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Massive Stars, Relativistic Polytropes, and Gravitational Radiation

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I. INTRODUCTION

Twenty-five years ago Robert Oppenheimer and his students Robert Serber,[1] George Volkoff,[2] and Hartland Snyder[3] investigated the equilibrium and gravitational contraction of massive stars in the advanced stages of stellar evolution when nuclear sources of energy have been exhausted. They concluded that "when the pressure within stellar matter becomes high enough, a new phase consisting of neutrons will be formed. . . . For masses greater than $\frac{3}{4}$ $M_\odot$ there are no static equilibrium solutions. . . . When all thermonuclear sources of energy are exhausted a sufficiently heavy star will collapse. Unless fission due to rotation, the radiation of mass, or the blowing off of mass by radiation, reduce the star's mass to the order of that of the sun, this contraction will continue indefinitely. . . . The total time of collapse for an observer comoving with the stellar matter is finite, . . . ; an external observer sees the star asymptotically shrinking to its gravitational radius." It is a tribute to Robert Oppenheimer's genius that these are the few statements about massive stars accepted as true today.

In recent times there has been a renewal of interest in massive stars kindled by the suggestion of Fred Hoyle[4] and myself that stars with mass of order $\sim 10^8$ $M_\odot$ may accumulate at the center of galaxies, or in intergalactic space, and may serve as the source of the prodigious energies involved in emission or storage in the radio *galaxies* and *stars*. [At our present state of knowledge we must italicize the words *galaxies* and *stars* when used in connection with radio sources.] A general discussion of the relativistic and astrophysical aspects of the situation has been given in collaboration with our colleagues Geoffrey and Margaret Burbidge.[5]

There is no convincing evidence that the radio *stars* have lifetimes in excess of 10^5 to 10^6 years. Thus the total energy radiated is $\sim 10^{59}$ ergs, as reported for 3C273 by Schmidt[6] and for 3C48 by Greenstein and Matthews,[7] corresponding to an observed and theoretical[4] luminosity of $\sim 10^{46}$ erg sec^{-1}. It is noteworthy that this energy requirement is well within the nuclear resources of a star with $M = 10^8$ $M_\odot$ since 10^8 $M_\odot c^2 \sim 10^{62}$ ergs and hydrogen burning supplies energy equivalent to $\sim 1\%$ of the rest mass energy. Conversion of $\sim 10\%$ of the hydrogen into helium in a star with $M = 10^8$ $M_\odot$ is adequate to meet the observed luminosity requirements of the radio *stars*. This leads to the interesting question concerning the properties of massive stars under circumstances such that the proviso mentioned in the first paragraph above "when nuclear sources of energy have been exhausted" is not applicable. Part II of this paper discusses a limited but important aspect of the properties of massive stars, namely the

[1] J. R. Oppenheimer and R. Serber, Phys. Rev. **54**, 540 (1938).
[2] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374, 413 (1939).
[3] J. R. Oppenheimer and H. Snyder, Phys. Rev. **56**, 455 (1939).
[4] F. Hoyle and W. A. Fowler, Monthly Notices Roy. Astron. Soc. **125**, 169 (1963); Nature **197**, 533 (1963).
[5] F. Hoyle, W. A. Fowler, G. R. Burbidge, and E. M. Burbidge, to be published in Astrophys. J. (This reference will be referred to subsequently as HFB^2.)
[6] M. Schmidt, Nature **197**, 1040 (1963).
[7] J. L. Greenstein and T. A. Matthews, Nature **197**, 1041 1963).

fact that hydrostatic equilibrium in such stars requires a positive total energy above the rest mass energy of the particle constituents or in nuclear parlance "a negative binding energy." In the discussion the internal structures of the massive stars will be approximated as relativistic polytropes.

It must be emphasized that the energy storage requirements ($\lesssim 10^{62}$ ergs) found by Maltby, Matthews, and Moffet[8] for the strong, extended radio sources associated with *galaxies* cannot be met from nuclear resources and that gravitational energy must be called upon in the ultimate collapse of the massive condensations. During the early, *stellar* stages of a radio source nuclear energy can supply the optical and radio luminosity requirements. The ultimate requirements for sources which extend far beyond the confines of the *galaxies* with which they are identified can be met only by gravitational energy transferred in some way from the collapsing core of the massive star to the envelope and eventually the external surroundings. Part III of this paper discusses a possible mechanism of this transfer. Part IV is a summary.

II. BINDING ENERGY OF A MASSIVE STAR IN HYDROSTATIC EQUILIBRIUM

Feynman[9] and Iben[10] have shown that the binding energy of a massive star must be negative when general relativistic terms in the equation for hydrostatic equilibrium are appreciable. This result applies in some cases even when the relativistic parameter $2GM/Rc^2$ is small compared to unity. Here, we will attempt to understand this result in the simplest possible way by investigating general relativistic effects in massive stars in the first-order approximation beyond the classical Newtonian terms.

The total energy E of a star exclusive of the rest mass energy when infinitely dispersed at zero temperature is equal but opposite in sign to the binding energy E_b and is given by

$$E = -E_b = (M - M_0)c^2 . \qquad (1)$$

In this expression M is the mass of the star given in terms of the stellar radius R and the mass M_r interior to r by

$$M = \int_0^R dM_r = \int_0^R \rho dV = \int_0^R 4\pi\rho r^2 dr . \qquad (2)$$

Spherically symmetric coordinates have been used and r has been chosen to give the "coordinate" element of volume dV, in the usual manner. Note that $V = \frac{4}{3}\pi r^3$ is the volume interior to r. In the third equality in Eq. (2) we have used

$$dM_r/dr = 4\pi\rho r^2 \qquad (3)$$

where

$$\rho = \rho_0 + u/c^2 \qquad (4)$$

is the mass-energy density measured by a local observer and includes both the rest mass ρ_0 of the "atoms" in the star plus the mass equivalent u/c^2 of the internal energy per unit volume of the atomic constituents and of radiation. The rest mass of the star is given by

$$M_0 = \int_0^R \rho_0 \left(1 - \frac{2GM_r}{rc^2}\right)^{-\frac{1}{2}} dV \qquad (5)$$

where a "proper" element of volume has now been employed.

By "atoms" in the previous paragraph we mean the nuclei plus the electrons necessary to balance the nuclear charges. The rest mass-energy as well as the kinetic energies of electron–positron pairs or other particle pairs created by the radiation field must be included in the internal energy u on the assumption that the pairs will be annihilated on dispersal to infinity. In principle we calculate the binding energy of the atomic constituents in the star at a given time. This is then the energy required to disperse these constituents to infinity and zero temperature without nuclear or atomic changes. Atomic energy changes in this dispersal can certainly be neglected and in most but not all cases nuclear changes will not occur during the dispersal. Any changes which do occur must be taken into account by calculating the binding energy relative to the nuclei which result upon dispersal. This aspect of the problem will not appear explicitly in what follows.

