

### 5. ACKNOWLEDGMENTS

All my years as a physicist have been informed by what Professor J. Robert Oppenheimer, as teacher, friend, and Director, has given me. I hope the spirit of those past years will be evoked if I mention in gratitude also our old Pasadena friend, Richard Tolman. It is a pleasure to recall more recent discussions

with my associates Ethel and Irwin Tessman of Purdue University, Malcolm Skolnick of the Institute for Advanced Studies, David Hawkins of Elementary Science Study and the University of Colorado, and V. F. Weisskopf of CERN. The quality of their help, like that of my old teachers, is fairly to be appraised not by what I have written, but by these earnest thanks.

## Structure, Substructure, and Superstructure\*

CYRIL STANLEY SMITH

*Massachusetts Institute of Technology, Cambridge, Massachusetts*

Anyone who works with the microscope for an intellectual or a practical purpose will frequently pause for a moment of sheer enjoyment of the patterns that he sees, for they have much in common with formal art. What follows is an attempt to extend into a more general field some views on the nature of organization and relationships that arose during many years of study of the microstructures of metals and alloys.<sup>1</sup> In a landscape painting of the Far East, a rock in the foreground with cracks and crystalline texture is often echoed in a distant mountain with cliffs, chasms, wrinkles, and valleys; a tree may be related to a distant forest or a turbulent and eddied stream to a distant tranquil pond. Each part with its own structure merges into a structure on a larger scale. Underlying structures are imagined as a necessary basis for the visible features. The connectivity of all is suggested by the branching treelike element of the design. Both separateness and continuity are interwoven, each necessary to the other and demonstrating the relationship between different features on a single scale and between units and aggregates on dif-

fering scales. There is an analogy between a work of art which suggests an interplay of dimensions and the real internal structure of a piece of metal or rock which results from physical interactions between the atoms and electrons composing it.

The study of microstructure on the scale within the range of the optical microscope (dimensions between a micron and a millimeter) is a somewhat old-fashioned branch of science, and it still involves a high degree of empirical observation and deduction. Far more "highbrow" is the rigorous science and simple elegant mathematics of the ideal crystal lattice considered as point groups in space. The whole field of crystal structure, mathematically developed in the nineteenth century by Bravais, Federov, and Schoenflies, was experimentally opened up by Von Laue and especially the Braggs in 1912-13, using the diffraction of x rays to reveal and to measure the periodicities and symmetries in the arrangement of planes of atoms in crystals. But the mathematical physicist must simplify in order to get a manageable model, and although his concepts are of great beauty, they are austere in the extreme, and the more complicated crystal patterns observed by the metallurgist or geologist, being based on partly imperfect reality, often have a richer aesthetic content. Those who are concerned with structure on a superatomic scale find that there is more significance and interest in the imperfections in crystals than in the monotonous perfection of the crystal lattice itself. Like the biologist, the metallurgist is concerned with aggregates and assemblies in which repeated or extended irregularities in the arrangement of atoms become the basis of major structural features on a larger scale, eventually bridging the gap between the atom and things perceptible to human senses.

\* Some readers of the *Reviews of Modern Physics* may feel affronted by the appearance of this elementary, seemingly naive, paper in their journal. But others may see a certain appropriateness, and may even enjoy this attempt to share with nonscientists the visual pleasures of the laboratory. Certainly, it would not have been written if its author had not for some years been exposed to Robert Oppenheimer's sensitive view of the world. The paper, with more illustrations, will appear in the *Vision and Value* series, a collection of essays on the contemporary scientific and artistic environment, edited by Gyorgy Kepes. This will be published by George Braziller, Inc., who have granted permission for the present publication.

<sup>1</sup> The converse relationship between aesthetics and metallurgy—the influence of the techniques discovered by craftsmen making works of art upon the development of the science of metals—was discussed at some length in C. S. Smith, *A History of Metallography* (University of Chicago Press, Chicago, 1960).

