

The Universal Fermi Interaction and the Conserved Vector Current in Beta Decay*

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INTRODUCTION

As early as 1948 the great similarity in the strength of the coupling constants in beta decay, muon decay, and muon capture had tempted several authors¹⁻⁴ to postulate a universal Fermi interaction. This view was later reviewed extensively in a series of lectures⁵ by Fermi. However, any universality should imply not only the equality of the coupling constants, but also the similarity of the structure of the interactions. For a long time there was no conclusive evidence to substantiate the latter. Then came the discovery of the nonconservation of parity in beta decay and also in μ decay in 1957. Following the tremendous activity of that period, overwhelming evidence showed that the interaction form of four fermions such as in nuclear beta decay and muon decay can be represented by a $(V-A)$ interaction.⁶⁻⁹ Recent evidence indicates strongly that this also holds for muon capture.¹⁰ Furthermore, from experiments on the $0^+ \rightarrow 0^+$ transition in O^{14} , the vector coupling constant g_V in beta decay was found to be very nearly (within 1% or 2%) equal to that of the Fermi constant g_μ of μ decay. The excellent agreement is by no means a blessing, but rather a puzzle! In beta decay one expects strong renormalization effects from the virtual emission and reabsorption of pions and baryons. On the other hand, the renormalization effect does not occur in muon decay because no pionic effect exists

there. Then, why should this agreement be so good?

To explain this unexpected good agreement, Feynman and Gell-Mann,¹¹ and earlier Gershtein and Zeldovich,¹² proposed the conserved vector current theory (CVC theory) based on its analogy in electromagnetism where the observed coupling strength “ e ” with electromagnetic field is the same for all particles coupled. This universality of electric charge follows from the fact that the electromagnetic current is conserved. If the weak vector current is similarly conserved, then the vector coupling constant would be a universal constant.

Because of the fundamental importance of the CVC theory, several types of experiments have been proposed, designed, and executed to verify the validity of the CVC hypothesis. So far, all the results are strongly in favor of the CVC theory.

In this paper we discuss only experimental results concerning beta decay. First, we show the theoretical formulation of the $(V-A)$ fermi interaction. The requirements of the continuity equation of the electromagnetic current are discussed in order to lead to the formulation of the CVC theory and the unique properties arising from it. Then we examine the present status of the question of equality between g_V and g_μ . Finally, we present and discuss the four different types of experiments and their results.

THEORETICAL FORMULATION OF THE $(V-A)$ FERMION INTERACTION

If the interaction were pure $(V-A)$,¹³ it would be interesting to explore some possible theoretical arguments which would lead to such a linear combination. In fact, the $(V-A)$ form has been reached independently by three different theoretical approaches, all based on the principal idea of representing the 4-component spinor ψ in terms of two 2-component spinors ϕ_+ and ϕ_- . To allow only one of the two 2-component spinors to appear in the interaction, dif-

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¹ T. D. Lee, R. Rosenbluth, and C. N. Yang, *Phys. Rev.* **75**, 905 (1948).

² O. Klein, *Nature*, **161**, 897 (1948).

³ J. Tiomno and J. A. Wheeler, *Rev. Mod. Phys.* **21**, 153 (1949).

⁴ G. Puppi, *Nuovo Cimento* **6**, 194 (1949).

⁵ E. Fermi, *Elementary Particles* (Yale University Press, New Haven, Connecticut, 1951).

⁶ C. S. Wu, *Rev. Mod. Phys.* **31**, 783 (1959); *Theoretical Physics in the Twentieth Century*, edited by M. Fierz and V. F. Weisskopf (Interscience Publishers, Inc., New York, 1960), pp. 249-303.

⁷ M. Deutsch and O. Kofoed Hansen, *Experimental Nuclear Physics*, edited by E. Segre (John Wiley & Sons, Inc., New York, 1959), Vol. 3.

⁸ E. J. Konopinski, *Ann. Rev. Nucl. Sci.* **9**, 99 (1959).

⁹ H. Schopper, *Fortschr. Phys.* **8**, 328 (1960).

¹⁰ L. Wolfenstein, in “Proceedings of International Conference on the Fundamental Aspects of The Weak Interactions at Brookhaven National Laboratory, 1963” (summary talk).

¹¹ R. Feynman and M. Gell-Mann, *Phys. Rev.* **109**, 193 (1958).

¹² S. S. Gershtein and J. B. Zeldovich, *Zh. Eksperim. i Teor. Fiz.* **29**, 698 (1955) [English transl.: *Soviet Phys.—JETP* **2**, 576 (1955)].

¹³ This is not exactly correct in nuclear beta decay where the axial vector coupling is slightly stronger $(V-1.2A)$, probably due to mesonic effects.

ferent hypothetical principles were proposed to justify its restriction.

These theoretical approaches are:

- (1) The chirality invariance conjectured by Sudarshan and Marshak.¹⁴
- (2) The two-component formulation of Dirac spinors by Feynman and Gell-Mann.¹⁵
- (3) The mass-reversal invariance proposed by Sakurai.¹⁶

(1) Chirality Invariance

The word “chirality” (pronounced as kirality) was derived from the Greek word “chir” (hand). Therefore, it can be used to imply handedness.

The chirality transformation is defined as $\psi \rightarrow \gamma_5 \psi$. For a particle of a given momentum, the Dirac equation has four solutions, each of them a 4-component spinor. Now for a mass zero particle, e.g., a neutrino, of these four solutions, two have positive chirality, i.e., $\gamma_5 \psi = \psi$, and two have negative chirality $\gamma_5 \psi = -\psi$.

Next let us consider the case $m \neq 0$. Here, of course, the general Dirac spinor is not an eigenstate of the operator γ_5 . However, it can be expanded in terms of such eigenstates.

We can write

$$\psi = \psi_+ + \psi_-, \tag{1}$$

where $\psi_{\pm} = \frac{1}{2} (1 \pm \gamma_5) \psi$.

In terms of 2-component spinors I, ϕ, ξ , we have

$$\gamma_5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$$

and

$$\psi = \begin{pmatrix} \phi \\ \xi \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \phi - \xi \\ -(\phi - \xi) \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \phi + \xi \\ \phi + \xi \end{pmatrix}. \tag{2}$$

Thus

$$\psi_+ = \begin{pmatrix} \phi_+ \\ -\phi_+ \end{pmatrix}, \text{ and } \psi_- = \begin{pmatrix} \phi_- \\ \phi_- \end{pmatrix},$$

where $\phi_{\pm} = \frac{1}{2} (\phi \pm \xi)$.

To express this in words; if one projects with the positive chirality operator, one obtains the 2-component spinor ϕ_- ; using the negative chirality operator yields the 2-component spinor ϕ_+ .

At this point, Sundarshan and Marshak¹⁴ made a bold conjecture that *the total 4-fermion interaction*

should be invariant under a γ_5 transformation on any of the ψ 's, $\psi_i \rightarrow \gamma_5 \psi_i$.

The interesting consequence of this assumption of chirality invariance is that the interaction is now uniquely determined to be (V,A) .

Consider the 4-fermion interaction

$$g(\bar{\psi}_2 \mathbf{O} \psi_1)(\bar{\psi}_4 \mathbf{O} \psi_3).$$

Make the transformations

$$\psi_i \rightarrow \gamma_5 \psi_i \text{ and } \bar{\psi}_j \rightarrow -\bar{\psi}_j \gamma_5. \tag{3}$$

Then chirality invariance implies $\bar{\psi}_j \mathbf{O} \psi_i \equiv -\bar{\psi}_j \gamma_5 \mathbf{O} \gamma_5 \psi_i$ where \mathbf{O} is any operator. We have

$$\mathbf{O} \gamma_5 = \mathbf{O} \text{ and } -\gamma_5 \mathbf{O} = \mathbf{O} \text{ or } [\mathbf{O}, \gamma_5]_+ = 0.$$

That means \mathbf{O} and γ_5 anticommute.

Of the five operators (S, V, T, A, P) only V and A anticommute with γ_5 . $S, T,$ and P commute with γ_5 . Therefore, the operator \mathbf{O} must be a linear combination of V and A ,

$$\mathbf{O} = a\gamma_\mu + b\gamma_\mu \gamma_5.$$

From the condition

$$\mathbf{O} \gamma_5 = \mathbf{O}, \quad \mathbf{O} = a\gamma_\mu \gamma_5 + b\gamma_\mu = a\gamma_\mu + b\gamma_\mu \gamma_5.$$

This gives $a = b$ or $\mathbf{O} = a\gamma_\mu(1 + \gamma_5)$.

The interaction is thus

$$g\{\bar{\psi}_2 \gamma_\mu (1 + \gamma_5) \psi_1\} \{\bar{\psi}_4 \gamma_\mu (1 + \gamma_5) \psi_3\}. \tag{4}$$

Of course, we could have retained $(1 - \gamma_5)\psi$ instead of $(1 + \gamma_5)\psi$; then we would have (S, T, P) instead of (V, A) . Theoretically, these two possibilities are equally good. It is the experimental evidence which has the final say.

(2) The Two Component Formulation of the Dirac Spinors¹⁵

As we have shown that for $m \neq 0$, the ψ_+ and ψ_- cannot be decoupled in the Dirac equation, it is rather mystical to write down the interaction involving only ψ_+ and not ψ_- for every particle. The great contribution by Feynman and Gell-Mann in this respect is to explain this situation by showing that the Dirac equation can also be expressed in terms of the 2-component wavefunction. However, the 2-component wavefunction must satisfy a second-order Klein-Gordon equation. Once one accepts this viewpoint, then the hypothetical principles which were proposed to restrict the interaction term to (V,A) seem to be more reasonable.

