path and κ. We believe that the theoretical agreement demonstrates the existence of the Abrikosov–Goodman[4,9] flux structure in cold-worked Nb for this regime of penetration. If this is so, the prediction that the flux structure is weakly dependent on temperature but strongly dependent on field should make this technique useful for improving the resolution of the details of the magnetization.

ACKNOWLEDGMENTS

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Axial Torque in Trained Superconducting Wires in a Transverse Magnetic Field

B. H. HEISE

Research Laboratories, Linde Division, Union Carbide Corporation, Tonawanda, New York

The work of several investigators has indicated that when a magnetic field penetrates certain hard superconductors the field is pinned in the material.[1–3] These superconductors, therefore, have pinning energies associated with them. Magnetic flux pinned in a superconductor gives rise to a trapped magnetic moment, and from the measured value of the magnetic moment the total pinned flux can be calculated. Furthermore, the pinning energies are derivable from measurements of the relaxation of the moment. These measurements can be made by means of a torsion balance which records the torque of the specimen as a function of the angular position of the applied magnetic field. Such a study is reported here.

APPARATUS

The specimen on which the torque study was made was a 5-ft length of Wah Chang Nb–25% Zr 10-mil wire. The wire was wound longitudinally on a 15-in. Bakelite rod in such a manner as to minimize the inductance of the circuit. The wire was enclosed in spaghetti and cemented firmly in place on the rod. The rod was attached, by the electrical contacts of the superconductor, to a stiff coaxial member which served to conduct the current and to transmit the torque to the outside of the Dewar system. Mercury cups were used as external electrical contacts. The member was suspended in the Dewar system by a single thread to reduce to a negligible value the torque from nonmagnetic contributions. Lever arms were attached to the member and were connected by a removable harness of light wires and pulleys to a balance and a counter weight. The Dewar system with its suspended torsion member was placed in a 0–10-kG Varian magnet which could be rotated in the horizontal plane through 180°.

EXPERIMENTAL

When the specimen is exposed to or trained in a magnetic field, it develops a magnetic moment which interacts with the field and gives rise to a torque. The torque is

$$\tau = MLg\,,$$

where M is the mass measured on the balance, L is the lever arm length, and g is the acceleration of gravity. The torque, in turn, is equal to the vector product of the magnetic moment $\mathbf{m}$ of the specimen and the field $\mathbf{H}$. Therefore,

$$\tau = mH \sin \alpha\,, \qquad (1)$$

where α is the angle between $\mathbf{m}$ and $\mathbf{H}$. Since the magnetic moment is equal to the pole strength or total flux times the average length of the trapped flux lines and since the flux is equal to B times the area of the specimen perpendicular to the flux, the following expression results

$$B = (\tau/\sin \alpha)/HV\,, \qquad (2)$$

where V is the volume of the specimen. The total number of flux quanta is, then,

$$n_T = BA/\varphi_0\,,$$

where A is the area of the specimen perpendicular to the flux and φ_0 is the amount of flux per quantum.

The amount of trapped flux retained by the specimen can be determined in the following manner. The specimen is trained in a field H by driving the wire normal several times with current. The field is then

[1] C. P. Bean and R. W. Schmitt, Science **140**, 26 (1963).
[2] P. W. Anderson, Phys. Rev. Letters **9**, 309 (1962).
[3] D. C. Freeman, Jr. (private communication).

reduced to zero and reset to 250 G. The torque, when recorded as a function of the field angle, is sinusoidal as predicted by Eq. (1). The trapped induction can, therefore, be calculated from any one of the points by Eq. (2). The trapped magnetic inductions for experiments where $H = 2$, 5, and 10 kG were calculated to be 1.79, 1.90, and 1.69 kG, respectively, for one set of experiments. The range in which these numbers fall is reproducible and extends from 1.6 to 1.9 kG for all investigations made. However, the amount of flux which a wire can trap at field H and retain at field zero is apparently random and independent of the field.