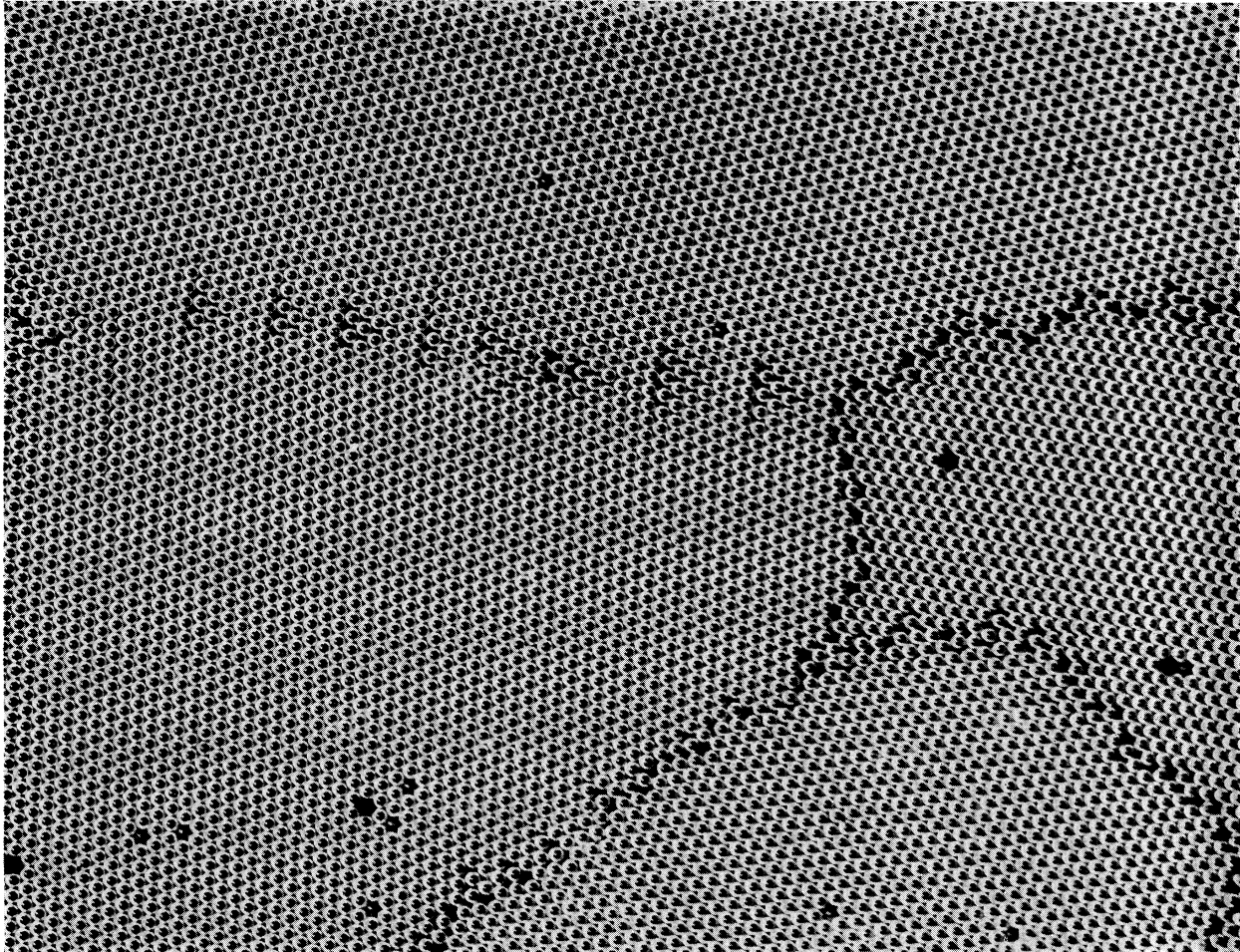


FIG. 1. Raft of tiny uniform soap bubbles showing "grain boundaries" where zones of differing orientation meet. Magnification  $\times 3$ .

The symmetry of crystals in relation to decorative ornament has been treated by many writers, none better than by Weyl in his *Symmetry* (Princeton University Press, 1952). The patterns of crystal imperfection are less commonly known, despite their prevalence and despite their relationship to so many aesthetically satisfying forms in which regularity and irregularity are intricately intertwined.

#### CRYSTALLINE AGGREGATES AND FOAM STRUCTURES

Aggregates of crystals have structures which are defined by the atomically thin layer of disordered material between the crystals. Many characteristics of their shape are shared with simple undifferentiated biological cells and the simplest common soap froth. In all these, the pertinent features are the two-dimensional surfaces that separate volumes of matter which, on this scale, is featureless. Two-dimensional interfaces are necessary to define the separate

identity of things in three dimensions. Junctions of the interfaces themselves produce linear (one-dimensional) features, and these, in turn, meet at points of zero dimension. This interaction between dimensions, the very essence of form, is expressed in mathematical beauty as Euler's Law. This simply states that, in a connected array, the number of points minus the number of line segments plus the number of surfaces and minus the number of polyhedral cells is equal to one, i.e.,

$$n_0 - n_1 + n_2 - n_3 = 1$$

where  $n_0$ ,  $n_1$ ,  $n_2$ , and  $n_3$  are the numbers of zero-, one-, two-, and three-dimensional features. There are no limitations to this, beyond the requirement of simple connectivity. Even more than Euclid, hath Euler gazed on beauty bare.

A pure metal, when cast (or, better, after a little working and heating) has a structure like that of Carrara marble—hosts of little crystals packed to-

gether irregularly. The units do not look like crystals, for they lack the symmetrical vertices and plane faces of a regular polyhedron, but internal order is there nevertheless. Although for centuries man has been fascinated by the geometrical shape and glitter of natural crystals, he has only recently come to see

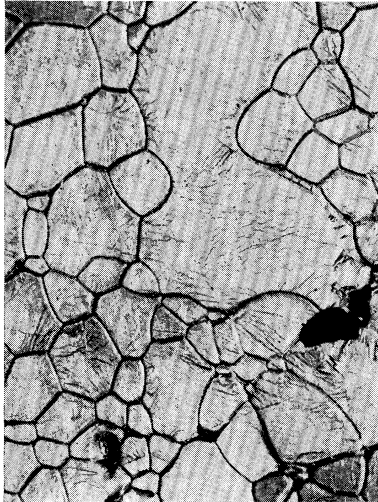


FIG. 2. Surface of overheated aluminum sheet showing the beginning of melting at the grain boundaries. Magnification  $\times 4$ . (Photo, Courtesy British Non-Ferrous Metals Research Association).

that the essence of crystallinity lies not in external shape but in the uniformity of the relationship of atoms to their neighbors within the crystal. A single isolated crystal growing from a solution or melt can grow uninterrupted in accordance with the dictates of the atomic steps on its surface. Usually this will result in a simple polyhedron, reflecting the internal order because of its effect on the rate of growth in different directions. If many crystals start to grow in the same region, sooner or later they will interfere with each other. Neighboring crystals differing in no way whatever save in the direction of their atom rows in space cannot join without some imperfection. Figure 1 illustrates this with a physical analogy. It is a magnified photograph of an array of tiny uniform bubbles floating on soapy water. The lines of disorder that form between the differently oriented areas of regularly arranged bubbles in this two-dimensional model are believed to be closely analogous to the planes of disorder constituting the boundaries between the three-dimensional crystal grains in metals, rocks, and other polycrystalline materials. The boundaries are a source of both strength and weakness and they provide sites for the beginning of any crystalline change. Though themselves invisible except at the extreme limit of resolution of modern electron and ion microscopy, they differ so much in

energy from the body of the crystals that they are easily revealed as lines of enhanced chemical attack, early melting (Fig. 2), or they can be inferred from the sudden change of crystal direction revealed by some kinds of chemical attack on the crystal surfaces (Fig. 3). Patterns like these can often be seen with the naked eye on the weathered surface of a cast brass doorknob or hand rail, or internally in clear ice which has been kept just at its melting point for several hours.