Let's write down the first-order Dirac equation for the 4-component field ψ in terms of Dirac matrices,

$$H\psi = \alpha \cdot p\psi + \beta m\psi. \tag{5}$$

¹⁴ R. Marshak and G. Sudarshan, Padua-Venice International Conference, 1957; Phys. Rev. **109**, 1860 (1958).

¹⁵ R. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

¹⁶ J. J. Sakurai, Nuomo Cimento **7**, 649 (1958).

We can express the 4-component wavefunction ψ in terms of the 2-component spinors ϕ and ξ . Then

$$H\phi = \sigma \cdot p\xi + m\phi, \quad H\xi = \sigma \cdot p\phi - m\xi. \quad (6)$$

Adding and subtracting these equations, we obtain

$$H\phi_+ = \sigma \cdot p\phi_+ + m\phi_-, \quad H\phi_- = -\sigma \cdot p\phi_- + m\phi_+, \quad (7)$$

where the ϕ_+ and ϕ_- were defined above.

If $m = 0$, these equations are decoupled, so the functions ψ_+ , ψ_- , which are eigenstates of the chirality operator γ_5 are also eigenfunctions of the Dirac equation. For $m \neq 0$, the two equations are coupled. However, the ϕ_+ and ϕ_- satisfy the Klein-Gordon equation since

$$\phi_+ = (1/m)(H + \sigma \cdot p)\phi_-, \quad (8)$$

we have

$$\begin{aligned} m^2\phi_- &= m(H - \sigma \cdot p)\phi_+ = (H - \sigma \cdot p)(H + \sigma \cdot p)\phi_- \\ &= [H^2 - (\sigma \cdot p)^2]\phi_- = (H^2 - p^2)\phi_-, \end{aligned} \quad (9)$$

or

$$[(\partial^2/\partial t^2) - \nabla^2 + m^2]\phi_- = 0. \quad (10)$$

This is the well-known Klein-Gordon equation. In other words, although ϕ_+ does not appear in the theory explicitly, nevertheless it appears implicitly via Eq. (8), expressed in terms of ϕ_- and its derivatives. This implies that the whole theory can be expressed in terms of a 2-component wavefunction, either ϕ_- or ϕ_+ , which, however, must satisfy the K-G equation.

On experimental grounds, we know that the 4-fermion interaction formulated in terms of ψ 's is *linear in the fields and does not contain derivatives*. (This was the reason that the Konopinski-Uhlenbeck modification was rejected.) Now, an arbitrary interaction form, even if it is linear in the ψ , in general, involves both ψ_+ and ψ_- , or in terms of 2-component wavefunctions, both ϕ_- and ϕ_+ . If expressed in terms of ϕ_- alone, the interaction must contain terms proportional to ϕ_- and also to $p\phi_- \sim \partial\phi/\partial x$. If, however, we *insist* that no such derivative terms should appear, then the interaction, formulated in terms of the ψ , must contain only ψ_+ and not ψ_- , i.e., only ϕ_- not ϕ_+ (or vice versa). This requirement is identical with that resulting from chirality invariance.

(3) Mass-Reversal Invariance

Consider the behavior of the Dirac equation

$$\gamma_\mu p_\mu \psi = im\psi \quad (11)$$

under the transformation $\psi \rightarrow \gamma_5 \psi$.

Since γ_5 anticommutes with each γ_μ , we have

$$\gamma_\mu p_\mu (\gamma_5 \psi) = -im(\gamma_5 \psi). \quad (12)$$

Thus, $\gamma_5 \psi$ is not an eigenfunction of the Dirac equation, unless we also make the transformation $m \rightarrow -m$. The Dirac equation is then invariant under the combined "mass reversal" transformation

$$\psi \rightarrow \gamma_5 \psi, \quad m \rightarrow -m. \quad (13)$$

When one applies this transformation to each of the four fermions simultaneously and demands that the interaction be invariant, then it is equivalent to γ_5 invariance.

(4) Connection with the $V-A$ Interaction

All these hypotheses discussed above are equivalent to the assumption that the beta interaction occurs only in states of positive chirality, i.e., negative helicity. The requirement of negative helicity, i.e., left-handed polarization for both neutrinos and electrons (in positive energy states) implies the existence of a (V,A) combination in beta decay, even though the coefficients are arbitrary. If we also require that the nucleons involved are left-handedly polarized (if their rest mass could be neglected), then the interaction is uniquely fixed as $V-A$.

By using the relations $\gamma_5(1 + \gamma_5) = (1 + \gamma_5)\gamma_5 = (1 + \gamma_5)$, the interaction can be rewritten as follows:

$$\begin{aligned} &g\{\bar{\psi}_2\gamma_\mu(1 + \gamma_5)\psi_1\}\{\bar{\psi}_4\gamma_\mu(1 + \gamma_5)\psi_3\} \\ &= g\{[(\bar{\psi}_2\gamma_\mu\psi_1)(\bar{\psi}_4\gamma_\mu(1 + \gamma_5)\psi_3)] \\ &\quad + [(\bar{\psi}_2\gamma_\mu\gamma_5\psi_1)(\bar{\psi}_4\gamma_\mu\gamma_5(1 + \gamma_5)\psi_3)]\}, \end{aligned} \quad (14)$$

since γ_μ and $i\gamma_\mu\gamma_5$ are usually the vector and axial vector operators, respectively, we then have the $(V-A)$ combination. This universal $(V-A)$ 4-fermion interaction gives the unique combination $(V-A)$, yields 2-component neutrinos of negative helicity, leads to the conservation of leptons, and is invariant under the combined inversion of "CP".

The universal $(V-A)$ interaction differs from the vector interaction originally proposed by Fermi

$$(\bar{\psi}_2\gamma_\mu\psi_1\bar{\psi}_4\gamma_\mu\psi_3) \quad (15)$$

only by the presence of the extra factor $(1 + \gamma_5)$. It is remarkable how close Fermi came to the correct beta decay interaction long before the understanding of beta decay was as far advanced as it is today.

THE CONSERVED VECTOR CURRENT HYPOTHESIS

As mentioned in the introduction, because of the strong coupling between the nucleons and pions, the coupling constants in the old theory of beta decay need renormalization. Nucleons can emit and absorb virtual pions such as $n \rightarrow n + \pi^0 \rightarrow p + \pi^- \rightarrow n + \pi^+ + \pi^- + \dots$. Therefore, a neutron exists for

only a fraction of its lifetime as a bare neutron; the rest of its life it exists as a proton surrounded by a negatively charged pion cloud or as a neutron surrounded by a neutral pion cloud, etc. The neutron in the latter state is called a dressed or physical neutron to differentiate it from a bare neutron. In the old beta theory, only the bare nucleon is assumed to undergo beta decay, not the dressed nucleon. Therefore, a nucleon undergoes beta decay for only a fraction of its lifetime. In the old theory, therefore, the effective coupling strength of a nucleon must be proportionately reduced or renormalized by the fraction of time spent as a dressed nucleon.

On the other hand, a muon does not have strong interactions. Its Fermi interaction strength needs no renormalization. Therefore the effective coupling constant in muon decay should equal the intrinsic one. What mystified people was that the effective interaction strength of the vector couplings in both beta decay and muon decay were found to be equal within $\sim 2\%$ (see the section on $g_V = g_\mu$). Therefore the question rose: why is no renormalization required between the effective and the bare interaction strength in beta decay? To explain the unexpectedly good agreement, Feynman and Gell-Mann's approach to explaining the equality of vector β -interaction strength in nuclear β decay and μ decay is to assume that the pions carry with them the beta interaction strength when they are virtually emitted from the nucleons (Fig. 1) and that the vector part of the

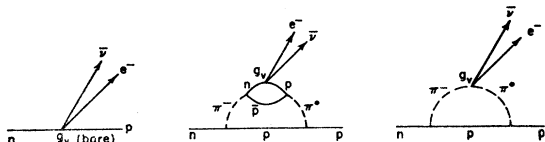


FIG. 1. Feynman diagram of the decay of the physical neutron.

nuclear beta interaction is so arranged as to have no renormalization effects.

(1) Analogy With Electromagnetism

The fact that the vector interaction in beta decay appears to be unaffected by pionic corrections has its analogy in electromagnetism. The electron is believed to be a simple Dirac particle with no charge distribution, i.e., essentially a point charge (except for small radiative corrections of order $\alpha/2\pi \sim 10^{-3}$), while the proton is a very complicated object containing a meson cloud surrounding a bare nucleon core. Yet the total charge of the proton, which one measures in electron-proton scattering at very low

energies, is the same as the proton charge one would measure if there were no pion interaction. As a matter of fact, *all* interactions are arranged in such a way that the equality between the physical electric charge and the bare charge is not disturbed, so that the electric charge of the proton is the same as the electric charge of the positron (of course in the presence of pion interactions, the charge of the nucleon core *alone* is not the same).

How is this equality achieved in electromagnetism? First, electric charge conservation holds in the process

$$p \leftrightarrow n + \pi^+,$$

i.e., the π^+ has the same charge as the proton. Secondly, even while the proton is in the "dissociated" state, the interaction of the π^+ with the electromagnetic field is the same as that of the proton (Fig. 2).

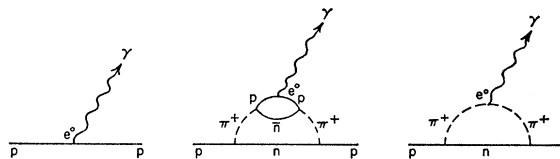


FIG. 2. Feynman diagram of the E-M interaction of the physical proton.

Mathematically, the vector potential A_μ couples to the *conserved charge current* which consists of the sum of the p and π^+ currents.

Of course, if the pion interaction with the electromagnetic field were different from the proton interaction, such as happens for the magnetic moment, this conservation law would not hold. Thus the magnetic moment of the physical proton differs from that of the bare proton.