When the specimen is trained as before in a field greater than 250 G and when this field is rotated about the specimen, a different set of data results. The torque, when plotted versus the magnet angle, reaches a maximum at less than 90° then tails off in a shallow descent toward some constant, nonzero value of torque. This type of behavior is expected if B is changing. That is, if a specimen is trained in a given field, and if the field is not removed, the specimen will enclose an amount of flux proportional to the field. This amount of flux is greater than would be trapped if the specimen were merely exposed to the field. If the field is then rotated, its component along the original training direction decreases and hence the trapped flux in the specimen must decrease through expulsion of flux. Together with the changing B of the specimen, the angle α also does not maintain its equality with θ. The relaxation of α is responsible for the fact that the torque approaches a nonzero value as the magnet angle increases. This problem will be considered later.

When the specimen is only exposed to the field H and not trained, the resulting torque versus magnet angle function rises monotonically to a maximum value during the rotation of the field. The maximum is maintained for all further rotation. Such functions are shown in Fig. 1(a) for $H = 2$ and 5 kG. The amount of flux penetrating the specimen in this instance can be expected to be constant with respect to α, and the following relation will hold true:

$$\tau/\sin \alpha = \text{const} = \tau_{\max}$$

for a given field. The magnetic induction is, therefore, constant and the deviation of the data of the untrained specimen from proportionality to $\sin \theta$ must be due solely to the relaxation of $\mathbf{m}$ and not its depletion. The explanation for this is that the interaction of $\mathbf{m}$ and $\mathbf{H}$ has become sufficiently strong that $\mathbf{m}$ is caused to slip by means of the breaking of the flux pins. The angle α must then be defined as the angle by which $\mathbf{m}$ lags $\mathbf{H}$.

The experimental procedure for obtaining torque versus α data for fields higher than 250 G is as follows: The untrained specimen is exposed to a magnetic field H at magnet angle zero. The field is then rotated to some position θ at which point the force is measured on the balance. The weighing harness is then lifted free of the torque member. This allows the specimen to rotate through the angle α and allows $\mathbf{m}$ to align itself with $\mathbf{H}$. Figure 1(b) shows graphs of the angle α versus the magnet angle as measured in the above manner for 2 and 5 kG. Notice these functions are nearly identical except for scale to their counterparts in Fig. 1(a).

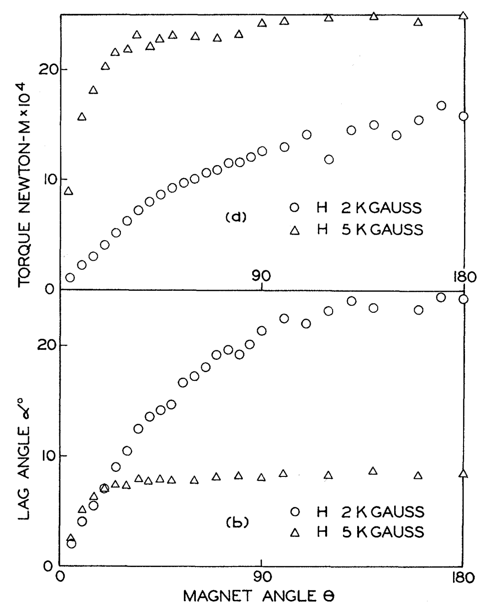

FIG. 1. (a) Torque values of the specimen measured as a function of the magnet angle for fields of 2 and 5 kG. (b) Angle between m and H measured as a function of the magnet angle for field of 2 and 5 kG.