Now these boundaries, which on an atomic scale are just imperfections in a uniform stacking array, on a larger scale themselves become the basis of structure. They are, in fact, films of matter, distinguished by structure rather than composition. They must completely surround every crystal in a mass and extend in foamlike fashion continuously throughout the entire mass. Having high energy and mobility, they tend to adjust to a configuration of small area, which makes them join each other always

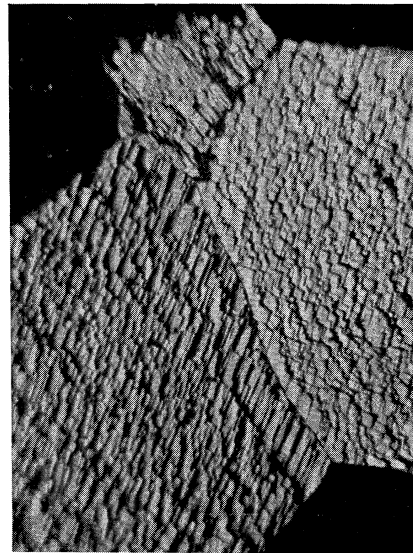


FIG. 3. Etched section of silicon iron alloy, showing the junction of three crystals. (This is an historic photograph, taken in 1898 by J. E. Stead.)

in groups of three at an angle of  $120^\circ$ , just as do the films in a froth of soap bubbles. In a mass of large bubbles of irregular size (Fig. 4) there will be differences in pressure between adjacent bubbles to match the surface tension in the curved films and to reconcile the  $120^\circ$  angle with the necessity to fill space. Since three bubbles meet at each junction, Euler's law requires the average bubble in an extended array to have exactly six sides, but there is

no requirement that each one be a hexagon, only that if there are some with more than six sides, there must be a matching number with less. The froth therefore,

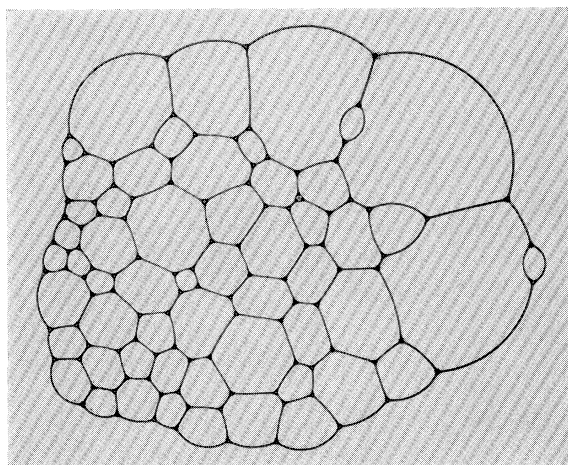


FIG. 4. Froth of irregular soap bubbles showing a cellular structure analogous to that of metals. These bubbles were blown between parallel glass plates and are essentially two dimensional. Magnification  $\times \frac{1}{2}$ .

though lacking long-range symmetry, nevertheless has very definite rules as to its composition. It is pleasing in appearance because the eye senses this interplay between regularity and irregularity. The topological requirements of space-filling rigidly determine the relationships of the whole, but allow any one cell to be of pretty much any shape, while surface tension equilibrium requires only that the films be at  $120^\circ$  to each other at the point of meeting, always three together, and it produces the pressure differences that are needed to balance the resulting curvatures. Beyond this, all depends on the accidents which brought a bubble of a particular size to a given place and surrounded it with its particular neighbors, each also with its private history.

It is interesting to compare a two-dimensional soap froth with the topologically similar but geometrically different pattern of craze marks in a ceramic glaze (Fig. 5). Though the cracks divide the surface into cells meeting three of each junction, the geometry is different from the froth because the cracks must follow the direction of stress in the glaze and a new crack joins an old one perpendicularly.

A foam in three dimensions is a bit more complicated, but depends on the same principles. To divide space into three-dimensional cells, at least six two-dimensional interfaces must meet at each point; and if surface tension dominates they will join in groups of three at  $120^\circ$  to each other along lines, forming

cell edges, which meet symmetrically at the tetrahedral angle of  $109.47^\circ$  (the angle whose cosine is  $-\frac{1}{3}$ ). This configuration of three-, two-, and one-dimensional junctions is repeated at every vertex. Curvature is necessary to connect adjacent vertices and to reconcile the short- and long-range needs. Because the polygons (cell faces) must be in groups which close around each three-dimensional cell, the average polygon will have a smaller number of sides than the hexagon which connectedly fills space in two dimensions. No single plane polygon can meet the requirements, for it would have to have 5.1043 sides in order to have corner angles of  $109.47^\circ$ . The best solution that has been proposed corresponds to a fourteen-sided body with six plane four-sided faces and eight doubly curved hexagonal faces, the mixture of polygons having on the average  $5\frac{1}{7}$  sides. This curious irrational number is of the utmost importance, though it is little appreciated. Certainly it is responsible for the prevalence of pentagons in nature, and it probably lies behind the five-fold symmetry of plants and the five fingers and toes of animals. Pentagonal faces are readily seen within a three-dimensional froth of bubbles on a glass of beer and they occur also in such disparate bodies as human