(2) The Conserved Electromagnetic Current

The charge current for a proton is a polar vector whose four components are given by

$$\begin{aligned} i_\mu &= \psi_p^+ \alpha_\mu \psi_p = \rho(v_\mu/c) \quad \text{for } \mu = 1,2,3 \\ &= i\psi_p^+ \psi_p = i\rho \quad \text{for } \mu = 4, \end{aligned} \tag{16}$$

in units of the electric charge e .

In covariant notation we have, apart from a factor i ,

$$i_\mu = \bar{\psi}_p \gamma_\mu \psi_p. \tag{17}$$

Of course, for neutrons there is no charge current.

We can combine the results for proton and neutron in terms of isotopic spin operators. For proton and neutron, we have, respectively, $\tau_z = +1$, and -1 . Thus

$$i_\mu = \bar{\psi}_N \gamma_\mu \frac{1}{2} (1 + \tau_z) \psi_N, \tag{18}$$

where ψ_N represents a general nucleon wavefunction. The nucleon current can be decomposed into an isotopic spin scalar and isotopic spin vector

$$i_\mu = \frac{1}{2} \underbrace{\tilde{\psi}_N \gamma_\mu \psi_N}_{\text{isoscalar}} + \frac{1}{2} \underbrace{\tilde{\psi}_N \gamma_\mu \tau_z \psi_N}_{\text{isovector}} = i_\mu^S + i_\mu^V. \quad (19)$$

The isoscalar term satisfies the continuity equation:

$$\partial i_\mu^S / \partial x_\mu = \nabla \cdot i_\mu^S + \partial \rho / \partial t = 0. \quad (20)$$

The conservation of isoscalar current implies the conservation of the number of nucleons. However, the second term which is the z component of an isotopic spin vector is not conserved by itself, but only if it is supplemented by the pion term, i.e.,

$$J_\mu^z = \frac{1}{2} \tilde{\psi}_N \gamma_\mu \tau_z \psi_N + [\pi \times (\partial \pi / \partial x_\mu)]_z + \dots \quad (21)$$

(3) The Formulation of the Conserved Vector Current Theory (CVC)

For a conventional vector beta interaction, the nucleon current is given by

$$J_\mu^+ = (1/\sqrt{2}) \tilde{\psi}_N \gamma_\mu \tau_+ \psi_N \quad \text{for } \beta^- \text{ decay}, \quad (22)$$

where

$$\begin{aligned} \tau_+ \psi_n &= [(\tau_x + i \tau_y) / \sqrt{2}] \psi_n = \sqrt{2} \psi_p, \\ \tau_+ \psi_p &= 0, \end{aligned} \quad (23)$$

and similarly

$$J_\mu^- = (1/\sqrt{2}) \tilde{\psi}_N \gamma_\mu \tau_- \psi_N \quad \text{for } \beta^+ \text{ decay}. \quad (24)$$

These currents are very similar to the electromagnetic isovector current J_μ^z . The J_μ^+ , J_μ^z , J_μ^- are the three components of one and the same isotopic spin current J_μ .

It was suggested by Feynman and Gell-Mann¹¹ that, just as for electromagnetism, we must supplement the nucleonic current by a pionic term, i.e., that not only J_μ^z , but also J_μ^+ and J_μ^- contain a pionic vector current,

$$J_\mu^+ = (1/\sqrt{2}) \tilde{\psi}_N \gamma_\mu \tau_+ \psi_N + [\pi \times (\partial \pi / \partial x_\mu)]_+ + \dots \quad (25)$$

Physically, this is equivalent to attributing the same beta interaction strength to the direct pion-lepton as to the baryon-lepton vertex, as shown in Fig. 1. Since the strong interactions are charge-independent, we have conservation of isotopic spin T , a generalization of conservation of charge, i.e., of T_z . Thus the Feynman-Gell-Mann hypothesis amounts to the assumption that the total isotopic spin current including both nucleonic and pionic terms, is conserved.

It is interesting to recall here the comments which Gershtein and Zeldovich¹² made at a time when the

Fermi part of the beta interaction was believed to be scalar rather than vector. They wrote that "It is of no practical significance but only of theoretical interest if the interaction is vector type; then g_V (bare) = g_V (effective). No renormalization can be foreseen by analogy with Ward's identity for the interaction of a charged particle with the electromagnetic field; in this case, virtual processes involving particles do not lead to charge renormalization of the particle."

The analogy between the β interaction and electromagnetism is illustrated by the following correspondences.

TABLE I. Correspondences between beta interaction and electromagnetism

	Electrodynamics	Vector-type β interaction
Coupling constant	e	$(1/\sqrt{2})g_V$
Current	J_μ^z	J^+
Field potential	A_μ	$\tilde{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_\nu$
Interaction Hamiltonian	$e J_\mu^z A_\mu$	$(1/\sqrt{2})g_V J_\mu^+ \tilde{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_\nu$

CONNECTION BETWEEN WEAK AND ELECTROMAGNETIC FORM FACTORS

The conserved vector current theory relates the isovector weak interaction form factors uniquely to the well-known isovector electromagnetic form factors which have been extensively measured by the electron scattering experiments.

Now the matrix element of the isotopic vector part of the electromagnetic interaction between two nucleons can be written as

$$M_V^{E-M} = e \bar{u}_2 \left[F_Q^{E-M}(q^2) \gamma_\mu + \frac{\mu_p^a - \mu_n^a}{2M} F_M^{E-M}(q^2) \sigma_{\mu\nu} q_\nu \right] u_1; \quad (26)$$

q is the momentum transfer; $\sigma_{\mu\nu} = \frac{1}{2} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$; μ_p^a and μ_n^a are the anomalous magnetic moments of the proton and neutron; $\mu_p^a \cong 1.79$ and $\mu_n^a = -1.91$ nuclear Bohr magneton. F_Q^{E-M} and F_M^{E-M} are the well-known isovector parts of the charge and magnetic form factors for the nucleon. In the limit $q^2 \rightarrow 0$, $F_Q^{E-M}(0) = 1$ and $F_M^{E-M}(0) = 1$.

In analogy, the matrix element for the conserved vector current in weak interaction is given by

$$M_V^W = (g_V/\sqrt{2}) \bar{u}_2 [f_1^V(q^2) \gamma_\mu + f_2^V(q^2) \sigma_{\mu\nu} q_\nu] u_1, \quad (27)$$

where $f_1^V(q^2)$ and $f_2^V(q^2)$ are the form factors in the vector part of the weak interaction. The unique correspondences between these two sets of form factors bring out significant consequences.

(1) Renormalizability of the Vector Current

$f_1^V(q^2)$ should behave like $F_0^{E-M}(q^2)$. Since $F_0^{E-M}(0) = 1$, which is the static limit of the charge, we should also have $f_1^V(0) = 1$. This implies that the vector coupling constant of the weak interaction in the CVC theory need not be renormalized.

(2) "Weak Magnetism Term"

$f_2^V(q^2)$ should behave like $(\mu_p^a - \mu_n^a)/2MF_M^{E-M}(q^2)$. This implies that the anomalous magnetic term which is due to the pion cloud must also appear in weak interactions. In fact, a relation between $f_2^V(0)$ and $f_1^V(0)$ in weak interactions can be obtained from the corresponding relation in electromagnetic interaction:

$$\frac{\text{Weak Interaction}}{f_2^V(0)} = \frac{\text{EM Interaction}}{(\mu_p^a - \mu_n^a)/2M} \cdot \frac{f_1^V(0)}{1} \quad (28)$$

Since the relation of the term f_2^V to the term f_1^V in weak interactions is equivalent to the relation of the magnetism term to the charge term in the electromagnetic interaction, the f_2^V term is the corresponding magnetism term in weak interactions and has been called "Weak Magnetism" by Gell-Mann. This second relation enables one to interrelate beta-decay transitions to electromagnetic transitions in nuclei. The investigation of the triad B^{12} , C^{12} , and N^{12} is interpreted mainly by this relation.

It can also be seen that, in high-energy neutrino processes where high q values enter, one can actually determine the form factors by measuring the high-energy neutrino-capture cross section. Preliminary results from CERN (reported at International Conference on the Fundamental Aspects of the Weak Interactions at Brookhaven National Laboratory, September, 1963, and Sienna Conference, October, 1963) appear to be consistent with the form factors deduced from electron scattering.

THE CONSEQUENCES OF THE CVC HYPOTHESIS AND THE SIGNIFICANT EXPERIMENTAL EVIDENCE

We now review the observed evidence from the five different types of experimentation. These are

1. The equality of the coupling constants g_V in β decay and g_μ of μ decay.
2. The equivalence of the weak vector form factors and the electromagnetic form factors. A possible evidence can come from a study of the beta spectra of B^{12} and N^{12} .
3. Determination of the absolute decay rate of $\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$.

4. $(\beta - \alpha)$ angular correlation measurements in Li^8 and B^8 and
5. $\beta - \gamma$ (circularly polarized) correlation of the mirror transitions $Na^{24} \xrightarrow{\beta^-} Mg^{24} \xleftarrow{\beta^+} Al^{24}$.

All these experiments are extremely difficult ones but their results are all strongly in favor of the CVC hypothesis.

(1) Equality of g_V and g_μ

How good is the equality between g_V for β decay and g_μ for μ decay? At present, it appears that the two coupling constants are indeed nearly equal, with g_μ perhaps 2% to 3% larger. However, the accuracy of some of the small theoretical corrections is not beyond question.

The situation can be briefly summarized as follows.

Fermi Coupling Constant in Beta Decay g_V

ft values:

The constant g_V can be deduced from the ft values of a series of superallowed $0^+ \rightarrow 0^+$ transitions by the following relation

$$g_V^2 |M_F|^2 ft = 2\pi^3 \hbar^7 (\ln 2) / m_0^5 c^4 \quad (29)$$

The $|M_F|^2$ for $0^+ \rightarrow 0^+$ transitions is equal to 2 in the absence of charge-dependent effects. In order to calculate the ft values accurately, one must have:

(a) *Accurate measurements of maximum beta energy and half-lives of these transitions:* Five of these transitions (O^{14} , Al^{26*} , Cl^{34} , V^{46} , and Co^{54}) have been carefully determined by measuring nuclear Q values.^{17,18,19} These results are summarized in Table II.