Obviously, since $\mathbf{m}$ can be made to relax and follow $\mathbf{H}$ for sufficiently high fields, then the breaking of flux pins must be responsible. The effective stress which a pinning site can withstand without breaking must be related to the reversible energy in the specimen due to the rotation of the field. The energy of the total number of pins is merely the integral of the torque function and is given by the following expression:

$$\Delta E = \int_0^{\alpha_{\max}} \tau(\beta) d\beta \,, \tag{3}$$

where β is an integration variable in the direction of

α. Since τ is proportional to the sin α, Eq. (3) becomes

$$\Delta E = (\tau/\sin \alpha_{\max})[1 - \cos \alpha_{\max}] .$$

Written on the fluxoid basis, this energy becomes

$$\Delta E_{\text{per fluxoid}} = [(\tau/\sin \alpha_{\max})/n_T][1 - \cos \alpha_{\max}] = \varphi_0 l H[1 - \cos \alpha_{\max}] . \quad (4)$$

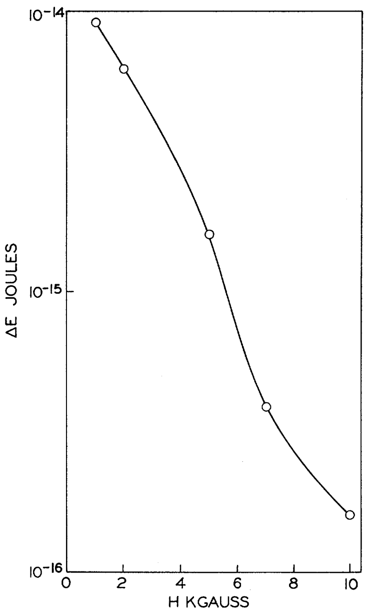

FIG. 2. Energy available to pin one fluxoid in the material measured as a function of the applied magnetic field.

The fact that $\alpha \neq \theta$ even for angles $\theta < \alpha_{\max}$ as shown in Fig. 1(b) implies that, as the torque increases, the flux seeks more stable pinning positions in the specimen. Since relaxation occurs regardless of the angle of the field, the fluxoids can be assumed to deform in such a manner as to equalize the energy on each element along the length of the fluxoid under the stress of the externally applied force couple. Hence, the energy given by Eq. (4) is taken to be equal to the elastic energy of the entire fluxoid under stress and not just the ends of the fluxoid under stress.

Measurements of the torque and $\alpha_{\max}$ have been made at H equal to 1, 2, 5, 7, and 10 kG. These data, together with values of B calculated from Eq. (2), are given in Table I. The energies, as calculated from Eq. (4), are shown in Fig. 2.

TABLE I. Measured values of the maximum torque, maximum lag angle, and calculated values of magnetic induction for various magnetic fields.

	H kG 1	2	5	7	10
$\tau_{\max}$ newton-m $\times 10^4$	2.97	22.0	193	412	592
$\alpha_{\max}{}^{\circ}$	44	26	8	3.3	1.8
B kG	0.45	1.6	5.8	8.8	8.9

DISCUSSION

Two parameters are measured by the experiment discussed here. These are the torque of the specimen as a function of the lag angle and the maximum lag angle. From these values, two physical properties of the specimen can be calculated, viz., the total trapped magnetic induction and the energy per fluxoid which holds the flux pinned to the material. The values calculated for B are not in good agreement with applied fields from which they arose; however, this discrepancy can be explained by the fact that at these low fields appreciable shielding of the sample occurs; therefore, the field which interacts with the magnetic moment is not the measured amount but rather the value at the surface of the specimen, which is somewhat higher. The effect of this variation on the energy calculations is less important since $1 - \cos \alpha_{\max}$ is far more rapidly varying than is the applied field.

In order to account for the presence of a pinning energy, the assumption may be made that the specimen is not of uniform critical field across its cross section, rather that some of the material, particularly regions of precipitation and high strain, remains normal at all times. Such regions would act as pinning sites since no energy is required in penetrating them with flux. The calculations based on this model are awkward since the energy change in driving the adjoining, already partially penetrated, superconductor normal is difficult to formulate in the absence of flux bundle size and distribution information. An attempt is being made, however, to construct such a model based on the data presented here.

Discussion 6

BEAN: Have you noticed any creep effects in the torque as a function of time in the saturated torque? Have you waited for long periods of time?

B. H. HEISE, *Linde Research Laboratories:* These measurements were made at a constant rate. That is, the magnet was rotated and the data taken at a set time. There is a time dependence, seemingly exponential, but I have not measured it.