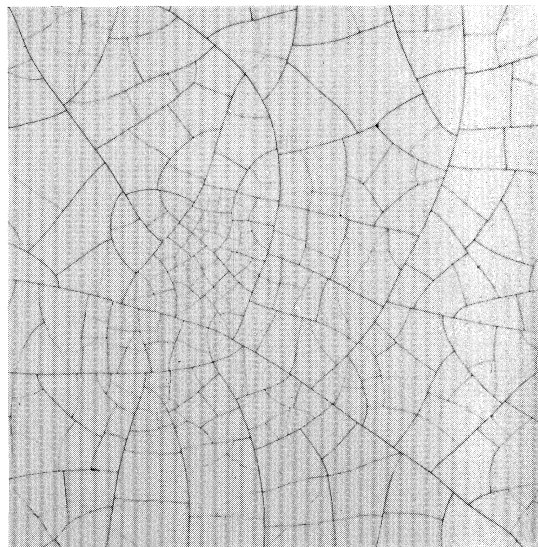


FIG. 5. Pattern of craze lines in the glaze on a ceramic surface. Magnification  $\times 0.6$ .

fat cells or metal grains (Figs. 6 and 7). Pentagons are frequent but not universal, for the ideal number is an irrational one and pentagonally faced polyhedra alone cannot fill space.

It should be noted that the external shape of the crystals in Fig. 7 reveals nothing of their inner order,



for the shape depends on the property of the disordered boundary, not the ordered crystals that are separated. A more obviously crystalline geometric structure occurs when a second crystalline phase originates in direct contact with a preexisting one, for it automatically forms in whatever definite orientation gives the lowest energy of the interface.

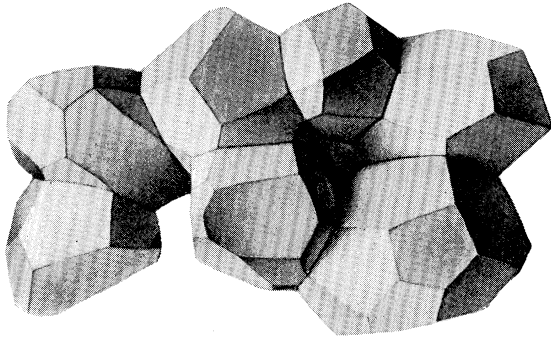


FIG. 6. The shape of cells in human fat tissue. Magnification  $\times 288$ . (F. T. Lewis, Courtesy American Academy of Arts and Sciences.)

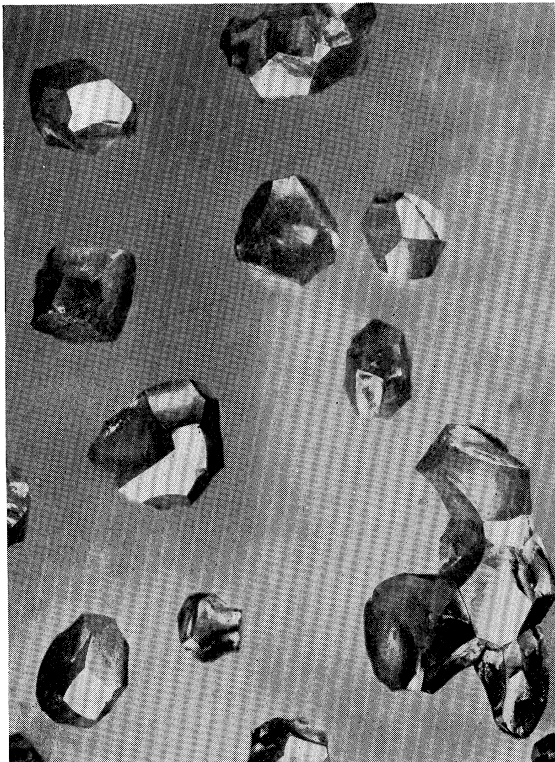


FIG. 7. Crystal grains separated from an aggregate (a coarse-grained piece of brass), showing the natural shape of crystals in random contact with each other. Note the frequency of pentagons and curved surfaces. Magnification  $\times 8$ .

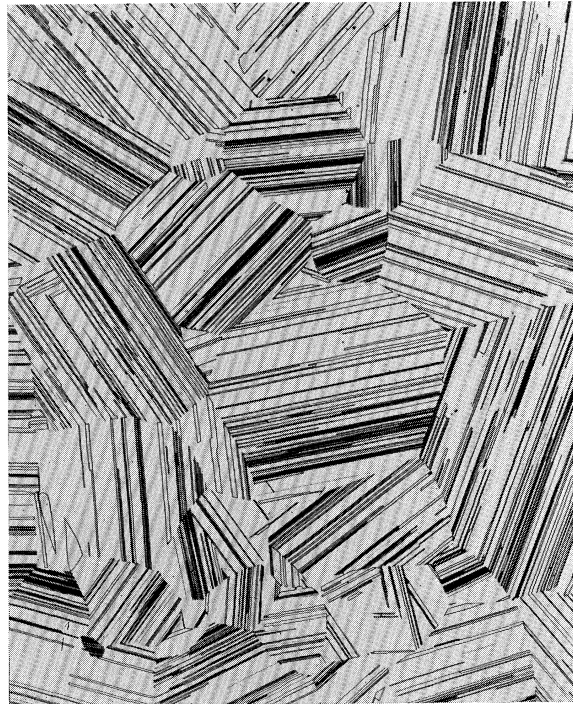


FIG. 8. Duplex crystals with bands of different composition in exact orientation relationship within one grain, but forming an overall foam structure like a pure metal, here seen in a copper-silicon alloy, worked and annealed. Magnification  $\times 120$ .