(b) *F functions:* To calculate the value f , one must integrate the β spectrum. To do this one has to rely on the computed tables of the Fermi function F . Unfortunately, the widely used Fermi function tables²⁰ by the National Bureau of Standards (NBS) were computed based on a point charge nucleus and retaining only the leading term in the expansion of the Fermi function. For precision results, several additional factors must be taken into account in evaluating the Fermi function, such as the finite charge distribution of the nucleus, the orbital electron screening, etc. The nuclear size correction has been recently

¹⁷ R. K. Bardin, C. A. Barnes, W. A. Fowler, and P. A. Seeger, *Phys. Rev.* **127**, 583 (1962). (O^{14}).

¹⁸ J. Jänecke, *Phys. Letters* **6**, 69 (1963) (V^{46}).

¹⁹ J. M. Freeman, J. H. Montagne, D. West, and R. E. White, *Phys. Letters* **3**, 136 (1962). J. M. Freeman, R. E. White, J. H. Montagne, G. Murray, and W. E. Burcham, in *Proceedings of Second International Conference on Nuclidic Masses*, Vienna, 1963.

²⁰ *Tables for the Analysis of Beta Spectra*, National Bureau of Standards, Applied Mathematics Series No. 13 (U. S. Government Printing Office, Washington, D. C., 1952).

TABLE II. The five accurately measured $0^+ \beta^+ 0^+$ transitions and their calculated ft values: $(ft)_I$ a point charge nucleus; $(ft)_{II}$ with nuclear size effect, Rose's screening formula and radiative correction from (30). The values for O^{14} , Al^{26*} , and Cl^{34} from (22); $(ft)_{III}$ with nuclear size effect, Reitz's screening correction, and radiative correction from (30).

$0^+ \beta^+ 0^+$ Transition	$t_{\frac{1}{2}}$ (sec)	E_{\max} (keV)	ft			Ref.
			I	II	III	
${}^8O^{14} \rightarrow {}^7N^{14}$	71.36 ± 0.09	1812.6 ± 1.4	3066 ± 10	3127	3173	17,22
${}^{13}Al^{26*} \rightarrow {}^{12}Mg^{26}$	6.374 ± 0.016	3208.0 ± 2.3	3015 ± 12	3086	3135	19,22
${}^{17}Cl^{34} \rightarrow {}^{16}S^{34}$	1.565 ± 0.007	4460 ± 4.5	3055 ± 20	3140	3176	19,22
${}^{23}V^{46} \rightarrow {}^{22}Ti^{46}$	2.424 ± 0.002	6041 ± 7	3011 ± 25	3138	3172	18,30
${}^{27}Co^{54} \rightarrow {}^{26}Fe^{54}$	0.1937 ± 0.0010	7229 ± 5	2966 ± 18	3134	3165	19,30

calculated by several authors²¹⁻²³ and the results are all in good agreement. There is no such accord for the screening corrections given by the detailed calculations of Reitz²⁴ and the approximate method of Rose.²⁵ The discrepancy in ft values resulting from screening corrections varies from 2% in O^{14} to 1% in Co^{54} . This screening correction is now being re-examined by several authors.^{25a}

(c) *Radiative corrections*^{26,27}: Because of various radiative processes—photon emission and absorption, both virtual and real, bremsstrahlung, etc.—one must also apply radiative corrections in the β decay. Unfortunately the theory diverges,

$$\frac{\Delta T_{\beta}}{T_{\beta}} = -\frac{3e^2}{2\pi} \ln \frac{\Lambda}{E_{\max}(\beta^+)}, \quad (30)$$

Λ is infinity, if you integrate to infinity. Fortunately the nucleons have a structure so the upper limit may be in the neighborhood of a proton mass. Kinoshita and Sirlin used the proton mass as the cut off of the integral and obtained

$$(\Delta ft)_{0^+ \beta^+ 0^+} / (ft)_{0^+ \beta^+ 0^+} \cong +1.7\%,$$

or

$$\Delta g_V / g_V \cong -0.8\%.$$

Finally, relativistic corrections,²⁸ competition from

²¹ B. S. Dzhelepov and L. N. Zyrianova, *Influence of Atomic Electrical Fields on Beta Decay* (Izvestia Akademii Nauk SSSR, Moscow, 1956), in Russian.

²² L. Durand, L. F. Landovitz, and R. B. Marr, *Phys. Rev.* **130**, 1188 (1963).

²³ C. P. Bhalla and M. E. Rose, *Table of Electronic Radial Functions at the Nuclear Surface and Tangents of Phase Shifts*. (ORNL 3207, 1961). Also *Phys. Rev.* **128**, 774 (1962). A more complete table is in press.

²⁴ J. R. Reitz, *Phys. Rev.* **77**, 10 (1950).

²⁵ M. E. Rose, *Phys. Rev.* **49**, 727 (1936).

^{25a} *Note added in proof.* The electron screening corrections to beta decay spectra have been reinvestigated recently by L. Durand III and are found to be in agreement with those obtained by Rose and by Longmire and Brown, but in marked disagreement with those obtained by Reitz. The latter results appear to be incorrect. The author wishes to thank Dr. Durand III for communicating his results to her prior to its publication.

²⁶ T. Kinoshita and A. Sirlin, *Phys. Rev.* **113**, 1652 (1959).

²⁷ S. M. Berman and A. Sirlin, *Ann. Phys. (N. Y.)* **20**, 20 (1962).

²⁸ A. Altman and W. M. MacDonald, *Phys. Rev. Letters* **1**, 456 (1958).

K capture, and contributions from second forbidden matrix elements²⁹ were estimated and found to be too small to be relevant.

To compare the magnitude of these different corrections, Freeman *et al.*³⁰ calculated the ft values for

$(ft)_I$ a point charge nucleus;

$(ft)_{II}$ with nuclear size effect, Rose's screening formula, and radiative correction of Kinoshita, Sirlin, and Berman;

$(ft)_{III}$ with nuclear size effect, Reitz's screening correction, and radiative correction of Kinoshita, Sirlin, and Berman.

These three different calculations of ft values are listed in Columns 4, 5, and 6 of Table II, respectively. It is interesting to note the extreme uniformity of the ft values of the five transitions (within 2%) in spite of the large range of Z . Apparently the atomic number has very little effect on the matrix element.

(d) *Impurity corrections to $|M_F|^2$* : We have mentioned that for $0^+ \rightarrow 0^+$ transitions, $|M_F|^2 = 2$. This is true only for perfect isotopic spin states. It is expected that the Coulomb interaction between protons, which violates isotopic spin conservation, will distort the nuclear wavefunctions to some extent, so that the nuclear matrix element is decreased. Detailed calculations of this effect^{31,32} indicate that the effect in O^{14} is likely to be very small. The upper limit of $\Delta|M|^2/|M|^2 < \frac{1}{4}\%$. However, Weidenmüller³² pointed out that, should an appreciable configuration mixing of higher states be present at the $1p$ -shell states involved in the decay, then the Coulomb effects may be boosted up to 1 to 2%. However, there have been no experimental demonstrations of the presence of such mixing yet. Meantime, the introduction of a small percentage of charge-dependent nuclear potential (i.e., different

²⁹ B. J. Blin-Stoyle, V. Gupta, and J. S. Thompson, *Nucl. Phys.* **14**, 685 (1960).

³⁰ J. M. Freeman, R. E. White, J. H. Montagne, G. Murray and W. E. Burcham, *Phys. Letters* **8**, 115 (1964).

³¹ W. M. MacDonald, *Phys. Rev.* **110**, 1420 (1958), and in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press, Inc., New York, 1959).

³² H. A. Weidenmüller, *Phys. Rev.* **128**, 241 (1962).

p - p and p - n potential of electromagnetic but non-Coulomb origin), was investigated by Blin-Stoyle and Le Tourneux.³³ It was shown that even a very small departure from charge symmetry could lead to corrections of the desired amount in the matrix element. However, MacDonald and Altman³⁴ calculated the possible magnitude of the charge-dependent nuclear potential from the existing experimental data and found it to be too small to give the required reduction of the nuclear matrix element. So the question is by no means resolved yet.

If $|M_F|^2 = 2$ is assumed and the unweighted average ft value from O^{14} , Al^{26*} , Cl^{34} , V^{46} , and Co^{54} is used in calculation, Freeman *et al.*³⁰ obtained

(1) with Rose's screening formula:

$$(ft)_{av} = 3125 \pm 10 \text{ sec};$$

$$g_V = (1.4029 \pm 0.0022) \times 10^{-49} \text{ erg cm}^3;$$

(2) with Reitz's screening corrections (see footnote 25a):

$$(ft)_{av} = (3164 \pm 8) \text{ sec};$$

$$g_V = (1.3943 \pm 0.0017) \times 10^{-49} \text{ erg cm}^3.$$

g_μ of Muon Decay

To obtain g_μ , one can use the muon lifetime equation

$$1/T_\mu = g_\mu^2 m_\mu^5 c^4 / 192 \pi^3 \hbar^7, \quad (31)$$

where the latest measurements^{35,36} gave

$$m_\mu = (206.768 \pm 0.003)m_0,$$

$$T_\mu \text{ (uncorrected)} = 2.198 \pm 0.001 \mu \text{ sec}.$$

The radiative correction was estimated^{26,27} to be

$$\Delta T_\mu / T_\mu \cong -0.42\% \quad \text{or} \quad \Delta g_\mu / g_\mu \cong +0.21\%.$$

Therefore g_μ with the radiative correction is

$$g_\mu = (1.4350 \pm 0.0011) \times 10^{-49} \text{ erg cm}^3.$$

Discussion

From the values of g_V and g_μ , we obtain

$$(g_\mu - g_V) / g_\mu = (2.2 \pm 0.15)\%$$

using Rose's screening correction,

$$= (2.8 \pm 0.12)\%$$

using Reitz's screening correction.

