Possible Method of Determining the Moment of Charge of ν_e*

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I. CHARGE FORM FACTOR OF NEUTRINO

The possibility that the neutrinos may have a nonzero electromagnetic form factor has been discussed recently.[1] If one assumes the existence of a charged intermediate boson $W^{\pm}$ for the weak interactions, then the neutrino can acquire a current distribution J_λ through the virtual transition

$$\nu_l \rightleftarrows l^- + W^+ , \qquad (1)$$

where $l^- = e^-$ or μ^-. The matrix element of J_λ between states of a single neutrino may be written as

$$i\gamma_4\gamma_\lambda(1+\gamma_5)F(q^2) , \qquad (2)$$

where $q^2 =$ (4-momentum transfer)^2.

Since the neutrino is a particle of zero mass, its electromagnetic current distribution has no static limit. Nevertheless, it is useful to consider the Fourier transform of $F(q^2)$ in a system where the 4-momentum transfer q_μ has no time-like component. Let

$$\rho(r) = (8\pi^3)^{-1}\int F(q^2)\exp(i\mathbf{q}\cdot\mathbf{r})d^3\mathbf{q} , \qquad (3)$$

where $\mathbf{q}$ and $\mathbf{r}$ are both 3-vectors, $|\mathbf{q}| = (q^2)^{\frac{1}{2}}$ and $|\mathbf{r}| = r$. The function $\rho(r)$ can be regarded as the "spatial" charge distribution of the neutrino. The total charge of ν_l is, of course, zero; i.e.,

$$\int \rho(r)d^3\mathbf{r} = F(q^2=0) = 0 . \qquad (4)$$

The moment of charge M of the neutrino may be defined[2] as

$$M = \int r^2\rho(r)d^3\mathbf{r} , \qquad (5)$$

and is related to $F(q^2)$ by

$$M = -6[\partial F/\partial(q^2)] \quad \text{at} \quad q^2 = 0 . \qquad (6)$$

From (1), it is expected that the charge distribution of the neutrino consists essentially of two parts: a positive core surrounded by a negative cloud. The explicit form of $F(q^2)$ has been calculated,[1] and the moment M is found to be

$$M \cong ea^2[\tfrac{5}{3}\ln(\alpha^{-1}) - \tfrac{8}{3}\ln(m_W/m_l) - 2] , \qquad (7)$$

where e is positive, $(e^2/4\pi) = \alpha = (137)^{-1}$, a^2 is related to the weak interaction Fermi coupling constant G by

$$a^2 = (8\pi^2\sqrt{2})^{-1}(3G) \cong (1.1\times10^{-17}\,\text{cm})^2 , \qquad (8)$$

and m_l, m_W are the masses of l^- and W^+, respectively. The spatial extension of this charge distribution is $\approx 10^{-13} - 10^{-14}$ cm, which is much larger than a. However, the probability of finding the physical neutrino in its virtual charge state $l^- + W^+$ is only $\approx 10^{-6}$. Thus, the magnitude of the moment M becomes only $\approx ea^2$. Because of the different masses of e and μ, the moment M for ν_μ is very different from that of ν_e. If m_W is set to be about its current lower limit[3] 1.5 BeV, then

$$M \cong -15.1\times ea^2 \quad \text{for} \quad \nu_e$$

and

$$M \cong -0.88\times ea^2 \quad \text{for} \quad \nu_\mu . \qquad (9)$$

The magnitude of M becomes larger if m_W turns out to be heavier than 1.5 BeV.

For a static charge distribution which is equal to $\rho(r)$, the electromagnetic field at any point outside the charge distribution is necessarily zero because of the spherical symmetry of $\rho(r)$ and the neutrality condition (4). The electromagnetic interactions between such a charge distribution and other charge particles would appear like a contact interaction, instead of the usual long-range type. The property

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[1] J. Bernstein and T. D. Lee, Phys. Rev. Letters 11, 512 (1963).

[2] For a charged particle such as the proton, the notion of charge radius R is a convenient one, and is related to the moment of charge by $R^2 = (M/e)$. For neutrinos R^2 can be negative and its magnitude does not correspond to the square of the physical extension of the charge distribution. It is possible that the term "charge radius of the neutrino" may lead to some minor confusion.

[3] Proceedings of the International Conference on Elementary Particles, Sienna, 1963 (to be published).

that the field vanishes outside the charge distribution is clearly invariant under any Lorentz transformation. That it is also true for the neutrino can be seen by either directly examining the matrix element (2) or by regarding the neutrino as the limit of a particle with a mass $m_\nu \to 0$. The electromagnetic interaction between the neutrino and any other charged particle, thus, resembles the usual weak interaction but with an amplitude about α^{-1} times weaker.