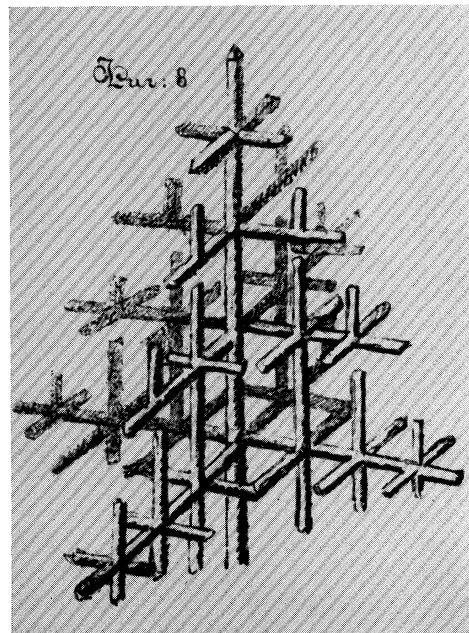


FIG. 9. Dendritic growth of iron crystal. Magnification about  $\times 300$ . (This historic drawing was made in 1876 by the Russian metallurgist D. K. Tschernoff.)

Not infrequently two different kinds of crystal will grow in close symbiotic relationship with each other, forming a duplex but well oriented unit, in a larger, irregular, foamlike aggregate. Examples of such oriented duplex structure are shown in Fig. 8.

#### BRANCHED STRUCTURES

The soap froth is the archetype for all cellular systems which for any reason are constrained toward a minimum area of interface. A quite different type of structure, though a common one, is the one that results from the growth of isolated individuals in the branched form best illustrated by a common tree. This occurs whenever a protuberance has an advantage over adjacent areas in getting more matter, heat, light, or other prerequisites for growth. Such structures occur in electric discharge, in corrosion, and even in crystal growth (Fig. 9) although in the last case the basic mechanism is given an over-all symmetry. All these branching structures start from

a point and grow lineally, but they eventually stop as the branches interfere with others already present. Until the structure encounters some extraneous obstacle (Fig. 10), the shapes are quite different from the interface-determined shapes discussed above. The structure is that of an individual, not of an aggregate.

There are other structures in which a branched treelike structure arises from an inverse mechanism. The best known example is the successive joining of many small streams to form a single large river. In brittle solids the merging together of many small cracks to form a single surface gives rise to a similar form (Fig. 11).

#### THE ROLE OF HISTORY IN STRUCTURES

In the typical process amenable to study by physics, the small number of units involved and the simplicity of their interactions gives a definiteness and reproducibility that is either invariant or is dependent in a simple way upon time. In other sciences



FIG. 10. The surface of an ingot of antimony showing dendritic crystals which have growth interference with each other. (This pattern was the mystic Star of Antimony of the alchemists.) About  $\frac{2}{3}$  natural size. (Courtesy The Science Museum, London.)

such as biology or metallurgy, the structure of complex matter must be dealt with, involving myriads of units and interacting interactions, with associ-

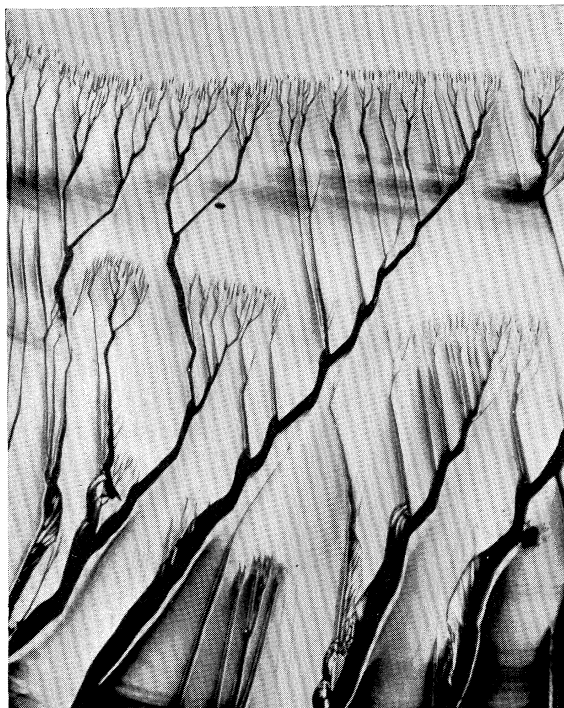


FIG. 11. Pattern of ridges formed by a crack moving in a crystal of a brittle compound of copper and magnesium,  $\text{Cu}_2\text{Mg}$ . The crack proceeded from the top to the bottom of the figure. Magnification  $\times 120$ . (Courtesy Duane Mizer, Dow Metal Products Co.)

ations of perfection and imperfection which can be combined in an almost infinite variety of ways. The structures which merit particular study because they happen to exist depend almost completely on their history—quite as much as does, though with more diversity, the present human condition. Although other structures might have been formed with equal *a priori* probability from the same units and unit processes, the whole unique sequence of atomic-scale events that actually did occur, each adding a little to a preexisting structure, was necessary to give rise to the particular array of molecules, crystals, or cells that form the final structure.<sup>2</sup> Although the ideal crystal lattice of a substance at equilibrium depends only on its composition and temperature, all other aspects of the structure of a given bit of polycrystalline matter depends upon history—the details of the

<sup>2</sup> I am indebted to John R. Platt for pointing this out. The idea that biology is essentially an historical science is elaborated by Ludwig von Bertalanffy, *Problems of Life* (John Wiley & Sons, Inc., New York, 1952).

nucleation of individual crystals, usually at sites where imperfections or heterogeneities preexist in the matrix; the locally varying rates at which the individual crystals grow into their environment, incorporating or rejecting matter as a result of the micro-processes of atom transfer; and the manner in which the crystals impinge to produce the grain boundary as a new element of structure which itself changes shape in accordance with its properties and the particular local geometry resulting from historical accidents. Far more complex, but in principle similar, things occur in biological and social organizations.