Indeed, if the equality of $g_V = g_\mu$ is required by the CVC theory, then the observed discrepancy of 2%–3% may appear to be in contradiction to the CVC theory. On the other hand, the ft values of the four transitions (O^{14} , Cl^{34} , V^{46} , and Co^{54}) are so remarkably constant, it makes one wonder how reliable the radiative corrections to the β decay or the impurity corrections to the $|M_F|^2$ are. After all, the radiative correction for β decay is calculated purely for bare nucleon and not for nucleons inside of a complex nucleus. How justified is this assumption? On the top of all these uncertainties, one now discovers that one also needs a more accurately computed Fermi function with proper electron screening corrections. Some time ago, it was also suggested that if the intermediate vector boson W does exist, then the lifetime of the muon is modified depending on the mass of the intermediate vector boson.³⁷ The effect due to the existence of W particle can be expressed by

$$[T_\mu]_W = [T_\mu] \times \left\{ 1 - \frac{3}{5} \left(\frac{m_\mu}{m_W} \right)^2 + \left(\frac{3\alpha}{\pi} \right) \ln \left(\frac{m_W}{m_N} \right) \right. \\ \left. + O \left[\left(\frac{m_N}{m_W} \right)^2 \alpha \right] \right\}.$$

The term $O[(m_N/m_W)^2\alpha]$ has been evaluated recently and found to be extremely small of the order of 0.1% or less.^{38a} The second term $\frac{3}{5} (m_\mu/m_W)^2$ could have saved the situation if the mass of the vector boson is only slightly heavier than the K meson. However, the recent CERN high-energy neutrino experiments have shown that the mass of the vector meson probably lies between 1.25–1.65 BeV.^{38b} Therefore the correction due to this term is now reduced to 0.2–0.4% on the lifetime of muon decay, corresponding to only 0.1–0.2% on the muon coupling constant. This certainly does not help the matter as one would have hoped for.

In view of these uncertain theoretical corrections involved in comparing $g_V = g_\mu$, one probably should not be lightly discouraged by the existing discrepancy of $\sim 2\%$.

On the other hand, some intriguing ideas to explain this observed discrepancy have been suggested by Cabibbo³⁹ based on an analysis of leptonic decays using unitary symmetry for strong interactions.

³³ R. J. Blin-Stoyle and J. Le Tourneux, *Ann. Phys.* **18**, 12 (1962); R. J. Blin-Stoyle and S. C. K. Nair, *Phys. Letters* **7**, 161 (1963).

³⁴ A. Altman and W. M. MacDonald, *Nucl. Phys.* **35**, 593 (1962).

³⁵ G. Charpak, F. J. M. Farley, R. I. Garwin, T. Muller, J. C. Sens, and A. Zichichi, *Phys. Letters* **1**, 16 (1962).

³⁶ F. J. M. Farley, T. Massam, T. Muller, and A. Zichichi, in *Proceedings of International Conference on High Energy Physics* (CERN, Geneva, 1962), p. 415.

³⁷ T. D. Lee and C. N. Yang, *Phys. Rev.* **119**, 1410 (1960); S. Oneda and J. C. Pati, *Phys. Rev. Letters* **2**, 125 (1959).

³⁸ (a) T. D. Lee (private communication). (b) CERN High Energy Neutrino Experiments. *Proceedings of International Conference on Weak Interactions*, Sept., 1963, at Brookhaven National Laboratory. Also *Sienna International Conference on Elementary Particles*, Sept. 30–Oct. 5, 1963 (in press).

³⁹ N. Cabibbo, *Phys. Rev. Letters* **10**, 531 (1963); **12**, 62 (1964).

Since this is outside of the scope of this review, I do not go into the details of Cabibbo's proposal. The general idea may be stated as follows. It is assumed that the weak current J_μ of strong interacting particles consists of two parts. One part is due to the strangeness conserving current of $\Delta S = 0$, $\Delta Q = 1$ and the other is due to the $\Delta S = \Delta Q = 1$ current. Their isospin selection rules are, respectively, $\Delta I = 0$ or 1 and $\Delta I = \frac{1}{2}$. The relations between these two currents are expressed in terms of a parameter " θ ." The vector coupling constant for β decay which represents only $\Delta S = 0$ decay, is, therefore, not the coupling const g but $g \cos \theta$ where θ is determined from leptonic hyperon decays. Since θ determined from experimental data is around 0.26, this gives a correction of 6.6% to the ft values of Fermi transitions in the right direction to eliminate the discrepancy between g_V and g_μ . While this is only a tentative exploration, it does seem to explain both (1) the smaller g for $\Delta S = 1$ transitions and (2) the apparent $g_\mu > g_V$. However, further developments are required for its justification.

One should now turn to an examination of the evidence for the CVC theory from other experimental investigations.

(2) The B^{12} - C^{12} - N^{12} Experiment

As we have pointed out, the CVC theory relates the nucleon current in the vector part of the beta interaction to the charge current in electromagnetism. Although the effective coupling strength is not renormalized by pionic corrections, a nucleon also possesses a magnetic moment which is greatly altered by the pion cloud. Physically, for a given charge, the bare pion carries a larger magnetic moment, on account of its smaller mass. This is responsible for the anomalous nucleon magnetic moment. The CVC theory implies that such anomalous magnetic moment terms must also appear in beta decay.

(a) Theoretical Expectation

A very ingenious way to test the effect of the CVC theory in beta decay was suggested by Gell-Mann⁴⁰ and was successfully observed in various laboratories.⁴¹⁻⁴³ This test involves the beta and gamma

⁴⁰ M. Gell-Mann, Phys. Rev. **111**, 362 (1958).

⁴¹ T. Mayer-Kuckuk and F. C. Michel, Phys. Rev. **127**, 545 (1962).

⁴² N. W. Glass and R. W. Peterson, Phys. Rev. **130**, 299 (1963).

⁴³ Y. K. Lee, L. W. Mo, and C. S. Wu, Phys. Rev. Letters **10**, 253 (1963). The Fig. 4 shown in this paper is recalculated by using Bhalla and Rose's²³ new table of Fermi functions and the new end-point energy of N^{12} .⁵¹

transitions in the $A = 12$ nuclei as indicated in Fig. 3. There are three transitions from the three levels of an isotopic multiplet $T = 1$, $I = 1$ to the common

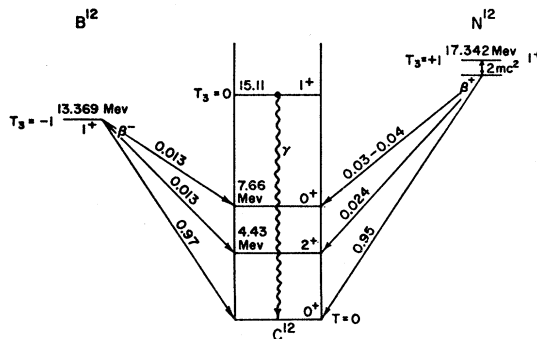


FIG. 3. Energy levels and decay schemes of B^{12} - C^{12} - N^{12} triad.

ground state of C^{12} : the β^- transition from B^{12} , the corresponding β^+ transition from N^{12} , and the γ transition from C^{12} .

Let us consider the β^- transition from the $T_z = -1$ level. The dominant contribution should come from the axial vector current, in allowed order. Its effective matrix element is

$$g_A M_{GT} \{ e^+ \sigma [(1 + \gamma_5) / \sqrt{2}] \nu \}_z. \quad (32)$$

Now we consider the forbidden corrections from both V and A interactions. However, the matrix element of the V interaction of J_μ^+ from the $T_z = -1$ state is now uniquely related to that of J_μ^z of the analogous electromagnetic transition from the $T_z = 0$ state. For the γ transition ($\Delta I = -1$, no), one expects magnetic dipole effects to dominate and since $\Delta I = -1$, only the isotopic vector part of electromagnetic current will contribute. All orbital currents are negligible as shown by the detailed calculations of Weidenmüller.⁴⁴

Therefore

$$M_{e1} = -(\mu e / 2M) (\nabla \times \mathbf{A})_z, \quad (33)$$

where μ is the transition magnetic moment in units of the proton Bohr magneton ($e/2M$) and $\nabla \times \mathbf{A}$ is evaluated at the nucleus.

The matrix element from the vector β interaction is

$$-\sqrt{2} \frac{\mu e}{2M} \frac{g_V}{e} \left[\nabla \times \left(e^+ \alpha \frac{1 + \gamma_5}{\sqrt{2}} \nu \right) \right]_z. \quad (34)$$

It is important to point out here that the forbidden vector interaction is now determined by the transition magnetic moment μ in the analogous gamma

⁴⁴ H. Weidenmüller, Phys. Rev. Letters **4**, 299 (1960).

transition. The transition magnetic moment μ can be calculated from the observed radiation width τ_γ of the $T = 1$ state in C^{12} by the relation $\Gamma_\gamma = [\mu^2/3(137)] \times (W^3/M^2)$, where W is the gamma ray energy, M is the mass of nucleon, and $\Gamma_\gamma = 53 \pm 11$ eV.⁴⁵ This gives $\mu = 2.2$. It is also important to demonstrate that the anomalous magnetism plays a dominant role in this M1 transition. This can be shown by using the relation

$$\mu = (1/\sqrt{2})(\mu_p - \mu_n) \int \underline{\sigma}.$$

The value of $(\mu_p - \mu_n)$ deduced from the empirical $\int \underline{\sigma} \cong 0.8$ and $\mu \cong 2.2$ is ~ 4.0 , which agrees fairly well with the difference $\mu_p - \mu_n = 4.7$. Since in the old theory of beta decay the pions were not coupled to leptons and therefore possessed no β -emitting power, the large anomalous magnetic moments coming from the pion clouds had no claim to be in beta decay.