It is convenient to eliminate ρ_0 by use of Eq. (4) so that in further calculations we will use

$$E = \int_0^R u \left(1 - \frac{2GM_r}{rc^2}\right)^{-\frac{1}{2}} dV + \int_0^R \rho c^2 \left[1 - \left(1 - \frac{2GM_r}{rc^2}\right)^{-\frac{1}{2}}\right] dV + E_{\text{dyn}} . \qquad (6)$$

We have now included the dynamical energy E_{dyn}, which arises from bulk motions throughout the star and which should properly be included in M. Classically $E_{\text{dyn}} = \int \frac{1}{2} \rho \dot{r}^2 \, dV$. In the classical approxima-

[8] P. Maltby, T. A. Matthews, and A. T. Moffet, Astrophys. J. **137**, 153 (1963).
[9] R. P. Feynman, private communication (1963).
[10] I. Iben, Jr., Astrophys. J. **138**, 1090 (1963).

tion one can neglect $2GM_r/rc^2$ in the first integral but not in the second. Then

$$E = \int_0^R u dV - \int_0^R \frac{GM_r}{r} \rho dV + E_{\rm dyn}$$
$$= \int_0^R 3\epsilon p dV - \Omega + E_{\rm dyn} , \quad (7)$$

where Ω is the gravitational *binding* energy taken as a positive quantity. For a polytrope of index n, it is well known that Ω is given by

$$\Omega = \frac{3}{5-n} \frac{GM^2}{R} \to \frac{3}{2} \frac{GM^2}{R} \text{ for } n = 3 . \quad (8)$$

In the first integral of the last part of Eq. (7) the ratio $\epsilon = u/3p$ is determined by the state of the matter and radiation in each shell in the star. We calculate the contributions to u and p for nuclei, ionization electrons, electron–positron pairs and radiation in the nondegenerate approximation which holds for massive stars and find that

$$\epsilon - 1 = \frac{u}{3p} - 1 = \frac{\beta}{3}\left[x - 3 - \frac{3}{2Z}\frac{n_0}{n_e} + z\left(1 - \frac{n_0}{n_e}\right)\right]\left[1 + \frac{1}{Z}\frac{n_0}{n_e}\right]^{-1} , \quad (9)$$

where β is the ratio of gas pressure to total pressure, x is the mean kinetic energy of the electrons and positrons in units of kT, n_0 is the original number of ionization electrons per cm^3 necessary to balance the nuclear charge Z, n_e is the total number of electrons and positrons per cm^3, and z equals $m_e c^2/kT$. The quantity x is tabulated by Chandrasekhar[11] as U/PV. Below $T = 10^9$ deg at sufficiently high densities, $x = \frac{3}{2}$, $n_e = n_0$, so that

$$\epsilon - 1 = -\tfrac{1}{2}\beta \leqslant 0 \qquad T \leqslant 10^9 \text{ deg} . \quad (10)$$

At higher temperature and sufficiently low densities, $n_e \gg n_0$, so that

$$\epsilon - 1 = \tfrac{1}{3}\beta[x + z - 3] \geqslant 0 . \quad (11)$$

The situation can also occur at high temperature and high density where $n_e \sim n_0$ and $x \sim 3$ so that $\epsilon - 1 \sim 0$. In general, ϵ starts at $\frac{1}{2}$ at very low temperatures, rapidly rises to unity in massive stars, reaches a maximum $\sim$1.2 around $T = 2 \times 10^9$ degrees and returns to unity at higher temperature. The internal energy per unit volume, $u = 3\epsilon p$, varies throughout a star primarily because of the rapid inward rise of pressure p and not because of the variations in ϵ. In what follows we will neglect variations in ϵ and introduce its average value $\bar{\epsilon}$ for order-of-magnitude estimates.

We now turn to the classical calculation of E through Eq. (7). The integration over $3\epsilon p\, dV$ can be carried out by use of the expression for the pressure gradient in the star. In the classical case

$$\frac{dp}{dr} = -\rho\left(g + \frac{dv}{dt}\right) = -f\rho g = -f\rho\frac{GM_r}{r^2} , \quad (12)$$

where ρ is the density, $g = GM_r/r^2$ is the acceleration due to gravity, and dv/dt is the actual acceleration of the material measured positively in the outward direction, i.e., increasing r. In this note we will not attempt to treat dynamic effects ($dv/dt \neq 0$) except through the use of the variable f which has been introduced in the last parts of Eq. (12). For implosion ($dv/dt < 0$), $f < 1$; for explosion ($dv/dt > 0$), $f > 1$. As in the case of ϵ we will introduce an average value $\bar{f}$.

The classical calculation then proceeds as follows:

$$\int_0^R 3\epsilon p\, dV = 3\epsilon p V\Big|_{V=0}^{p=0} - \int_0^R 3\epsilon V\, dp - \int_0^R 3pV\, d\epsilon$$
$$\approx + \int^R 4\pi r^3 \epsilon f \rho g\, dr \approx \overline{\epsilon f} \int \frac{GM_r}{r}\rho\, dV = \overline{\epsilon f}\,\Omega , \quad (13)$$

where the approximation in neglecting variations in ϵ and f are now apparent, in particular $d\epsilon \approx 0$. Thus

$$E \approx (\overline{\epsilon f} - 1)\,\Omega + E_{\rm dyn} \backsim (\bar{\epsilon}\bar{f} - 1)\,\Omega + E_{\rm dyn} . \quad (14)$$

Note that the product ϵf must be averaged over $p\, dV$ and that the second approximation must be used with caution. In the case of hydrostatic equilibrium throughout a star, $f = 1$ everywhere, $E_{\rm dyn} = 0$, and

$$E_{\rm eq} \approx (\bar{\epsilon} - 1)\,\Omega \approx -\tfrac{1}{2}\bar{\beta}\Omega \text{ for } T \leqslant 10^9 \text{ deg} . \quad (15)$$

Thus a classical star is bound by $\frac{1}{2}\,\Omega$ for $\bar{\beta} = 1$ where radiation pressure can be neglected (small stars) and has zero binding for $\bar{\beta} = 0$ where radiation pressure is dominant (very massive stars). At temperatures above 10^9 deg electron–positron pair formation can lead to stars with total positive energy or negative binding as indicated by Eq. (11). If this energy cannot be supplied after the star has passed through bound states of quasi-hydrostatic equilibrium at low temperatures then $\overline{\epsilon f}$ must remain less than unity. This means that at least part of the star must contract fairly rapidly and it can be argued that the inner regions where the temperature is highest and ϵ is the largest will be most susceptible to rapid contraction.

[11] S. Chandrasekhar, *An Introduction to the Study of Stellar Structure* (University of Chicago Press, Chicago, Illinois, 1938), p. 347, Table 24.