In the space-filling aggregate, the individuals limit each other. They may be arrayed randomly or regularly, but however undetermined the shape of an individual, the conditions of joining at the points where three or more meet are defined. Structure on one level, by its imperfections or variations, always gives rise to a new kind of structure on a larger scale. (Inversely, it may even be that there is no detectable structure without some underlying structure on a smaller scale. The validity of atomism depends on the tool used to find it). A local configuration will always have some connection to neighboring ones. In ever-decreasing degree every part is dependent on the whole and vice versa.

#### ON SECTIONS AND SURFACES

The structures usually observed on metals and rocks are those of plane sections cut through a three-dimensional structure, slicing through the crystal planes and boundaries at various angles, and thus introducing distortions of shape and hiding connections that may exist in the third dimension. We have become very adept at interpreting things from two-dimensional representation, indeed most of our thinking is in such terms. The two-dimensional surface of a painting can represent a straight or distorted projection or a point-perspective view of either real or imaginary things. In sculpture, the surface can be the natural surface of an object, but it is usually a cut through a body of material which has a three-dimensional structure and it reveals a surface texture with its own aesthetic qualities. Sections are subtly different from the same structures when formed against a preexisting surface. Sectioning is simpler than three-dimensional representation because there is no superposition as in projection and no change of scale as in perspective; it gives a single-elevation contour map, with volumes reduced to areas, surfaces to lines, and lines to points. If the structure is cellular and randomly oriented, representations of all possible views will be seen at various places in the sec-

tion. Depending on the orientation, certain features will be magnified in one direction. Convexity or concavity of a surface in relation to the sectioning plane produces closed isolation or extended connectivity of the linear traces on it. If the structure is not random but irregularly lamellar, e.g., the grain of wood, the variations in the third dimension can be seen as a distribution of texture in the two-dimensional slice. Some examples in which such structures are exploited are wood-veneer textures, marbled ceramics, the Damascus sword (Fig. 12) and Japanese swords

sensed or thought pattern with a preexisting one, a pattern formed in the brain's physical structure by biological inheritance and the imprint of experience. Could it be that aesthetic enjoyment is the result of the formation of a kind of moiré pattern between a newly sensed experience and the old; between the different parts of a sensed pattern transposed in space and in orientation and with variations in scale and time by the marvelous properties of the brain? The parts of a sensed whole form many patterns suggesting each other in varying scale and aspect, with

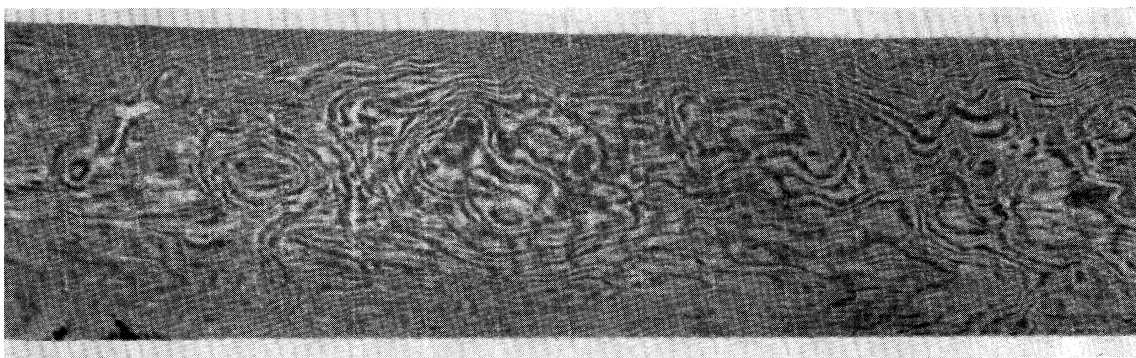


FIG. 12. Detail of a "Damascus" sword blade from the Wallace collection, London. The surface of the blade had been formed by cutting through the irregular lamellar structure which originated in the crystallization of the high-carbon steel and had maintained its identity during the forging. Magnification  $\times 1.6$ .

and tsuba (Fig. 13). These all owe much of their charm to the combined aspects of both design and texture that they possess, with effects not unlike those of woven textiles but more natural in origin and with three-dimensional overtones.

#### CONCLUSION

Do not these simple structures of crystals and the simpler ones of bubbles graphically illustrate some important features of the world and our appreciation of it, aesthetically as well as intellectually? It is the Chinese principal of *yang* and *yin*, balanced positive and negative deviations from uniformity, which, if occurring at many places must form a foam structure of cells no matter what material- or idea-space is involved. The freedom of a structural unit inflicts and suffers constraints whenever its closer interaction with some neighbors makes cooperation with others less easy. Social order intensifies the interfacial tension against a differently ordered group. Everything that we can see, everything that we can understand, is related to structure, and, as the gestalt psychologists have so beautifully shown, perception itself is in patterns, not fragments. All awareness or mental activity seems to involve the comparison of a

patterns of imperfection and disorder of one kind forming the partially ordered framework of another with an almost magical diversity depending on the degree to which local deviations from the ideal pattern are averaged out. Somehow the brain perceives the relationship and actively enjoys the rich interplay possible in patterns composed of the simplest parts, an interplay between local and long-range, between branching extension and consolidation, between substance and surface, between order and disorder.