For completeness, one must also consider the lowest "forbidden" corrections coming from the axial vector current, for example, corrections due to the gradient of the lepton fields. Summing up, β^\pm -transition matrix element in the above cases is made up from the allowed A interaction plus forbidden corrections from the A interaction plus or minus a forbidden matrix element, from the V interaction.

$$-g_A M_{GT} \{ e^+ (\underbrace{\sigma}_{\text{allowed } A \text{ term}} - i \underbrace{a(\mathbf{k} \times \boldsymbol{\alpha})}_{\text{interference term between } A \text{ and } V} - b \underbrace{\mathbf{k} \gamma_5}_{\text{correction in } A \text{ term}}) \cdot \underline{\nu} \} \quad (35)$$

where

$$\alpha \equiv \frac{\mu}{\sqrt{2}M} \left| \frac{g_V}{g_A} \right| \left| \frac{1}{M_{GT}} \right|.$$

The spectrum can be represented by the standard allowed shape, multiplied by a correction factor

$$1 + \frac{8}{3} a \left(E - \frac{E_0}{2} - \frac{m^2}{2E} \right) + \frac{2}{3} b \left(E_0 - \frac{m^2}{E} \right). \quad (36)$$

For fast β^- particles, the b term fortunately does not contribute to the spectrum shape. One reduces the correction factor to

$$(1 + \frac{8}{3} aE).$$

For the β^+ transition, one obtains the same result except the sign in front of the energy term changes to negative. The correction factor is $(1 - \frac{8}{3} aE)$ for β^+ .

⁴⁵ E. Hayward and E. G. Fuller, Phys. Rev. 106, 991 (1957).

This change of sign comes about because the coefficient " a " results from a $(V - A)$ interference which changes sign from β^- to β^+ transition. Therefore the ratio of the B^{12} and N^{12} correction factors is

$$R(E) = 1 + \frac{16}{3} aE.$$

More extensive calculations on the spectral deviation from the allowed Fermi shapes for B^{12} and N^{12} were also carried out by Morita,⁴⁶ Gell-Mann and Berman,⁴⁷ and Huffaker and Grueling.⁴⁸ All the contributions due to weak magnetism, electromagnetic correction, finite de Broglie wavelength effect and second forbidden matrix elements of various coordinate and momentum types were included. These calculations turned out to be in very good accord. The curves showing correction factor vs energy in Morita's and Huffaker's calculations exhibit very slight curvatures. The slopes $a^-(B^{12})$ and $a^+(N^{12})$ are about equal and opposite in sign. The final theoretical factors

$$\begin{aligned} A(\text{CVC}) &= a^-(B^{12}) - a^+(N^{12}) = (1.10 \pm 0.17)\% \\ &\text{per MeV,} \\ A(\text{Fermi}) &= 0.10 \text{ per MeV.} \end{aligned}$$

The $A(\text{Fermi})$ is the calculated correction factor ratio based on old Fermi theory. The expected difference between $A(\text{CVC})$ and $A(\text{Fermi})$ is very striking. This experiment, therefore, is a very suitable test for the CVC hypothesis.

(b) *Experimental Confirmation*

The shape factors for the β spectra of B^{12} and N^{12} have been measured by several laboratories. The B^{12} and N^{12} nuclei are produced on electrostatic accelerators using the reactions $B^{11}(d,p)B^{12}$ and $B^{10}(\text{He}^3,p)N^{12}$, respectively.

The first measurements were made by Mayer-Kuckuk and Michel at California Institute of Technology⁴¹ and by Glass and Peterson at Los Alamos.⁴² Both groups found a ratio A in general agreement with the CVC, i.e., $1.30 \pm 0.31\%/MeV$ and $1.62 \pm 0.28\%/MeV$, respectively. However, the deviations of the shape factors of the *individual* spectra were not accurately determined in the early measurements.

Recently, Lee, Mo, and Wu⁴³ at Columbia University have measured the two spectra with their iron-free intermediate-focusing magnetic spectrometer and found that the deviation of the shape

⁴⁶ M. Morita, Phys. Rev. 113, 1584 (1959).

⁴⁷ M. Gell-Mann and S. M. Berman, Phys. Rev. Letters 3, 99 (1959).

⁴⁸ J. N. Huffaker and E. Grueling, Trans. N. Y. Acad. Sci. 24, 591 (1962).

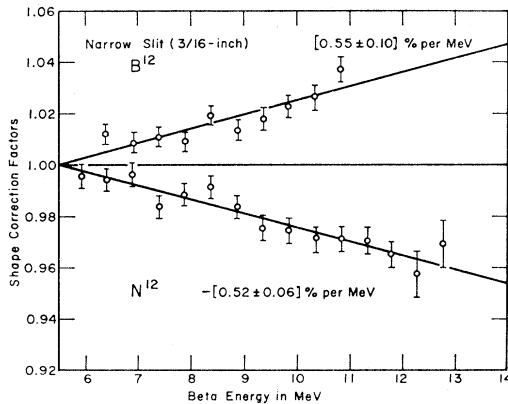
TABLE III. The experimentally determined Shape correction factors for B^{12} and N^{12} and their ratios.

	Lee, Mo, and Wu ⁴³	Mayer-Kuckuk and Michel ⁴¹	Glass and Peterson ⁴²	Theoretical prediction ⁴⁶⁻⁴⁸
$a^-(B^{12})$	$+0.55 \pm 0.10\%$ (narrow) $+0.52 \pm 0.09\%$ (wide)	$+1.82 \pm 0.08\%$...	$+0.55 \pm 0.12\%$
$a^+(N^{12})$	$-0.52 \pm 0.06\%$ (narrow) $-0.50 \pm 0.09\%$ (wide)	$+0.52 \pm 0.20\%$...	$-0.55 \pm 0.12\%$
$A = \frac{a^-(B^{12})}{a^+(N^{12})}$	$1.07 \pm 0.24\%$ (narrow) $1.02 \pm 0.24\%$ (wide)	$1.30 \pm 0.31\%$	$1.62 \pm 0.28\%$	$1.10 \pm 0.17\%$

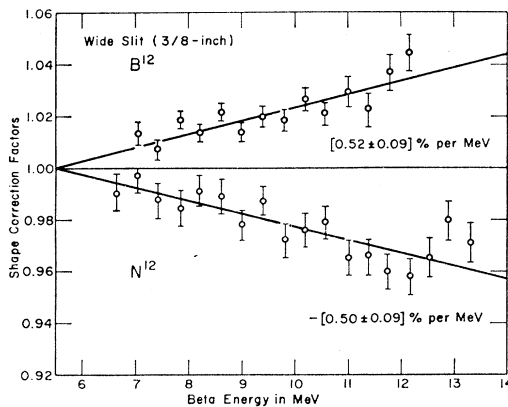
factor for the B^{12} spectrum is $+0.55 \pm 0.10\%$ per MeV and for the N^{12} spectrum $-0.52 \pm 0.06\%$ per MeV, as shown in Fig. 4. As predicted, the deviations

different slit systems and the same conclusions were obtained. The evidence, thus, strongly supports the theory of conserved vector current.

The results are summarized in Table III.



(a)



(b)

FIG. 4. The measured shape correction factors $a^-(B^{12})$ and $a^+(N^{12})$: (a) for the narrow annular slit ($\frac{3}{16}$ in.), and (b) for the wide annular slit ($\frac{3}{8}$ in.).

are in opposite directions to each other. The ratio of the two shape factors is $1.07 \pm 0.24\%$ per MeV, which compares favorably with the theoretically predicted value of $A + \delta A = 1.10 \pm 0.17\%$ per MeV. Furthermore, these curves have also been run with

(c) ft Values of B^{12} and N^{12}

The careful study of the β decays of B^{12} and N^{12} has led to the possible discovery of some new unexpected effects. From the very accurate measurements of decay energies and lifetimes for these β transitions⁴⁹⁻⁵¹ it is possible to calculate the ft values quite accurately. On this basis it is found that $ft = 117\,000$ for B^{12} and $12\,900$ for N^{12} (the uncertainties are 0.5% to 1%). Thus, the ft value for N^{12} is 10% larger than that for B^{12} . This difference has not yet been satisfactorily explained.

It might appear that the weak magnetism term which changes the spectrum shape for β^- and β^+ decay in opposite directions could also change the ft values, but this is not the case. For the average of E (integrated over the allowed spectrum) is in fact just $\frac{1}{2} E_0$; thus the correction in the shape factor [Eq. (36)] averages 0. Consequently, in the approximation considered above, there is no difference between ft values. However, there exist other correction terms besides the weak magnetism term, the "induced couplings," (e.g., the induced pseudoscalar coupling and possibly also an induced tensor coupling). These terms might conceivably account for the difference of ft values,^{52,53} though they have negligible effect on allowed spectrum shapes.

(3) The $\pi^+ \rightarrow \pi^0 + e^+ + \nu$ Decay

An important consequence of the CVC theory is the predicted occurrence of decay of π^+ into π^0 .

As was pointed out above, according to the CVC

⁴⁹ T. R. Fisher, Phys. Rev. **130**, 2388 (1963).

⁵⁰ R. W. Peterson and N. W. Glass, Phys. Rev. **130**, 292 (1963).

⁵¹ W. Whaling and T. R. Fisher, Bull. Am. Phys. Soc. **8**, 598 (1963).

⁵² S. Weinberg, Phys. Rev. **112**, 1375 (1958).