For the general relativistic case we replace Eq. (12) by

$$\frac{dp}{dr} = -f\rho\,\frac{GM_r}{r^2}\left(1+\frac{p}{\rho c^2}\right)\left(1+\frac{4\pi pr^3}{M_r c^2}\right) \times\left(1-\frac{2GM_r}{rc^2}\right)^{-1} \equiv -f\rho\,\frac{\mathcal{G}M_r}{r^2} \qquad (16)$$

Note that the *effective* gravitational constant $\mathcal{G}$ diverges as $2GM_r/rc^2 \to 1$. *Gravity changes from the weakest to the strongest interaction under appropriate circumstances!* The first integrand in Eq. (6) contains $(1 - 2GM_r/rc^2)^{\frac{1}{2}}$ in the denominator so we divide Eq. (16) by this term and expand the two sides to obtain

$$dp\left(1+\frac{GM_r}{rc^2}\right) \approx -f\rho\,\frac{GM_r}{r^2}\left(1+\frac{p}{\rho c^2} + \frac{4\pi pr^3}{M_r c^2}+\frac{3GM_r}{rc^2}\right)dr\,. \qquad (17)$$

Just as in the classical case, we use Eq. (17) to evaluate the first integral in Eq. (6). In addition the second integrand can be expanded and the final result is

$$E = (\overline{\epsilon f}-1)\Omega + \frac{4\pi G}{c^2}\,\overline{\epsilon(f+1)}\int prM_r dr + \frac{16\pi^2 G}{c^2}\,\overline{\epsilon(f-1)}\int \rho pr^4 dr + \frac{12\pi G^2}{c^2}\,\overline{(\epsilon f-\tfrac{1}{2})}\int \rho M_r^2 dr + E_{\mathrm{dyn}} \qquad (18)$$

where the various averages, $\overline{\epsilon f}$ and $\bar{\epsilon}$, involve different "weighting" functions but we ignore these differences.

We first investigate Eq. (18) in the case of hydrostatic equilibrium where $f = 1$, $E_{\mathrm{dyn}} = 0$. It is also possible to make the approximation $\bar{\epsilon} = 1$ except in the first term where, when the term is important, $\epsilon - 1 = -\frac{1}{2}\,\bar{\beta}$ with $\bar{\beta}$ the appropriate average over

$$\beta \approx (\eta_n^{\frac{1}{2}}/\mu)(T/T_c)^{(n-3)/(n+1)} \qquad T \leqslant 10^9\ \mathrm{deg}\,. \qquad (19)$$

Here μ is the mean molecular weight taken to be constant throughout the star and T_c is the central temperature. Equation (19) can be shown to hold approximately for massive stars ($M \gg 10^3\,M_\odot$) at central temperatures less than 10^9 degrees. The quantity η_n is given by

$$\eta_n = \frac{3}{4\pi}\,(n+1)^3\,\frac{\mathfrak{R}}{aG^3}\left(\frac{M_n}{M}\right)^2 \qquad (20)$$

$$= 335(M_\odot/M)^2 \ \text{for}\ n = 3\,, \qquad (21)$$

where M_n is the constant of integration for the mass scale for the second order differential equation for a polytrope of index n. For example, $M_3 = 2.018$. The second constant applies to the radius scale and will be designated by R_n in what follows. For example, $R_3 = 6.897$. For the polytrope of index 3, β is constant throughout the interior and

$$\bar{\beta} = \beta = \frac{4.3}{\mu}\left(\frac{M_\odot}{M}\right)^{\frac{1}{2}} \ll 1$$

$$\backsim 10^{-2},\ M \backsim 10^6\,M_\odot;\ \backsim 10^{-4},\ M \backsim 10^{10}\,M_\odot. \quad (22)$$

It will be found for all massive polytropes that $\bar{\beta}$ is small compared to unity.

In any case for hydrostatic equilibrium Eq. (18) becomes

$$E_{\mathrm{eq}} = -\tfrac{1}{2}\,\bar{\beta}\Omega + \frac{8\pi G}{c^2}\int prM_r\,dr + \frac{6\pi G^2}{c^2}\int \rho M_r^2\,dr\,. \qquad (23)$$

Massive stars are highly convective and with $\beta \sim 0$ this corresponds most closely to a polytropic structure with index $n = 3$. For $n = 3$, $\Omega = \frac{3}{2}\,GM^2/R$ and the integrals in Eq. (23) can be integrated numerically using Table 6 in Eddington's *The Internal Constitution of the Stars*.[12] The result is

$$\frac{E_{\mathrm{eq}}}{Mc^2} = -\tfrac{3}{4}\,\beta\left(\frac{GM}{Rc^2}\right) + 5.1\left(\frac{GM}{Rc^2}\right)^2 = -\tfrac{3}{8}\,\beta\left(\frac{R_g}{R}\right) + 1.3\left(\frac{R_g}{R}\right)^2 \ \text{for}\ n = 3\,, \qquad (24)$$

where $R_g = 2GM/c^2 = 3.0 \times 10^5\,(M/M_\odot)$ cm is the limiting gravitational radius of the polytrope.

Equation (24) gives the first two terms in a general expansion in terms of the dimensionless parameter

$$\frac{R_g}{R} = \frac{2GM}{Rc^2} = \left(\frac{32\pi\,G^3}{3c^6}\,M^2\bar{\rho}\right)^{\frac{1}{3}} = \left[\frac{\bar{\rho}(M/M_\odot)^2}{1.8\times 10^{16}}\right]^{\frac{1}{3}}. \qquad (25)$$

In general it is assumed that general relativity becomes important when this parameter is the order of unity, or

$$\bar{\rho} = 1.8\times 10^{16}(M_\odot/M)^2\ \mathrm{g\ cm^{-3}}\,, \qquad (26)$$

so that for stars near one solar mass the critical mean density is the order of 10^{16} g cm^{-3} which exceeds nuclear densities ($\sim 2 \times 10^{14}$ g cm^{-3}). However, for massive stars, e.g., $M = 10^8\,M_\odot$, the critical mean density is the order of unity, the central densities are only $\sim$100 g cm^{-3} and *general relativity is seen to be important even in the range where the atomic and*

[12] A. S. Eddington, *The Internal Constitution of the Stars* (Cambridge University Press, Cambridge, England, 1930).

nuclear properties of matter are fairly well understood. Equation (24) shows in addition that the general relativistic second order term in the binding energy of a star is comparable to the nonrelativistic first order term when

$$2GM/Rc^2 \approx 0.3\,\beta \approx (1.3/\mu)(M_\odot/M)^{\frac{1}{2}} \approx 3\times10^{-4} \quad \text{for } \mu=\tfrac{1}{2},\quad M=10^8 M_\odot,\quad n=3, \tag{27}$$

or

$$\begin{aligned}\bar{\rho} &\approx (4.0\times10^{16}/\mu^3)(M_\odot/M)^{\frac{3}{2}}\ \text{g cm}^{-3}\\ &\approx 3.2\times10^{17}(M_\odot/M)^{\frac{3}{2}} \quad \text{for } \mu=\tfrac{1}{2}\\ &\approx 3.2\times10^{-11}\ \text{g cm}^{-3} \quad \text{for } M=10^8 M_\odot,\quad n=3.\end{aligned} \tag{28}$$

The minimum in $E_{\rm eq}/Mc^2$ occurs at one-eighth this density or $2GM/Rc^2 = 0.15\beta$. It will be apparent that general relativistic considerations cannot be neglected even during relatively early stages of the contraction of massive stars. It will also be clear that the perturbation approximation used in evaluating the second order terms in Eq. (24) is quite accurate under these circumstances. The structure of the star is that of a classical polytrope. At the same time the classical result for massive stars supported by radiation pressure is a near-zero binding energy. Thus the relativistic second order term becomes dominant in the binding energy calculation at anomalously large radii and low densities.