The very nature of life is pattern-matching, whether in the simple acceptance or rejection of "food" units to fit the RNA molecules within a cell or the joining together of conforming and differentiated cells in the over-all pattern of the organism which the parts themselves both dictate and conform to. The growth of ordered but lifeless matter typically occurs by the addition of atoms or molecules to the very surface of a crystal. A not dissimilar process of structural matching is involved in the duplication of protein within a living cell, but a complete organism grows by *internal* multiplication, and the consequent burgeoning of outward movement produces the differing environments for cells which



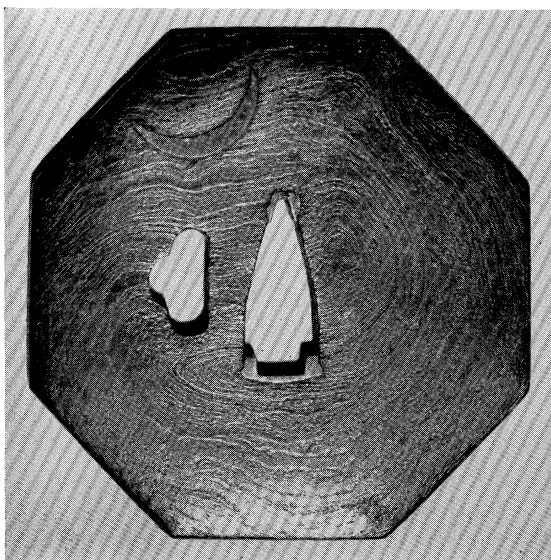


FIG. 13. A Japanese Mokumé sword guard from the collection of G. E. Hearn. The texture arises from the intentional incorporation of innumerable layers of slightly different steels into a single mass by repeated welding and forging, and then chemically etching the final surface which was cut through the forged lamellae. The moon is inlaid in silver. About natural size.

is an essential characteristic of a living organism.

There is a kind of indeterminacy, quite different in essence from the famous principle of Heisenberg

but just as effective in limiting our knowledge of nature, which lies in the fact that we can neither consciously sense nor think of very much at any one moment. Understanding can only come from a roving viewpoint and sequential changes of scale of attention. The current precision in science will limit its advance unless a way can be found for relating different but interwoven scales and dimensions. The elimination of the extraneous, in both experiment and theory, has been the veritable basis of all scientific advance since the 17th century, and has led us to a point where practically everything above the atom is understood "in principle." Sooner or later, however, science in its advance will have exhausted the supply of problems that involve only those aspects of nature that can be freshly studied in simple isolation. The great need now is for concern with systems of greater complexity, for methods of dealing with complicated nature as it exists. The artist has long been making meaningful and communicable statements, if not always precise ones, about complex things. If new methods, which will surely owe something to aesthetics, should enable the scientist to move into more complex fields, his area of interest will approach that of the humanist, and science may even once more blend smoothly into the whole range of human activity.

## Comparison of Observed and Theoretically Calculated Intensities in the Continuous Spectra of Main-Sequence B Stars\*

BENGT STRÖMGREN

*The Institute for Advanced Study, Princeton, New Jersey*

### I. INTRODUCTION

Observations of the radiation from O and B stars at wavelengths below 3000 Å have been made in recent years from rockets. Comparisons of the observed intensities with intensities calculated on the basis of model atmospheres for high-temperature stars have been carried out in a number of investigations (see

Stecher and Milligan<sup>1</sup>, Underhill<sup>2</sup>, Chubb and Byram<sup>3</sup>, Pecker<sup>4</sup>, Meinel<sup>5</sup>). For wavelengths in the range 1300–2100 Å the actual intensities appear to be smaller than the intensities predicted from model atmospheres by a considerable factor.

<sup>1</sup> T. P. Stecher and J. E. Milligan, *Astrophys. J.* **136**, 1 (1962).

<sup>2</sup> A. B. Underhill, *Space Science Reviews*, edited by C. de Jager (D. Reidel Publishing Company, Dordrecht-Holland, 1963), Vol. 1, p. 749.

<sup>3</sup> T. A. Chubb and E. T. Byram, *Astrophys. J.* **138**, 617 (1963).

<sup>4</sup> J. C. Pecker, *Ref. 2*, p. 729.

<sup>5</sup> A. B. Meinel, *Astrophys. J.* **137**, 321 (1963).

\* Supported in part by a grant from the National Science Foundation and in part by the U. S. Office of Naval Research (Contract Nonr 1858).