⁵³ J. N. Huffaker and E. Grueling, Phys. Rev. **132**, 738 (1963).

theory, pions are endowed with β -interaction strength. Thus charged π^+ should be able to decay into π^0 . For example we can have

$$\pi^+ \rightarrow \pi^0 + e^+ + \nu.$$

This decay is very similar, in principle, to a nuclear $0^+ \rightarrow 0^+$ transition such as for example, the $O^{14} \rightarrow N^{14}$ decay. In both cases the decaying particle has spin 0 and isotopic spin $T = 1$, and T_z changes from $+1$ to 0. As in nuclear beta decay, we find

$$\frac{1}{\tau} = \frac{mc^2}{\hbar} \frac{G^2}{2\pi^3} |M|^2 f. \quad (37)$$

Now expressing the dimensionless coupling constant in terms of the g_V and using

$$|M|^2 = \left| \int 1 \right|^2 = 2,$$

and

$$f \sim \frac{1}{30} W_0^5 \text{ for } W_0 \gg 1, \quad (38)$$

where $W_0 =$ maximum energy in units mc^2 , the electron rest energy, we obtain

$$1/\tau = g_V^2 (m^5 c^4 / \hbar^7) (W_0^5 / 30\pi^3). \quad (39)$$

The decay energy $= (m_{\pi^+} - m_{\pi^0}) c^2 = 4.6$ MeV.

We find

$$1/\tau = 0.43 \text{ sec}^{-1} \quad (40)$$

and a branching ratio

$$R = \frac{\pi^+ \rightarrow \pi^0 + e^+ + \nu}{\pi^+ \rightarrow \mu^+ + \nu} = (1.07 \pm 0.02) \times 10^{-8}. \quad (41)$$

Actually this process could have also occurred in the old form of the theory, i.e., by

$$\pi^+ \rightarrow p + \bar{n} \rightarrow \left\{ \begin{matrix} p + \bar{p} \\ n + \bar{n} \end{matrix} \right\} + e^+ + \nu \rightarrow \pi^0 + e^+ + \nu. \quad (42)$$

In this theory the calculation of the decay rate is beset with mathematical difficulties due to the strong coupling of pions to nucleons, and divergent integrals. The calculated R is between 5×10^{-9} and 10^{-8} depending on the assumptions made.⁵⁴ It should be pointed out that the processes, [Eq. (42)] as well as direct decay, are included in the CVC calculation, since the beta-decay current is the same for a bare pion or a nucleon-antinucleon pair, just as for the vector part of nuclear beta decay.

The $\pi^+ \rightarrow \pi^0 + e^+ + \nu$ decay can be detected by observing the following series of events: A slow

neutral π^0 is created after a positive pion π^+ is stopped and decays. The π^0 immediately decays into two 70-MeV γ rays. Meanwhile, the e^+ produced in the π^+ decay annihilates into two 0.5-MeV γ rays. This sequence of events, two 70-MeV γ rays and two 0.5-MeV γ rays created practically simultaneously, is so typical that it is possible to distinguish it by multiple coincidence experiments from the tremendous background (by factor 10^8) of normal $\pi \rightarrow \mu \rightarrow e$ decays. Even the rare $\pi \rightarrow e + \nu$ decays occur 10^4 times as frequently! Several laboratories have carried out experiments to measure this ratio. The reported results are summarized in Table IV. It can be seen that the

TABLE IV. Recent measurements of the decay rate of $\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$.

	R	Events observed
CERN ^a	$(1.15 \pm 0.22) \times 10^{-8}$ $(1.08 \pm 0.22) \times R_{CVC}$	52 ± 3
Lawrence Lab ^b	$(2.0 \pm 0.6) \times 10^{-8}$	10
Dubna ^c	$(1.30 \pm 0.35) \times 10^{-8}$	40 ± 8
Columbia ^d	$(1.0 \pm 0.3) \times 10^{-8}$	33

^a P. DePommier, J. Heintze, C. Rubbva, and V. Soergel, Phys. Letters 5, 61 (1963).

^b R. Bacastow, T. Elliott, R. Larsen, C. Wiegand, and T. Ypsilantis, Phys. Rev. Letters 9, 400 (1962).

^c A. F. Duntitsev et. al. (reported by I. V. Chuvilo) in Proceedings of the International Conference on the Fundamental Aspects of Weak Interactions at Brookhaven National Laboratory, September, 1963.

^d D. Bartlett, S. Devons, S. Meyer, and J. Rosen, Bull. Am. Phys. Soc. 9, 71 (1964).

measured ratio is in good agreement with the ratio predicted by the CVC theory.

(4) The Beta-Alpha Angular Correlations in the Li^8 and B^8 Beta Decays

Another test of the CVC theory, similar in principle to that for the $A = 12$ nuclei, can be made by studying the β - α angular correlation in the decays of Li^8 and B^8 .

Although the radiations following allowed β decay of unoriented nuclei are uncorrelated in direction with β rays, forbidden effects may produce correlations of the form

$$W(\theta_{\beta\alpha}) = 1 + B \cos^2 \theta_{\beta\alpha}, \quad (43)$$

where the small coefficient B depends on the details of the matrix elements involved. Bernstein and Lewis⁵⁵ and Morita⁵⁶ suggested that second forbidden vector interference terms should lead to deviations from isotropy in the β - α angular correlations of the Li^8 and B^8 beta decays [$Li^8 (\beta^-, \nu^-) Be^{8*} (\alpha) He^4$ and $B^8 (\beta^+, \nu) Be^{8*} (\alpha) He^4$]. These mirror nuclei decay pri-

⁵⁴ E. Feenberg and H. Primakoff, Phil. Mag. 3, 328 (1958).

⁵⁵ J. Bernstein and R. R. Lewis, Phys. Rev. 112, 232 (1958).

⁵⁶ M. Morita, Phys. Rev. 113, 1584 (1959).

marily by an allowed Gamow–Teller transition to the 2.90-MeV level of Be^8 which promptly decays into two alpha particles. The mass-8 triplet is illustrated in Fig. 5 where the dashed line represents the

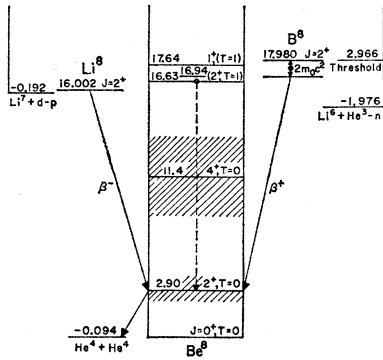


FIG. 5. Energy-level diagram of the mass-8 triad. The Li^8 electron decay and the B^8 positron decay are followed by the α -particle breakup of the 2.90-MeV Be^{8*} level.

$M1$ gamma transition to the 2.90-MeV level of Be^8 from the $I = 2^+, T = 1$ level, analogous to the Li^8 and B^8 ground states.

Due to the recoil of Be^{8*} from the β decay, the β - α angular correlation transformed from the Be^{8*} rest system to the laboratory system is of the form

$$W(\theta_{\beta\alpha}) = 1 + A \cos \theta_{\beta\alpha} + B \cos^2 \theta_{\beta\alpha}, \quad (44)$$

where A is equal to $-W_\beta/p_\alpha c$ to the first order. W_β and p_α are the electron total energy and the alpha-particle momentum, respectively. The coefficient B may be represented by $B = aW_\beta$, where a is defined by Gell-Mann:

$$a = (\mu/\sqrt{2}M \sigma^2)(g_V/g_A). \quad (45)$$

The situation here is similar in principle to that for the $A = 12$ sequence, but is not possible to draw as definite conclusions.

First of all, the radiation width of the $M1 \gamma$ ray is not known experimentally. It has been calculated theoretically by Weidenmüller⁴⁴ and Kurath⁵⁷ using the intermediate coupling shell model. Weidenmüller's estimate gives Γ_γ between 1 and 4 eV, while Kurath estimates it to be between 3 and 5 eV.

Proceeding similarly as for the $A = 12$ sequence and using Weidenmüller's result, it can be shown that the anisotropy parameter B is given by

$$0.0025W < B < 0.0045W \text{ for CVC,}$$

$$\text{and } 0.0005W < B < 0.002W$$

for conventional Fermi theory,

⁵⁷ D. Kurath, Phys. Rev. Letters 4, 180 (1960).

for Li^8 and with the same magnitude but opposite sign for B^8 .

The coefficient B is extremely small indeed. This difficult comparison has been carried out by Nordberg, Morinigo, and Barnes⁵⁸ with high precision. Their measurements were made by counting coincidences between α particles detected in a gold-silicon surface-barrier detector and electrons or positrons detected in a plastic scintillator which could be rotated about the target to positions of $0^\circ, 90^\circ$, or 180° , relative to the alpha detector. The alpha-particle pulse-height spectra in coincidence with electrons from the Li^8 beta decay and B^8 beta decay at various angles are shown in Figs. 6 and 7, respectively. The

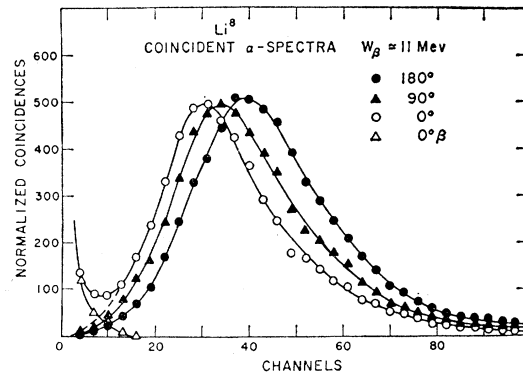


FIG. 6. The α -particle pulse-height spectra in coincidence with electrons from the Li^8 beta decay at various angles. A prominent feature of the curve is the energy shift with angle which is the result of the Be^{8*} recoil from the beta decay. This experiment measured the total areas of the curves to determine the coefficients of the angular distribution, $W(\theta_{\beta\alpha}) = 1 + A \cos \theta_{\beta\alpha} + B \cos^2 \theta_{\beta\alpha}$. The points marked " $0^\circ\beta$ " were the response of the gold-silicon detector to the electrons.

total areas of the curves are used to determine the coefficients A and B in Eq. (43). Their final results were

$$B_{\text{Li}^8} = (0.00316 \pm 0.00060)W_\beta,$$

and

$$B_{\text{B}^8} = (-0.00386 \pm 0.00100)W_\beta.$$

A recent independent determination of the $\text{Li}^8 \beta$ - α correlation⁵⁹ gives $B_{\text{Li}^8} = (0.0037 \pm 0.0010)W_\beta$ in good agreement with the results of Nordberg *et al.* This one may conclude that the measured value lies within the range of the theoretical values of B for the CVC theory and disagrees with the prediction of the old Fermi theory.