For many purposes, particularly regarding the rates of nuclear processes, it is advantageous to replace the radius R in the collapse parameter by the central temperature. This can be done quite simply since the radius of a polytrope of index n is related to the central temperature T_c by

$$\begin{aligned}R &= \frac{GMR_n}{(n+1)M_n}\left(\frac{\mu\beta}{\Re T}\right)_c\\ &= \left[\frac{3G}{4\pi(n+1)a}\right]^{\frac{1}{4}}\left(\frac{M}{M_n}\right)^{\frac{1}{2}}\frac{R_n}{T_c}\\ &= \frac{5.83\times10^9}{(T_9)_c}\left(\frac{M}{M_\odot}\right)^{\frac{1}{2}}\ \text{cm} \quad \text{for } n=3.\end{aligned} \tag{29}$$

Using Eqs. (23) and (29) the result for $E_{\rm eq}/Mc^2$ for three polytropes is

$$\begin{aligned}\frac{E_{\rm eq}}{Mc^2} &= -\frac{9}{4}\,\frac{\Gamma(3/2)\Gamma(5/4)}{\Gamma(11/4)}\,\frac{\Re}{\mu c^2}\,T_c + \frac{19}{35}\,(2\pi)^{\frac{1}{2}}\,\frac{a^{\frac{1}{2}}G^{\frac{3}{2}}M}{c^4}\,T_c^2\\ &= -2.1\times10^{-13}\,T_c + 5.0\times10^{-27}(M/M_\odot)T_c^2 \quad \text{for } n=0,\end{aligned} \tag{30}$$

$$\begin{aligned}&= -3\,\frac{M_3}{R_3}\,\frac{\Re}{\mu c^2}\,T_c + 3\,\frac{M_3}{R_3}\,\frac{a^{\frac{1}{2}}G^{\frac{3}{2}}M}{c^4}\,T_c^2\\ &= -1.6\times10^{-13}\,T_c + 3.3\times10^{-27}(M/M_\odot)T_c^2 \quad \text{for } n=3,\end{aligned} \tag{31}$$

$$\begin{aligned}&= -\frac{9}{4}\,\frac{\Gamma(3/2)\Gamma(7/4)}{\Gamma(13/4)}\,\frac{\Re}{\mu c^2}\,T_c + \frac{2}{15}\,(6\pi)^{\frac{1}{2}}\,\frac{a^{\frac{1}{2}}G^{\frac{3}{2}}M}{c^4}\,T_c^2\\ &= -1.3\times10^{-13}\,T_c + 2.2\times10^{-27}(M/M_\odot)T_c^2 \quad \text{for } n=5.\end{aligned} \tag{32}$$

In the numerical expressions we have set $\mu = \frac{1}{2}$ since the first term is important at low temperatures before hydrogen burning sets in. Equations (30), (31), and (32) show that the ratio of the second order term to the first order term is relatively independent of the polytropic structure.

The first term in Eqs. (30), (31), or (32) shows the linear decrease with central temperature of the total energy of the star (increase in binding energy) as the star begins contraction from the dispersed stage. It corresponds to the classical case, Eq. (15). The second term arises from the first order general relativistic approximation. This term is positive and leads at high enough temperatures to positive total energies and negative binding energies as shown by Iben[10] by more exact theoretical and numerical treatment of the problem. Equations (30), (31), and (32) reproduce Iben's numerical results in good approximation. The minimum total energy is reached at

$$T_c = 2.5\times10^{13}\,M_\odot/M \quad \text{for } n=3 \tag{33}$$

and the energy returns to zero and goes positive at

$$T_c = 5\times10^{13}\,M_\odot/M \quad \text{for } n=3. \tag{34}$$

This behavior is illustrated for $M = 10^6\,M_\odot$, $10^7\,M_\odot$, $10^8\,M_\odot$ in Fig. 1. In all three cases the energy necessary to establish hydrostatic equilibrium becomes large and positive before hydrogen burning sets in at 8×10^7 deg as determined by Hoyle and Fowler.[4] The zero energy temperatures are 5×10^7, 5×10^6, 5×10^5 degrees, respectively.

The second order terms in Eqs. (24), (30), (31), and (32) show that hydrostatic equilibrium under general relativistic conditions ($T_c > 5\times10^{13}\,M_\odot/M$ degrees) requires large amounts of internal energy for pressure support. Can nuclear reactions provide this energy? The conversion of hydrogen into helium releases 0.7% of the rest mass energy of that fraction of the star consumed. An upper limit is set by the fraction of the hydrogen in the central regions

($\sim$0.3 M) which can be burned before gravitational red shifts terminate energy release. It is of the order of 15%. Thus 10^{-3} Mc^2 can be made available in hydrogen burning and consequently

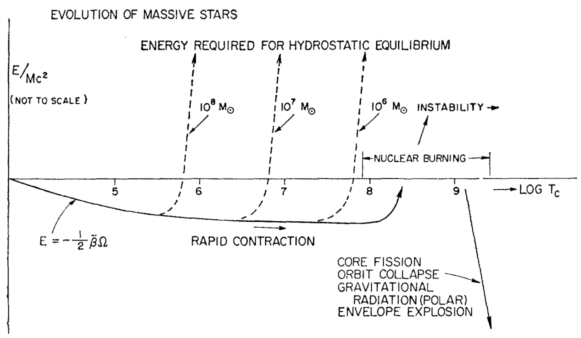

FIG. 1. The energy required for hydrostatic equilibrium in massive stars is shown schematically (dashed lines) as a function of the central temperature during contraction. The possible evolution during rapid contraction before nuclear burning and collapse afterward is also shown. The instability during nuclear burning is emphasized.

$$E_{\text{eq}}/Mc^2 \approx 3.3 \times 10^{-27}(M/M_\odot)T_c^2 \leqslant 10^{-3} \quad \text{for } n = 3\,. \qquad (35)$$

Hoyle and Fowler (1963) estimate that hydrogen burning through the CNO-cycle in massive stars where $\rho_c \sim 0.01$ to 0.1 g cm^{-3} occurs at $T_c \sim 8 \times 10^7$ deg so that

$$M/M_\odot \leqslant 10^8 \qquad (4\,{}^1\text{H} \rightarrow {}^4\text{He})\,. \qquad (36)$$

Helium burning and subsequent exothermic reactions in the core release only 3×10^{-4} Mc^2 and occur at still higher temperatures where the equilibrium energy required is very great indeed. Equation (35) thus indicates that for $M \gtrsim 10^8$ $M_\odot$ nuclear reactions cannot supply the internal energy necessary for hydrostatic equilibrium.

