic mean free path, and obtained

$$H_{c2}(0) = 3.03\kappa_1 H_c(0)\,. \tag{7}$$

He found that the temperature dependence of H_{c2} is nearly linear, with a positive second derivative.

It is thus evident that there are three different theoretical predictions both for $H_{c2}(0)$ and for $H_{c2}(t)$. The present experiments were undertaken to see which of the three theories, if any, corresponded most closely to reality.

II. EXPERIMENTAL RESULTS AND DISCUSSION

The alloys which were studied represented a wide range of compositions in the systems, Nb–Ta, Nb–Ti, Nb–Zr, Mo–Re, and V–Ti. The detailed experimental results and their discussion will be published else-

[1] A. A. Abrikosov and I. M. Khalatnikov, Advan. Phys. **8**, 45 (1959).
[2] A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. **32**, 1442 (1957) [English transl.: Soviet Phys.—JETP **5**, 1174 (1957)].
[3] V. L. Ginzburg and L. D. Landau, Zh. Eksperim. i Teor. Fiz. **20**, 1064 (1950).
[4] L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. **37**, 835,1407 (1959) [English transl.: Soviet Phys.—JETP **10**, 593, 998 (1960)].
[5] V. L. Ginzburg, Zh. Eksperim. i Teor. Fiz. **30**, 593 (1956) [English transl.: Soviet Phys.—JETP **3**, 621 (1956)].
[6] M. Tinkham, Phys. Rev. **129**, 2413 (1963).
[7] E. A. Shapoval, Zh. Eksperim. i Teor. Fiz. **41**, 877 (1961) [English transl.: Soviet Phys.—JETP **14**, 628 (1962)].

where; here we merely summarize the results, and compare them with the theories.

The upper critical field was defined as the transverse field at which the smallest detectable voltage appeared across the sample carrying a current of 10 A/cm^2; this corresponded for steady field measurements to 1/1000th of the normal resistance, and for pulsed fields to somewhat higher fractions. The critical field so defined was insensitive to the metallurgical state of the sample (cold work or annealing), and appears to be truly characteristic of the alloy composition. Nevertheless, more experiments are needed to clearly establish the equality of this field to the H_{c2} of the theory. For the present, we assume that the resistive critical field measured as above is equal to H_{c2}, as has also been done by Berlincourt and Hake.[8] The measurements were made for t ranging from about 0.15 to 1.0. Typical results for the Nb–Ti and Nb–Zr alloys are shown in Fig. 1.

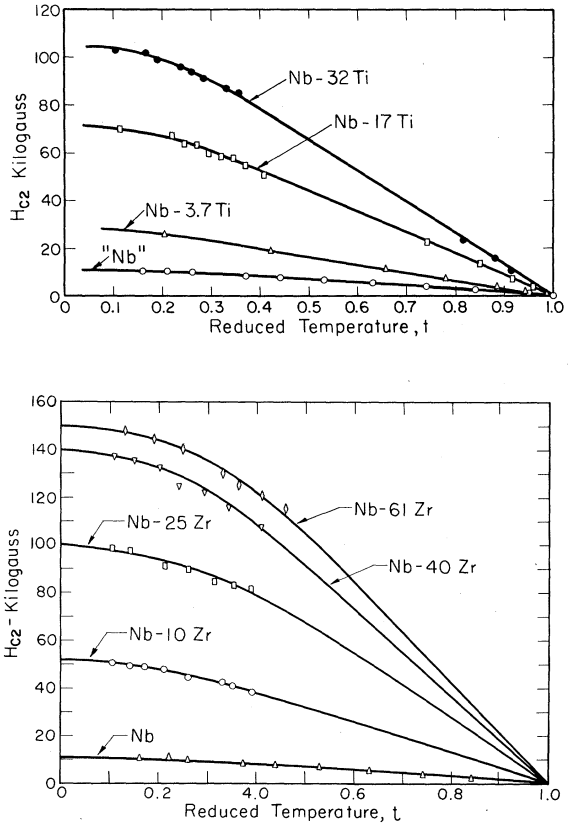

FIG. 1. $H_{c2}(t)$ vs t for Nb–Zr and Nb–Ti alloys.

The comparison with theory was done in two parts: (a) the temperature dependence, and (b) the absolute magnitude of H_{c2}.

[8] T. G. Berlincourt and R. R. Hake, Phys. Rev. **131**, 140 (1963).

(a) The temperature dependence of H_{c2}. This was done by plotting $H_{c2}(t)/H_{c2}(0)$ versus t, where $H_{c2}(0)$ was obtained by a smooth extrapolation of the experimental curves to $t = 0$. The results are shown in Fig. 2 for the Nb–Ti and Nb–Zr alloys. For these, as