⁵⁸ M. E. Nordberg, F. B. Morinigo, and C. A. Barnes, Phys. Rev. 125, 321 (1962).

⁵⁹ W. Gruhle, K. H. Lauterjung, and B. Schimmer, Nucl. Phys. 42, 321 (1963).

(5) β - γ (circularly polarized) Correlation of the Mirror Transitions: $\text{Na}^{24} \beta^- \text{Mg}^{24} \beta^+ \text{Al}^{24}$

The ingenious experiment suggested by Bouchiat⁶⁰ was stimulated by the observation of the large discrepancy between the experimentally determined Fermi matrix element M_F and the theoretically estimated isotopic spin impurities in the case of Na^{24} . It is well known that the isotopic spin selection rule for the Fermi type of β interaction is given by $\Delta T = 0$; where T is the total isotopic spin. Therefore, in β transitions, where $\Delta T = 1$ and $\Delta T_z = \pm 1$, only two possible sources other than $\Delta T = 0$ can

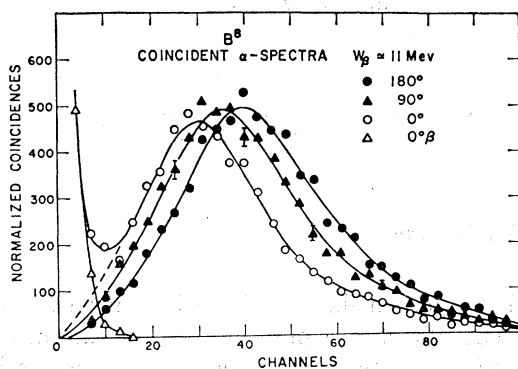


FIG. 7. Same as Fig. 6, except this is for B^8 .

contribute to the Fermi part of β interaction. One is the already mentioned isotopic spin impurity which is introduced by the charge dependent forces in nucleon-nucleon interaction such as the Coulomb interaction between the protons. Such an asymmetrical potential clearly violates the charge-independent condition for the total isotopic spin T to be a good quantum number. Therefore the charge-dependent potential perturbs and mixes states of different isotopic spins. The other source occurs only in the conventional theory of β decay where the pion current does not take part in the interaction, so the virtual pions in the physical nucleon state can induce a Fermi transition with $\Delta T \neq 0$. However, as was pointed out by Wigner,^{61,65} under the CVC theory, the contribution of virtual pion states to the $\Delta T = 0$ Fermi transition is strictly zero as it is required that the β -interaction Hamiltonian commute with the total isotopic spin operator. Thus the observed deviations from the $\Delta T = 0$ selection rule in Fermi transitions have to be explained only in terms of isotopic spin impurities.

⁶⁰ C. C. Bouchiat, *Phys. Rev. Letters* **3**, 516 (1959).

⁶¹ E. P. Wigner, in *Proceedings of the Welsh Foundation Conference on Chemical Research*, 1957 (unpublished).

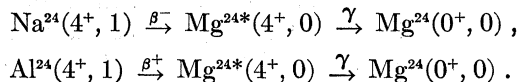
Let us take the case of β decay from the $T = 1$, $J = 4^+$ state of Na^{24} to the $T = 0$, $J = 4^+$ state of Mg^{24*} . This is a case of $\Delta T = 1$. However, the contribution of M_F as determined by β - γ (circularly polarized) angular correlation is found to be less than 10^{-3} while the theoretical estimate by $j-j$ coupling shell model yields a value of $(1.3 - 1.7) \times 10^{-2}$,^{62,63} an order of magnitude larger.

There are two possible explanations of this discrepancy:

- (1) The Coulomb matrix element has possibly been overestimated by using $j-j$ coupling model wavefunctions; or
- (2) The CVC theory may break down in the complex nuclei; then the virtual pion currents do induce Fermi transitions. Furthermore, the mesonic term and the Coulomb term interfere and cancel each other in Na^{24} , thus resulting in a much smaller value of M_F .

In order to make a choice between these two possibilities without having to use any nuclear model, the following mirror experiment was suggested by Bouchiat.

Let us consider the $T = 1$ multiplet which consists of the ground state of Al^{24} and Na^{24} and of the 9.5-MeV level of Mg^{24*} . The β^- decay of the ground state of Na^{24} has a 99.9% branch to the $T = 0$, $J = 4^+$ level of Mg^{24*} , and the β^+ decay of Al^{24} has a 10% branch to the same level:



The angular distribution of the circularly polarized γ ray of Mg^{24*} relative to the direction of emission of the β^+ particles is given by

$$W^\pm(\theta, \tau) = 1 + A^\pm(v/c)\tau \cos \theta; \quad (46)$$

the $+$ sign is for the β^- , the $-$ sign for the analogous β^+ transition. The asymmetry coefficient A^\pm which is determined by the β - γ angular correlation contains essentially the ratio of M_F to M_{GT} . Under charge conjugation (i.e., $\beta^+ \leftrightarrow \beta^-$), M_{GT} is odd and changes sign. If the CVC theory is valid, M_F is even because the contribution comes solely from the Coulomb impurity. The contribution to M_F from the mesonic effect would be odd, but it only exists in conventional theory of beta decay.

Thus the CVC theory predicts that exactly

$$A^+ + A^- = 0. \quad (47)$$

This difficult experiment has now been performed

⁶² C. C. Bouchiat, *Phys. Rev.* **118**, 540 (1960).

⁶³ R. J. Blin-Stoyle and L. Nováková, to be published.

independently by two different groups.^{64,65} Their results are in substantial agreement:

$$A^+(\text{Na}^{24}): + 0.104 \pm 0.026 \text{ (63)} ; \\ + 0.091 \pm 0.017 \text{ (64)} , \\ A^-(\text{Al}^{24}): - 0.089 \pm 0.057 \text{ (63)} ; \\ - 0.086 \pm 0.054 \text{ (64)} .$$

Therefore, it implies that β decay in complex nuclei

⁶⁴ E. L. Hasse, H. A. Hill, and D. B. Knudson, *Phys. Letters* **4**, 338 (1963).

⁶⁵ L. G. Mann, S. O. Bloom, A. Scott, R. Polichas, and J. R. Richardson, in *Proceedings of the Manchester Conf. on Low- and Medium-Energy Physics, 1963*; and also *Phys. Rev.* (to be published).

is consistent with CVC theory well within experimental error. However, it is not necessary to imply that it is in disagreement with the old Fermi theory since the Fermi matrix element, due to mesonic effect, could be very small. On the other hand, the isotopic spin purity seems to be definitely better than theoretical estimate by the j - j coupling model.

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Electromagnetic Interactions of a Yang-Mills Field

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1. INTRODUCTION

Since it is possible that the weak interactions are mediated by vector bosons, associated with a conserved vector current, it is of some interest to investigate a possible genetic relationship between these hypothetical particles and the photon. In particular it has been conjectured that the photon is the neutral member of a multiplet which embraces the charged W -mesons.¹ Since one is then faced with a parity violating generalization of quantum electrodynamics, it is perhaps also natural in such speculations to examine the possibility of magnetic poles which, like the W -mesons, may also violate parity² and may also be produced only at very high energies. In a theory of this kind it is clear that the existence, or the non-existence, of magnetic poles would have important implications for the structure of the weak interactions.

The Maxwell equations may be written

$$\partial_\mu F^{\alpha\mu} = J^\alpha, \quad (\text{A})$$

$$\partial_\mu \mathbf{F}^{\alpha\mu} = 0, \quad (\text{B})$$

where $\mathbf{F}^{\alpha\mu}$ is the dual field:

$$\mathbf{F}^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\mu\nu} F_{\mu\nu}. \quad (1.1)$$

The conservation of electric current follows from

¹ A. Salam and J. Ward, *Nuovo Cimento* **11**, 568 (1959).
² N. Cabibbo and E. Ferrari, *Nuovo Cimento* **23**, 1147 (1962).

the first equation since $F^{\alpha\beta}$ is antisymmetric. Equation (B) directly expresses the nonexistence of magnetic poles and permits the usual representation of the six-vector field $F_{\alpha\beta}$ in terms of the four-vector A_α .

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha. \quad (1.2)$$

This representation is invariant under gauge transformations:

$$A'_\alpha = A_\alpha + \partial_\alpha \Lambda. \quad (1.3)$$

In this way the nonexistence of magnetic poles leads to the gauge group. If one generalizes either by allowing magnetic poles or by expanding the gauge group, the single 4-vector potential must be replaced by a field with more degrees of freedom.

2. GEOMETRICAL BASIS

Since the electromagnetic field is so important for determining our present ideas about physical space time, one would expect any generalization of the former to have important geometrical implications; or turning the argument around, one might seek a geometrical basis for generalizing Maxwell's equations. In the context of general relativity this has often been done, but until recently without reference to the weak interactions.

In this paper we postulate a local gauge group which is non-Abelian, compact, and contains the electromagnetic gauge transformation. Therefore a general vector field may be designated by $A_{\mu M}$ where