There is an additional question. For $M < 10^8$ $M_\odot$ is the hydrostatic equilibrium stable or unstable? The problem is a difficult one when relativistic considerations are taken into account but the custommary classical argument[12] indicates that a star in equilibrium is not stable to sudden (adiabatic) contraction or expansion when $E_{\text{eq}} > 0$. On sudden contraction, the adiabatic increase in pressure is not as great as that required for the new equilibrium so contraction continues. Similarly, sudden expansion is followed by further expansion.

For stars with $M > 10^6$ $M_\odot$, Fig. 1 indicates that the contraction begins before the onset of nuclear burning. It is thus reasonable to assume that contraction continues during the nuclear burning although it may well be slowed somewhat by the release of nuclear energy. Under these circumstances we return to Eq. (18) and let f be less than unity. The equation then shows that the positive relativistic terms can be balanced by the first term and the total energy held constant as must be the case when the nuclear energy is small compared to that required for equilibrium. In Eq. (18) one can now set $\bar{\epsilon} \sim 1$ in all terms. Numerically it is found that

$$\frac{E}{Mc^2} = -\tfrac{3}{2}\,(1 - \bar{f})\left(\frac{GM}{Rc^2}\right) + (9.2\,\bar{f} - 4.1)\left(\frac{GM}{Rc^2}\right)^2 + \frac{E_{\text{dyn}}}{Mc^2} \quad \text{for } n = 3 \qquad (37)$$

and

$$\frac{E}{Mc^2} = -3.8 \times 10^{-14}(1 - \bar{f})\left(\frac{M}{M_\odot}\right)^{\frac{1}{2}} T_c + 6.0 \times 10^{-27}(\bar{f} - 0.45)\left(\frac{M}{M_\odot}\right) T_c^2 + \frac{E_{\text{dyn}}}{Mc^2}\,. \qquad (38)$$

For $M = 10^8$ $M_\odot$ the first and second terms cancel for $\bar{f} = 0.7$ at $T = 10^9$ degrees and for $\bar{f} = 0.6$ at $T = 2 \times 10^9$ degrees. Note that the general relativistic second term vanishes and becomes negative for $\bar{f} \leqslant 0.45$.

The above results indicate that a situation intermediate between hydrostatic equilibrium ($\bar{f} = 1$) and free fall ($\bar{f} = 0$) can lead to a constant total energy. As $\bar{f}$ decreases it is only necessary that E_{dyn} increase, as indeed will be the case. Thus, as shown in Fig. 1, rapid contraction at some fraction of the free fall rate occurs for massive stars with $M \geqslant 10^6$ $M_\odot$ early in their evolutionary history.

The characteristic free fall time for the outer layers of a star originally at radius R is given by

$$\tau_{ff} \approx \left(\frac{R^3}{2GM}\right)^{\frac{1}{2}} \approx 6 \times 10^{-14}\, R^{\frac{3}{2}} \left(\frac{M_\odot}{M}\right)^{\frac{1}{2}} \text{sec}\,. \qquad (39)$$

Choose R at the moment when the minimum in E_{eq}/Mc^2 is reached, namely

$$R \approx \frac{1}{0.15\beta} R_g \approx 0.8\, R_g \left(\frac{M}{M_\odot}\right)^{\frac{1}{2}} \approx 2.5 \times 10^5 \left(\frac{M}{M_\odot}\right)^{\frac{3}{2}} \text{cm}\,. \qquad (40)$$

This radius is 2.5×10^{14} cm for $M = 10^6$ $M_\odot$, 2.5×10^{17} cm for $M = 10^8$ $M_\odot$, and 2.5×10^{20} cm for $M = 10^{10}$ $M_\odot$. (These values are approximate for a polytropic structure with index 3 and can be somewhat larger if the index exceeds 3, e.g., $R \sim 10^{18}$ cm for $M \sim 10^8\ M_\odot$.) The free fall time for the outer radiating shell of a star is thus

$$\tau_{ff} \approx 2.5 \times 10^{-13}(M/M_\odot)^{\frac{3}{2}}\ \text{yr}\,. \tag{41}$$

This time is 7.5×10^{-3} yr for $M = 10^6\ M_\odot$, 25 yr for $M = 10^8\ M_\odot$, and 7.5×10^4 yr for $M = 10^{10}\ M_\odot$, and can be somewhat increased if polytropes with index >3 are considered. In addition, from the arguments advanced in the previous paragraph, the actual time will be somewhat longer than the free fall time especially for the outer layers where f may be only slightly less than unity during the initial stage of the contraction where the greater part of the time is spent. However, even for $f = 0.99$, an increase only by a factor $(1 - f)^{-\frac{1}{2}} = 10$ is obtained. Taking all factors into account the collapse time for $M = 10^8\ M_\odot$, for example, could be as long as 10^3 years.

Thus, it will be clear that only for the very highest masses under consideration, namely $10^{10}\ M_\odot$, are the collapse times comparable to the 10^5 to 10^6 year lifetimes associated [6,7] with the stellar stage of radio sources. For the smaller mass range it is necessary to give up the special symmetry inherent in spherical collapse and to look to other mechanisms which will lead to a period of quasi-stability for the radio *stars*. One possibility is rotation. A large initial rotation can lead to a flattened disk configuration which can fragment into smaller stars with characteristic times of stable evolution comparable to the observed lifetimes. Somewhat smaller rotations can certainly slow contraction along the two axes normal to that of the rotation. Contraction of the outer material along the third coordinate will be impeded if turbulence is set up by the catastrophic collapse of the central regions of the star described in Part III to follow.

If it is granted that rotation, turbulence, convection, or other mechanisms set time scales for the collapse of the outer layers of massive stars comparable to those suspected for radio *stars*, then it is possible to use the original estimates of Hoyle and Fowler[4] to demonstrate that the nuclear resources of the star can meet the luminous energy requirements if not the requirements for hydrostatic equilibrium. They estimated the luminosity of a massive star to be

$$\begin{aligned} L &\approx 5 \times 10^4 \frac{M}{M_\odot} L_\odot \\ &\approx 2 \times 10^{38} \left(\frac{M}{M_\odot}\right) \text{erg sec}^{-1} \\ &\approx 2 \times 10^{46}\ \text{ergs sec}^{-1} \quad \text{for } M = 10^8\ M_\odot\,. \end{aligned} \tag{42}$$

This estimate was based on a polytropic structure of index 3 but, unlike the stellar radius, is relatively independent of the index or for that matter of more complicated possibilities in internal structure. When the calculation was published,[4] optical luminosities of this great magnitude had not been reported and radio luminosities had been found only up to 10^{45} erg sec^{-1}. Subsequently[6,7] it was shown that the radio *stars* are extragalactic and that their optical luminosities are indeed of order 10^{46} erg sec^{-1}.