II. SCATTERING OF NEUTRINO BY ELECTRON

The smallness of this electromagnetic interaction makes it extremely difficult to be observed. For example, the cross section for $\nu_l + p \to \nu_l + p$ is expected to be about α^2 times that for $\nu_l + n \to \mu^- + p$. A possible way to determine experimentally the moment M for the e neutrino is to investigate the reactions[4]

$$\nu_e + e^- \to \nu_e + e^- \tag{10}$$

or

$$\bar{\nu}_e + e^- \to \bar{\nu}_e + e^- . \tag{11}$$

In Fig. 1, we list the various graphs for these re-

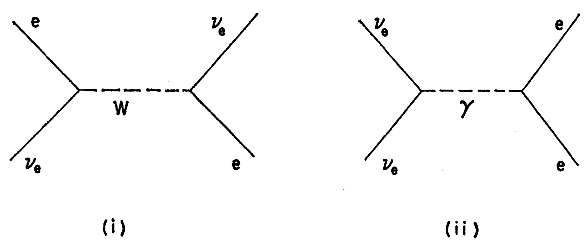

FIG. 1. Diagrams for $\nu_e + e^- \to \nu_e + e^-$ and $\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$.

actions. Diagram (i) represents the process through the exchange of a $W^\pm$ particle and diagram (ii) is that due to the electromagnetic interaction. The interference term between these two graphs makes the effect due to the charge distribution of the neutrino to be only α^{-1} times smaller than that due to the weak interactions.

To the same order in α, we must also consider the radiative correction and the inner bremsstrahlung process

$$\nu_e + e^- \to \nu_e + e^- + \gamma \tag{12}$$

or

$$\bar{\nu}_e + e^- \to \bar{\nu}_e + e^- + \gamma . \tag{13}$$

In this paper we discuss only the emission of soft photons of energy $\leqq \omega_{\max}$. The differential cross section can be written as

$$d\sigma = d\sigma_0 + d\sigma_{\mathrm{M}} + d\sigma_{\mathrm{rad}} + d\sigma_{\mathrm{soft}\gamma} ,$$

where $d\sigma_0$ is due to diagram (i) only, $d\sigma_{\mathrm{M}}$ represents the interference term between diagrams (i) and (ii), $d\sigma_{\mathrm{rad}}$ is the radiative correction to diagram (i) and $d\sigma_{\mathrm{soft}\gamma}$ is due to the emission of soft photons.

The calculations of these cross sections are straightforward. Let $d\sigma_0(\nu_e)$ and $d\sigma_0(\bar{\nu}_e)$ denote the cross sections for reactions (10) and (11), respectively[5]; we find

$$d\sigma_0(\nu_e) = \pi^{-1}G^2 dq^2 \tag{14}$$

and

$$d\sigma_0(\bar{\nu}_e) = \pi^{-1}G^2[1 - (2mk_\nu)^{-1}q^2]^2 dq^2 , \tag{15}$$

where m is the mass of electron, k_ν is the incident neutrino energy in the laboratory system, and q^2 is the (4-momentum transfer)^2 given by the neutrino to the electron. Let E be the energy of the final electron in the laboratory system, then ($\hbar = c = 1$)

$$q^2 = 2m(E - m) . \tag{16}$$

The integrated values $\sigma_0 \equiv \int d\sigma_0$ are given by

$$\sigma_0(\nu_e) = (4G^2/\pi)(m + 2k_\nu)^{-1}(mk_\nu) \tag{17}$$

and

$$\sigma_0(\bar{\nu}_e) = (4G^2/\pi)(m + 2k_\nu)^{-3} \times [mk_\nu^2(m^2 + 2mk_\nu + \tfrac{4}{3}k_\nu^2)] . \tag{18}$$

The interference term $d\sigma_M$ is proportional to the charge moment M of the neutrino:

$$d\sigma_M(\nu_e) = (\pi^{-2}\alpha G^2 dq^2)[(4ea^2)^{-1}M][1 - (4k_\nu^2)^{-1}q^2] \tag{19}$$

and

$$d\sigma_M(\bar{\nu}_e) = (\pi^{-2}\alpha G^2 dq^2)[(4ea^2)^{-1}M]\{[1 - (2mk_\nu)^{-1}q^2]^2 - (4k_\nu^2)^{-1}q^2\} . \tag{20}$$

The effects of the radiative correction are given by

$$d\sigma_{\mathrm{rad}}(\nu_e) = (\pi^{-2}\alpha G^2 dq^2) \times \{I_{\mathrm{rad}} - [1 + (4mk_\nu^2)^{-1}q^2(m - 2k_\nu)][\varphi/\sinh 2\varphi]\} \tag{21}$$

and

$$d\sigma_{\mathrm{rad}}(\bar{\nu}_e) = (\pi^{-2}\alpha G^2 dq^2)\{[1 - (2mk_\nu)^{-1}q^2]I_{\mathrm{rad}} - [1 + (4mk_\nu^2)^{-1}q^2(m - 2k_\nu)][\varphi/\sinh 2\varphi]\} , \tag{22}$$