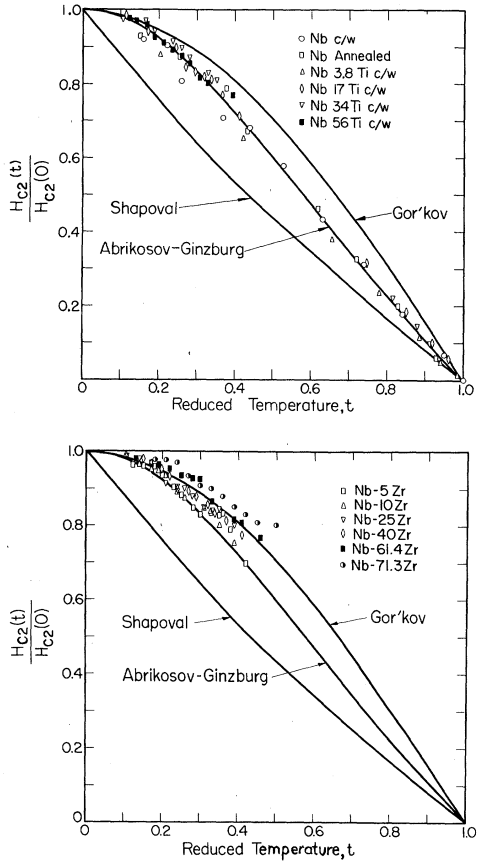

FIG. 2. $H_{c2}(t)/H_{c2}(0)$ vs t for Nb–Zr and Nb–Ti alloys.

well as for the other alloys systems that we studied, it is quite clear that the Shapoval curve is inapplicable. The alloys with lower critical fields lie closer to the Abrikosov–Ginzburg curve, while those with the higher H_{c2} approach the Gor'kov curve. It should be pointed out here that this comparison is independent of the values assumed for H_c and κ_1.

(b) The absolute magnitude of H_{c2}. For this purpose, we need to know $H_c(0)$ and κ_1. We estimated $H_c(0)$ from the BCS relation

$$H_c(0) = (0.17)^{-\frac{1}{2}}\gamma^{\frac{1}{2}}T_c\,, \tag{8}$$

where λT is the electronic specific heat. To estimate κ_1 is somewhat less easy; we chose to use Goodman's[9] extension of the Gor'kov treatment, and wrote

$$\kappa_1 = \kappa_0 + 7.5 \times 10^{-6}\gamma^{\frac{1}{2}}\rho\,, \tag{9}$$

[9] B. B. Goodman, IBM J. Res. Develop. **6**, 63 (1962).

where γ is in ergs cm^{-3} deg^{-2} and ρ is the resistivity in emu; κ_0 was obtained from the value for the pure metal for dilute alloys, and was negligible for the more concentrated ones. It may be remarked here that one can go back to the original expression for κ, Eq. (2), and note that, for a concentrated alloy in which $l \ll \xi_0$, one has[10], where λ_L is the London penetration depth,

$$\lambda_e^2 \approx \lambda_L^2(\xi_0/l) \,. \tag{10}$$

Substitution of Eqs. (10) and (8) into (2) leads essentially to Eq. (9) without the term κ_0.

Using κ_1 as calculated above, $H_{c2}(t)/\kappa_1 H_c(0)$ was plotted as a function of t for all the alloys. The results for the Nb–Ti and Nb–Zr systems are shown in Fig. 3. The conclusions previously drawn are confirmed here: that the Shapoval curve does not fit the experimental results, and that with increasing κ_1 the points shift from the Abrikosov–Ginzburg to the Gor'kov curve. It was also observed that the change-over occurred for most of the alloys at a ρ of about 30 $\mu\Omega$-cm. This leads to a final, somewhat speculative remark that perhaps the change-over occurs when $l \approx \xi_0$. This would explain why Nb-based alloys, with ξ_0 of a few hundred Å, require more heavy alloying before they show Gor'kov behavior, while indium- and lead-based alloys (mentioned in Ref. 8), with their much higher ξ_0, show the Gor'kov behavior at much lower κ_1.

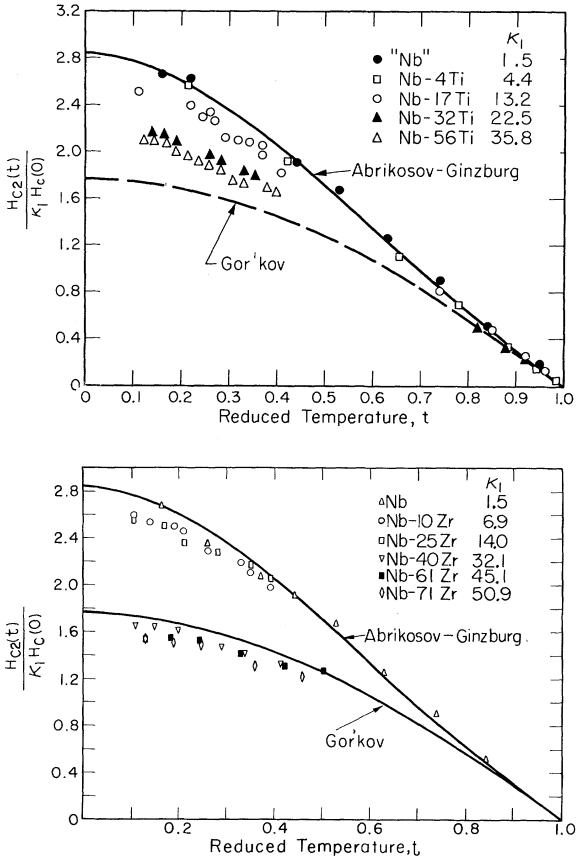

FIG. 3. $H_{c2}(t)/\kappa_2 H_c(0)$ vs t for Nb–Zr and Nb–Ti alloys.

ACKNOWLEDGMENTS

We are grateful to G. Comenetz, J. E. Kunzler, and W. T. Reynolds for the samples, and to A. Patterson for assistance with the experiments.

[10] P. B. Miller, Phys. Rev. **113**, 1209 (1959).