Since $10^{-3}\ Mc^2 \approx 2 \times 10^{51}\ (M/M_\odot)$ ergs can be made available by hydrogen burning it will be clear from comparison with Eq. (42) that the duration is independent of mass and is given by

$$\tau(4\,{}^1\text{H} \rightarrow {}^4\text{He}) \approx 10^{13}\ \text{sec} \approx 3 \times 10^5\ \text{yr}\,. \tag{43}$$

This interval matches the estimated lifetimes for radio *stars* quite well. It is not necessary that the hydrogen burning in the central regions extend over this interval. The stellar dimensions are such that heat transfer by convection from the interior to the surface and by radiation to the exterior actually sets the time scale. In the model discussed in Part III the nuclear energy from hydrogen burning is released in a short interval. However, during the 3×10^5 year interval when contraction is impeded by rotation, etc., the luminosity requirements must be met *in toto* by the nuclear rather than by the gravitational resources of the star.

Another point of interest during the quasi-stable stellar stage concerns the classical pulsation period of massive stars. This period is given by

$$\Pi \backsim (G\bar{\beta}\bar{\rho})^{-\frac{1}{2}} \backsim 10\ \text{yr} \tag{44}$$

for $\bar{\beta} \sim 10^{-3}$ and $\bar{\rho} \sim 10^{-7}$ g cm^{-3}. This mean density occurs for $M = 10^8\ M_\odot$ just beyond the point in the contraction where $E_{\text{eq}} = 0$. Smith and Hoffleit[13] observe periods of this order in the luminosity fluctuations of the radio *star* 3C273.

III. ENERGY RELEASE IN COLLAPSE OF THE CORE OF A MASSIVE STAR

Michel[14] has recently discussed the collapse of massive stars after the exhaustion of nuclear energy. He argues on general grounds that the central core of the star will collapse much more rapidly than the outer regions so that a separation of core from envelope will characterize the event. Moreover, if energy is transferred in some manner from core to envelope as suggested by Hoyle and Fowler,[4] then the envelope may actually explode away from the imploding core.

[13] H. S. Smith and D. Hoffleit, Nature **198**, 650 (1963).
[14] F. C. Michel, Astrophys. J. **138**, 1097 (1963).

These ideas can be clarified by reference to Fig. 2 which shows the run of certain variables: density ρ, effective temperature $T/\mu\beta$, mass M_r interior to

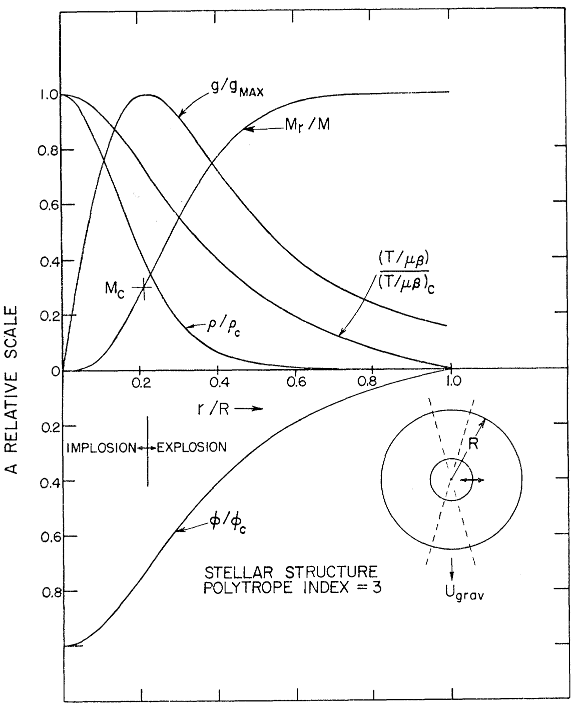

FIG. 2. The run of the variables, ρ, $T/\mu\beta$, φ, M_r, and g vs radius in a polytrope with index $n = 3$. The mass of the core containing all the material below $g = g_{\max}$ is seen to be $M_c = 0.3\ M$. The polar loss of gravitational radiation accompanying implosion of the core and explosion of the envelope is shown.

radius r, and gravitational potential φ, all in the classical approximation. Figure 2 has been drawn for a polytrope of index $n = 3$ on the grounds that this polytrope will match the internal structure at least approximately during the quasi-static period in which contraction is impeded by rotation, turbulence, and convection. The decrease in ρ, $T/\mu\beta$, and $|\varphi|$ with increasing R is illustrated as well as the increase in M_r. The acceleration due to gravity is seen to rise linearly at small r, reach a maximum at $r = 0.22\ R$ and decrease thereafter. Upon the failure of internal pressure support the linear region will collapse homologously ($\ddot{r} \approx g \propto r$) and it can be argued that the region within $g = g_{\max}$ will collapse in an approximately similar fashion at a much greater rate than the outer regions where g is smaller. The figure shows that $M_r \sim 0.3\ M$ at $g_{\max}$ and in what follows it will be taken that the collapsing core has this mass, $M_c \sim 0.3\ M$.

Michel[14] suggested that a loss of energy by the collapsing core, as for example by neutrino emission, reduces its gravitational mass and in turn the absolute value of the gravitational potential throughout the envelope is reduced. For radiation support dominant ($\beta \ll 1$), the envelope binding energy is approximately zero before core collapse so the decrease in gravitational potential energy results in an excess energy which is dissipated in explosion of the envelope.

On the other hand, HFB[2] showed that the energy loss by the most effective neutrino–antineutrino production mechanism

$$e^+ + e^- \rightarrow \nu + \bar{\nu} \tag{45}$$

does not lead to a substantial decrease in the mass-energy content of the core of stars with $M > 10^6\ M_\odot$. Their calculation can be simplified somewhat for such large masses. For an observer comoving with a representative sample of the internal matter, the energy loss from reaction (45) is

$$\frac{dU_\nu}{dt} \backsim \frac{4.3 \times 10^{15}}{\rho}\, T_9^9 \text{ erg g}^{-1}\text{ sec}^{-1} \quad \text{for } T_9 > 2\,. \tag{46}$$

For this same observer the density–temperature relation in the material at hand is very closely that given by the adiabatic relation (as the final result demonstrates) namely

$$\rho \backsim 2.8 \times 10^5 (M_\odot/M_c)^{\frac{1}{2}}\, T_9^3\,. \tag{47}$$

The pressure gradient in the material is sufficient to balance the classical gravitational term in Eq. (16). However, during the collapse the relativistic terms become comparable to the classical terms, imbalance results and inward acceleration comparable to that in classical free fall follows. This can be seen from Eq. (12) by increasing g to $\sim 2g$, setting $dp/dr \sim -\rho g$, thus finding $dv/dt \sim -g$ just as for $dp/dr \sim 0$ classically. This permits the use of the classical free fall relation

$$dt \backsim \frac{1}{(24\pi G\rho)^{\frac{1}{2}}}\,\frac{d\rho}{\rho}\,. \tag{48}$$