[4] The possibility that the processes (10) and (11) can occur through the weak interactions have been discussed by R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[5] Apart from a misprint of a factor 2, expressions for $d\sigma_0$ and σ_0 have been given by R. P. Feynman and M. Gell-Mann, Ref. 4.

where

$$I_{\rm rad} = -\varphi \tanh \varphi - (\tanh 2\varphi)^{-1} \int_0^{\varphi} 4\alpha \tanh \alpha d\alpha + 2[1 - (\tanh 2\varphi)^{-1} 2\varphi][-1 + \ln (m/\lambda_{\rm min})] , \quad (23)$$

and

$$q^2 = 4m^2 \sinh^2 \varphi . \quad (24)$$

In the above formulas we neglect (q^2/m_W^2) and (k_ν^2/m_W^2) as compared to 1. The calculation of radiative correction is very similar to that in electrodynamics. For example, $(2\pi)^{-1}\alpha I_{\rm rad}$ is identical with the α-order term in the renormalized vertex function[6] (excluding the vacuum polarization effect) of the $e^- - \gamma$ interaction. As in electrodynamics, $I_{\rm rad}$ is infrared divergent. A fictitious mass $\lambda_{\rm min}$ of the photon is included in (23). The infrared divergence is cancelled by the soft photon emission.[7] For either the ν_e or $\bar{\nu}_e$ process, the rate of emission of soft photons with energy $\leqq \omega_{\rm max}$ is proportional to $d\sigma_0$:

$$d\sigma_{{\rm soft}\gamma} = \pi^{-1} \alpha I_\gamma d\sigma_0 , \quad (25)$$

where[8]

$$I_\gamma = [(\tanh 2\varphi)^{-1} 4\varphi - 2] \ln (\omega_{\rm max}/\lambda_{\rm min}) + [1 - 2 \ln 2] + (2 \tanh 2\varphi)^{-1} \{4\varphi[1 - 2 \ln (\sinh 2\varphi)] + [L(e^{4\varphi}) - L(e^{-4\varphi})]\} , \quad (26)$$

and $L(x)$ is the Spence function

$$L(x) = \int_0^x t^{-1} \ln (|1 - t|) dt . \quad (27)$$

In the above expression we assume the maximum soft photon energy $\omega_{\rm max}$ is much smaller than either m or the final momentum of the electron.

The sum $(I_{\rm rad} + I_\gamma)$ and, therefore, also $(d\sigma_{\rm rad} + d\sigma_{{\rm soft}\gamma})$ are free from infrared divergence. We have

$$I_{\rm rad} + I_\gamma = [1 - (\tanh 2\phi)^{-1} 2\phi] 2 \ln (m/\omega_{\rm max}) - [1 + 2 \ln 2] - \phi \tanh \phi + 2(\tanh 2\phi)^{-1}[-2\phi \ln (2 \cosh \phi \sinh 2\phi) + 3\phi(1 + \phi) - L(e^{-2\phi}) - 2L(-e^{-2\phi})] . \quad (28)$$

To obtain the order of magnitude of these correction terms, we may examine their limiting forms at $q^2 = 0$. In this limit

$$d\sigma_0(\bar{\nu}_e) \to d\sigma_0(\nu_e) , \quad (29)$$

and

$$- d\sigma_M(\nu_e) \to -d\sigma_M(\bar{\nu}_e) = -[(4\pi e a^2)^{-1} \alpha M] d\sigma_0(\nu_e) , \quad (30)$$

which is $\geqq (8.8 \times 10^{-3}) d\sigma_0(\nu_e)$ provided $m_W \geqq 1.5$ BeV. The corresponding value of the radiative correction is smaller than $d\sigma_M$. As $q^2 \to 0$,

$$[d\sigma_{\rm rad}(\bar{\nu}_e) + d\sigma_{{\rm soft}\gamma}(\bar{\nu}_e)] \to [d\sigma_{\rm rad}(\nu_e) + d\sigma_{{\rm soft}\gamma}(\nu_e)] = -(2\pi)^{-1} \alpha d\sigma_0(\nu_e) . \quad (31)$$