The total energy loss can be calculated by substituting Eq. (47) into Eq. (46) and integrating the resulting dU_ν/dt over dt as given by Eq. (48). It might be argued that this loss should be calculated in the time coordinate of an external observer. However, the red shift in du_ν/dt and the time dilation in dt will compensate at least in first order. It is necessary only to set the upper limit on t equal to that finite value measured by the comoving observer when the core has reached the gravitational radius $2GM_c/c^2$. This time can be evaluated in terms of the limiting density given by Eq. (26). It should be recalled that the density throughout the core is substantially constant before collapse ($0.4 < \rho/\rho_c < 1$ from Fig. 1) and remains so during the homologous implosion. Thus the integrated energy loss is

$$E_\nu = \int_0^{t_{\max}} \frac{dU_\nu}{dt}\, dt \backsim \int_0^{\rho_{\max}} \frac{dU_\nu/dt}{(24\pi G\rho)^{\frac{1}{2}}} \frac{d\rho}{\rho}$$

$$\backsim 90 \left(\frac{M_c}{M_\odot}\right)^{\frac{1}{2}} \int_0^{\rho_{\max}} \rho^{\frac{1}{2}}\, d\rho$$

$$\backsim 1.5 \times 10^{26} \left(\frac{M_\odot}{M_c}\right)^{\frac{1}{2}} \text{erg g}^{-1}\,. \tag{49}$$

In terms of the fractional loss per unit of rest mass energy, this becomes

$$\frac{\Delta M_c}{M_c} = \frac{E_\nu}{c_2} \backsim 1.7 \times 10^5 \left(\frac{M_\odot}{M_c}\right)^{\frac{1}{2}}. \tag{50}$$

For $M_c = 0.3\, M$, $M = 10^8\, M_\odot$ the result is a fractional loss in mass by the core equal to $\sim 10^{-6}$ and even smaller for greater masses. Reaction (45) is not effective in the Michel mechanism for the "transfer" of energy from core to envelope.

Gravitational radiation was suggested as a mode of energy loss by Hoyle and Fowler[4] and by Hoyle[15] but numerical estimates were not given. Gell-Mann[16] emphasized the importance of gravitational radiation particularly in the case of a rotating star where the rotation can lead to fission of the collapsing core. The interval during which a prolate deformation of the core develops and ultimately results in fission will not be discussed in detail here and only one feature will be emphasized. It is during this interval that the nuclear energy of the core is considered to be transferred by convection and radiative transfer to the *inner* part of the envelope. Convection is rapid, temperature gradients are large and the distances are short compared to those involved in the previous problem of bringing energy to the stellar surface during the collapse interval for a spherical nonrotating star.

The amount of energy lost on the rotating binary model by gravitational radiation can be calculated in first order using equations given by Landau and Lifshitz.[17] Let the fission result in two spherical components of mass $\frac{1}{2}\, M_c$ in contact with distance between their centers equal to R_f. Let R_f be substantially larger than the limiting gravitational radius GM_c/c^2 of each component of the binary. Just after fission assume that each component collapses at the free fall rate to this radius. To a local observer this time is given approximately by

$$\tau_{ff} \backsim \left(\frac{R_f}{R_g}\right)^{\frac{1}{2}} \left(\frac{R_f}{c}\right) \backsim 6 \times 10^{-14} R_f^{\frac{3}{2}} \left(\frac{M_\odot}{M_c}\right)^{\frac{1}{2}} \text{sec}$$

$$\backsim \left(\frac{R_f}{R_g}\right)^{\frac{3}{2}} \left(\frac{R_g}{c}\right) \backsim 10^{-5} \left(\frac{R_f}{R_g}\right)^{\frac{3}{2}} \left(\frac{M_c}{M_\odot}\right) \text{sec}\,, \tag{51}$$

where R_g is now $2GM_c/c^2$ and is equal to the sum of the gravitational radii of the two components. Gravitational radiation is not emitted during the spherical collapse of each binary since there is no quadrupole moment.

It will be seen in the sequel that the free fall time for each binary is short compared to the collapse time for the binary orbit and thus at this point the model becomes a binary of two collapsed stars rotating with the angular velocity at the time of fission. The square of this angular velocity is

$$\omega_f^2 = GM_c/R_f^3\,. \tag{52}$$

Landau and Lifshitz[17] give the rate of gravitational energy radiation as

$$dU_g/dt = (2G/5c^5)M_c^2 r^4 \omega^6 = \tfrac{2}{5}\, G^4 M_c^5/c^5 r^5 \tag{53}$$

and the resulting rate of contraction of the orbit as

$$-dr/dt = \tfrac{2}{5}\, (2GM_c/rc^2)^3 c = \tfrac{2}{5}\, (R_g/r)^3 c\,. \tag{54}$$

The local time for collapse of the orbit is

$$\tau_{\text{orb}} = \frac{5}{2c} \int_{R_g}^{R_f} \left(\frac{r}{R_g}\right)^3 dr \backsim \frac{5}{8} \left(\frac{R_f}{R_g}\right)^3 \frac{R_f}{c}$$

$$\backsim \frac{5}{8} \left(\frac{R_f}{R_g}\right)^4 \frac{R_g}{c} \backsim 10^{-5} \left(\frac{R_f}{R_g}\right)^4 \left(\frac{M_c}{M_\odot}\right) \text{sec}\,. \tag{55}$$

In the approximations we have set $R_f > R_g$. If the orbit collapse time is to match the lifetime of radio *stars*, $\sim 10^{13}$ sec, then $R_f \sim 500\, R_g$ for $M_c = 0.3 \times 10^8\, M_\odot$. The value for R_f is determined by the angular velocity ω_0 and radius R_0 at the time the condensing material is decoupled from the surrounding medium so that its angular momentum is conserved thereafter. Thus, with $\rho_0 = M/(\frac{4}{3}\, \pi R_0^3) \sim M_c/R_0^3$

$$\omega_f^2 R_f^4 = GM_c R_f = \omega_0^2 R_0^4$$

or

$$\frac{R_f}{R_g} = \frac{2\omega_0^2 R_0^4}{c^2 R_g^2} = \frac{\omega_0^2 c^2}{2G^2 M_c^{\frac{2}{3}} \rho_0^{\frac{4}{3}}} \backsim 500\,. \tag{56}$$

For $\rho_0 = 10^{-24}$ g cm^{-3} and $M_c = 0.3 \times 10^8\, M_\odot$, Eq. (56) yields $\omega_0 = 3 \times 10^{-19}$ rad sec^{-1}. This relatively low value indicates that initially the condensing sys-

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[15] F. Hoyle, New Scientist **17**, 681 (1963).
[16] M. Gell-Mann, private communication (1963).
[17] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962), p. 366.

tem must have low angular velocity or must lose it before decoupling from the surrounding medium.

The integrated energy loss is

$$U_g = \int_{R_f}^{R_g} \frac{dU_g}{dt}\, dt = \tfrac{1}{8}\, GM_c^2 \int_{R_g}^{R_f} \frac{dr}{r^2}$$

$$= \tfrac{1}{16}\,(1 - R_g/R_f) M_c c^2 \backsim 0.02\, Mc^2 \,. \quad (57)$$

In this last approximation we have set $M_c = 0.3\, M$ and $R_g < R_f$.

It will be noted that one-half of the energy loss occurs in the collapse from R_f to $2R_g$. Thus this first order calculation is probably correct in order of magnitude. Even so, the major part of the gravitational radiation (90% while $10R_g > r > R_g$) is emitted during the short interval at the end of the binary orbit collapse. It marks a sudden injection of energy just when the nuclear energy has been exhausted by radiation during the quasi-stellar period. It is suggested that this injection may lead to the development of an extended radio source.