Assuming that the weak interaction is symmetric with respect to μ and e, the absolute value of G can be determined from the lifetime[8] τ_μ and the mass m_μ of the muon:

$$\tau_\mu^{-1} = (3 \times 2^6 \times \pi^3)^{-1} m_\mu^5 G^2 \times [1 + \tfrac{3}{5} (m_\mu/m_W)^2 - (8\pi)^{-1} \alpha (4\pi^2 - 25)] , \quad (32)$$

in which we neglect terms of the order of $(m_e/m_\mu)^2$, $\alpha(m_e/m_\mu)$, $(m_\mu/m_W)^2 \alpha \ln \alpha$ and α^2. The magnitude of the charge moment M of ν_e can be determined by either an absolute measurement of the total cross sections for reactions (10) and (11), or by a study of the q^2 dependence of the differential cross sections. It must be emphasized that the above expressions [Eqs. (21)–(28)] for radiative corrections and photon emissions remain *unchanged* if we use the usual Fermi theory, instead of the intermediate boson theory. This is not unexpected since we neglect (m_e^2/m_W^2) and (q^2/m_W^2) in our calculations. The further determination of G from the muon lifetime removes all the difference between these two theories. From an experimental point of view, the proposed method of measuring M, although an indirect one, seems to be relatively independent of our specific theoretical assumptions.

III. EXPERIMENTAL POSSIBILITY

The actual determination of the charge moment of ν_e is undoubtedly difficult, since neither the fundamental process (10) nor (11) has been observed. Nevertheless, we may envisage[9] using a very strong K-capture or $\beta^{\pm}$-radioactive source which is surrounded first by a thick shield and then by a massive layer of detectors. For definiteness, we may consider the K-capture of Zn^{65} which emits ν_e at

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[6] See, e.g., R. P. Feynman, Phys. Rev. **76**, 769 (1949).

[7] F. Bloch and A. Nordsieck, Phys. Rev. **52**, 54 (1937); H. A. Bethe and J. R. Oppenheimer, Phys. Rev. **70**, 451 (1946). See D. R. Yennie, S. C. Frantschi, and H. Suura, Ann. Phys. **13**, 379 (1961) for more recent references.

[8] T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959). S. M. Berman, Phys. Rev. **112**, 267 (1958). The I_γ is identical with Eq. (C.1.) in the paper by Kinoshita and Sirlin.

[9] The considerations given in this section are results of our discussions with R. Novick, M. Schwarz, and C. S. Wu.

1.4 MeV with a ~50% branching ratio. A megacurie of Zn^{65} can give, after a shield of ~1 m thickness, a rate of ~100 reactions per day per ton of detector weight. If the problem of background can be successfully solved, the determination of the moment of charge of ν_e may become feasible after an accumulation of about a few times 10^4 events.

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An Approximation Method for Diffraction Problems*

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I. INTRODUCTION

Many of the problems of physics result in integral equations with displacement kernels where the integration is over some restricted region of space. Usually a simple closed form solution is not obtainable. The best approach, in general, is then probably to find the solution by numerical computation. However, in case the region is either very small or very large (in some sense) it is to be expected that simple and accurate approximate formulas may be obtained by analytic methods. The "small" case has been somewhat more intensively investigated than the "large" one. It is to the latter that we devote our attention here.

As a kind of "minus first" approximation we might take the region to cover all space. This is, however, frequently much too crude. The effect to be investigated is then identically zero. For example, in the problem of the diffraction by a large circular disk discussed below such an approximation would consist of considering the disk to be infinite. Then, of course, there is no field on the side away from the incident direction. The effects we are looking for exist precisely because there are edges to the region.

A suitable zeroth-order approximation does suggest itself whenever the boundary of the region is such that its radius of curvature can always be considered "large." Then, on separating variables, the problem is roughly that of solving a one-dimensional displacement integral equation with limits going from zero to infinity. (The appropriate variable is the coordinate normal to the boundary surface.) Such Wiener–Hopf integral equations are known to be soluble. The approach to be discussed leads automatically to such a zeroth-order approximation. Higher order approximations are systematically and simply obtainable.

While the method has been found useful for a large class of problems it turns out that minor, though obvious, modifications must be made depending on the particular kernels involved. To avoid cumbersome general discussions we felt the best exposition would be a detailed treatment of some of the very simplest and most familiar problems. These arise in considering the diffraction of waves by simple obstacles. [These problems are "simplest" in that: (a) The necessary Wiener–Hopf decompositions can be found by inspection; and (b) The resulting solutions can be expressed in terms of elementary functions.]

Accordingly we treat below two problems. The first involves the diffraction produced by a "large" slit or strip, i.e., we ask for the solution of the scalar wave equation subject to the conditions of having a given incident plane wave and satisfying Dirichlet or Neumann boundary conditions on a strip (or the walls of a slit). This problem is discussed in considerable detail. The complete solution is obtained including zeroth and first approximations and shown to involve only known elementary functions. Historically, the zeroth approximation obtained here was first found by Schwarzchild.[1] However, Schwarzchild's approach is so cumbersome as to be difficult to carry beyond this approximation. Further, his method seems not to be generalizable to other geometrical situations.

The second problem we treat, diffraction by a circular disk or aperture, is designed to show that the

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[1] K. Schwarzchild, Math. Ann. 55, 177 (1920).