It will be noted that the energy loss given by Eq. (57) is just the difference in *classical* binding energy of the two components between R_f and R_g. The energy available is classical but the radiation mechanism is relativistic; gravitons carry off the energy from the core. The angular distribution of the gravitational radiation from a rotating binary is $1 + 6\cos^2\theta + \cos^4\theta$ so the polar emission is eight times the equatorial. The interaction of the gravitons with the envelope will be complicated but the expulsion of the envelope should be polar although not to the degree just quoted. (The gravitational force is a long range one.) This suggests some correspondence with the observation that the strong, extended radio sources consist of two components which have conserved momentum in moving apart.

The efficiency of energy transfer to the envelope is estimated[14] to be about 40% so that the energy of explosion of the envelope is

$$E_{\text{env}} \backsim 8 \times 10^{-3}\, Mc^2 \backsim 10^{52}\, M/M_\odot \text{ erg}$$

$$\backsim 10^{60}\text{–}10^{62} \text{ erg} \quad \text{for } M = 10^8\text{–}10^{10}\, M_\odot \,. \quad (58)$$

In the nuclear case we found $E \sim 10^{-3}\, Mc^2$. Here the fractional release is almost ten times as great but is still not quite 1% of the full rest mass energy.

This result meets the energy storage requirements of the strongest known radio sources only if the mass of the primary energy emitting region approaches $10^{10}\, M_\odot$. Gravitational radiation may well play the principal role in the release of energy from massive collapsing stars in that it is about ten times as effective as nuclear mechanisms before the ultimate collapsed state is reached. On conventional gravitational theory no radiation of any kind can be emitted once the gravitational radius R_g is reached. On the other hand, Hoyle and Narlikar[18] have proposed a theory in which mass energy can be radiated after a massive star has reached the collapsed state. Only through additional observations and studies will it become clear whether some modification of the orthodox Einstein theory of gravitation is necessary.

At the same time many important problems remain. What is the origin of the large amounts of matter involved? Do massive stars form in the center of galaxies or in intergalactic space? Do they form from gas clouds or collections of stars? Do radio *stars* ultimately become extended radio sources, the possibility discussed in this paper, or is there no connection between them? In conventional theory, do the first order calculations give the correct order of magnitude for the loss by gravitational radiation? How is the energy of the exploding envelope transformed with high efficiency into the magnetic fields and the high energy particles which are required if the radio emission is synchrotron radiation? Are shock wave phenomena involved or is high energy physics involved through the energetic decay of some still undiscovered particle?

Twenty-five years ago Robert Oppenheimer and his students began the study of massive stars. Today, the strong radio sources may indicate that such massive stars exist and, if so, further study of these peculiar objects may reveal answers to difficult but interesting questions.

IV. SUMMARY

The contents of this summary pertain in particular to a star with mass equal to $10^8\, M_\odot$ and in general to stars with mass in the range excess of $10^6\, M_\odot$.

(1) In a nonrotating, spherically symmetric, massive star, general relativistic considerations become important and gravitational collapse sets in at radius $R \sim 10^{18}$ cm and central conditions $\rho_c \sim 4 \times 10^{-10}$ g cm^{-3}, $T_c \sim 2.5 \times 10^5$ deg. Collapse to the gravitational radius $R_g \sim 3 \times 10^{13}$ cm occurs in a local time interval $\sim 10^3$ years for the outer regions and ~ 1 year for the inner regions. Large red shift effects preclude the release of significant amounts of energy from such a rapidly collapsing system.

(2) Rotation, assisted by internal turbulence and convection, is suggested as a possibility in impeding rapid gravitational collapse in massive stars. Rota-

[18] F. Hoyle and J. V. Narlikar, Proc. Roy. Soc. (London) **A273**, 1 (1963).

tion can lead to fission of the rapidly collapsing core before collapse of the envelope has reached appreciable velocity. During the development of the prolate deformation which leads to fission, the core releases nuclear energy in amount $\sim 10^{-3}\, Mc^2 \sim 10^{59}$ ergs into the envelope. This energy is sufficient to meet the luminosity requirement of the radio *stars* for 10^5 to 10^6 years. Upon fission the binary components collapse in $\sim$0.1 year to their gravitational radii. A turbulent, quasi-stable envelope of convecting, radiating material surrounds the rotating binary system. Other more complicated nonspherical internal structures could conceivably support the radiating envelope.

(3) Appropriate choices for the parameters involved can be made which lead to lifetimes for the binary system also in the range 10^5 to 10^6 years. In a relatively short interval ($\sim$0.1 year) at the end of this period, gravitational radiation from the rotating binary, which does have a quadrupole moment, injects energy into the envelope material in amount $\sim 10^{-2}\, Mc^2 \sim 10^{60}$ ergs. It is suggested that the resulting polar explosion may lead to the development of the strong, extended radio sources with at least two components.

(4) On the model discussed it is found that the gravitational resources of a massive star exceed the nuclear resources by only a factor of ten. Only 1% of the rest mass energy is made available for all forms of radiation. This and other problems are noted briefly at the end of Part III.

ACKNOWLEDGMENTS

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The Calculation of Stellar Pulsation*

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INTRODUCTION

In this paper we report methods of computation which have been developed to provide a theoretical understanding of the RR Lyrae and Cepheid type pulsating stars. The results reported are intended to illuminate the methods of calculation and to provide insight into the physical processes in these stars. A survey of pulsation in RR Lyrae models[1,2] using these methods has also been carried out and will be reported soon in another journal. A survey of pulsation in Cepheid models has been initiated and will be continued.[3]

The methods reported here arose from investigations[4] (referred to as I) on the energy transport in the hydrogen ionization zone of giant stars. In that paper, some preliminary numerical integrations of the equations of motion were reported, and the possibility of spontaneous generation of oscillations or pulsation was demonstrated. The machine code used at that time was, however, not suitable for more extensive calculations and the work reported on here is the refinement and extension of the earlier calculations.

The general idea behind these calculations is that the observed pulsation motions in Cepheids and RR Lyrae (and other related) stars arise spontaneously because of the particular physical properties of the envelopes. The relevant physical properties are the equation of state and the opacity. The method of attack is to integrate the time-dependent equations of hydrodynamics (with spherical symmetry) and heat flow by numerical means.

* Work supported in part by the Office of Naval Research and the National Aeronautics and Space Administration.

[1] R. F. Christy, Astron. J. **68**, 275 (1963).

[2] R. F. Christy, Astron. J. **68**, 534 (1963).

[3] A. N. Cox, K. H. Olsen, and J. P. Cox [Astron. J. **68**, 276 (1963)] have reported some somewhat similar calculations on Cepheid models. Unfortunately, they have not included the deeper regions of the envelope or the hydrogen ionization region near the surface. As a result, their calculations cannot be compared in detail with the observations.

[4] R. F. Christy, Astrophys. J. **136**, 887 (1962).