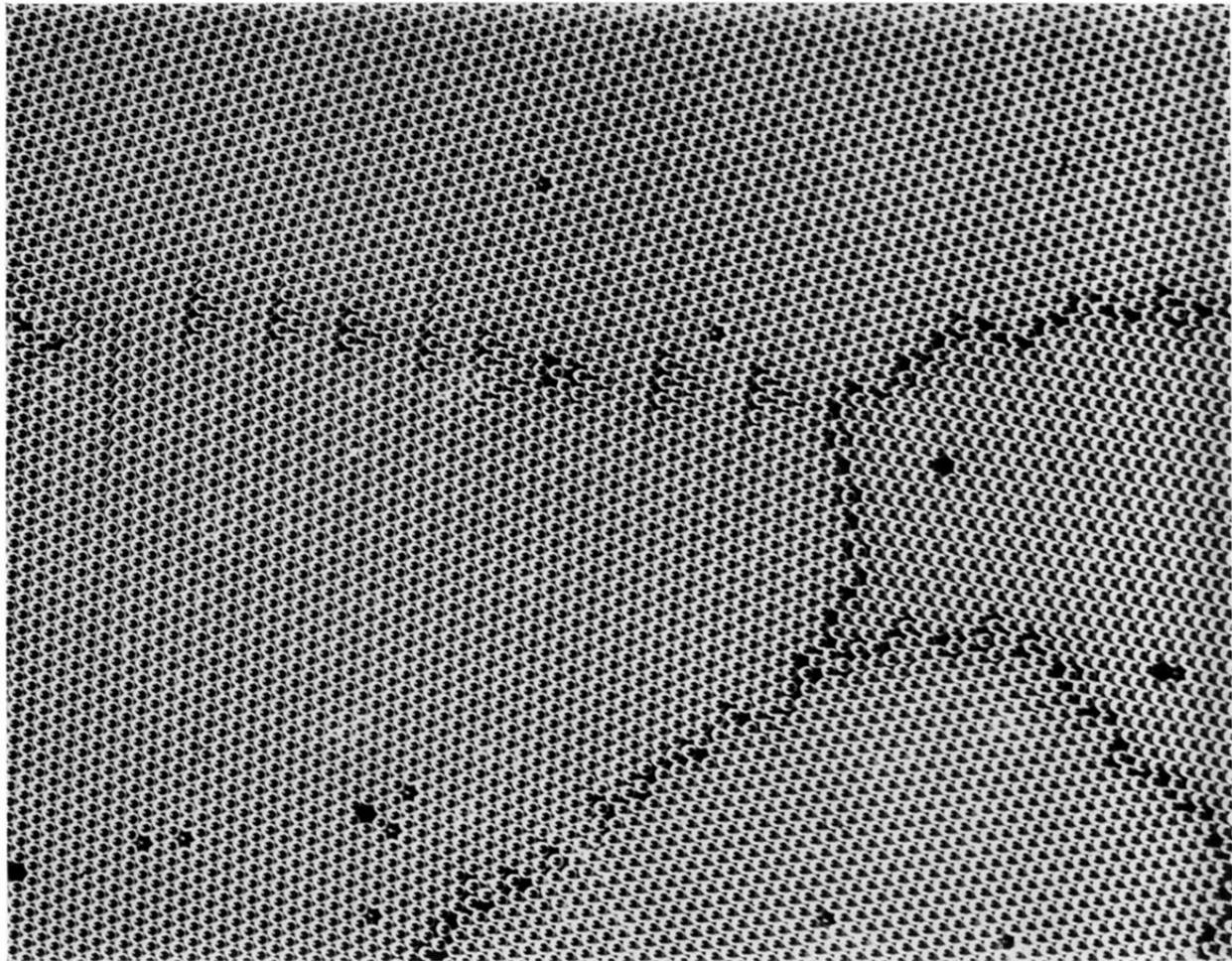


FIG. 1. Raft of tiny uniform soap bubbles showing "grain boundaries" where zones of differing orientation meet. Magnification  $\times 3$ .



FIG. 10. The surface of an ingot of antimony showing dendritic crystals which have growth interference with each other. (This pattern was the mystic Star of Antimony of the alchemists.) About  $\frac{2}{3}$  natural size. (Courtesy The Science Museum, London.)

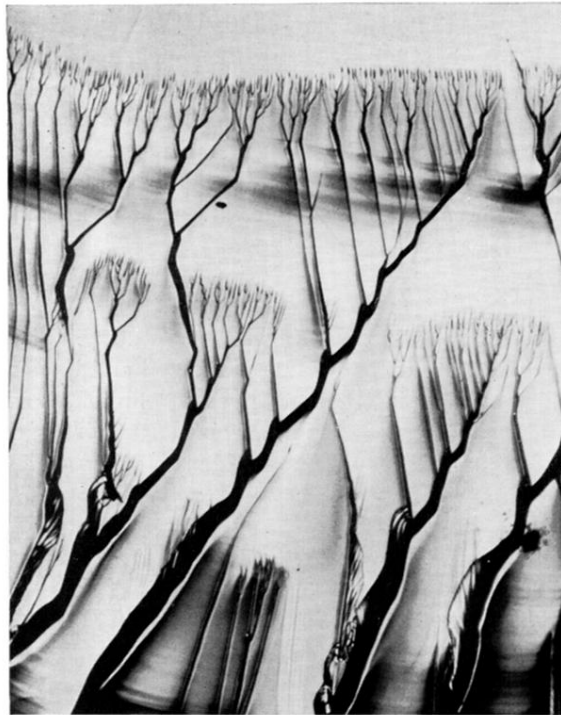


FIG. 11. Pattern of ridges formed by a crack moving in a crystal of a brittle compound of copper and magnesium,  $\text{Cu}_2\text{Mg}$ . The crack proceeded from the top to the bottom of the figure. Magnification  $\times 120$ . (Courtesy Duane Mizer, Dow Metal Products Co.)



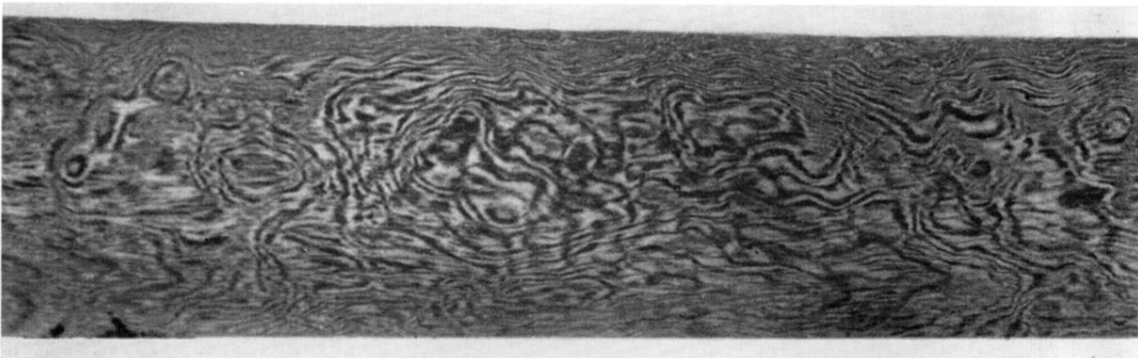


FIG. 12. Detail of a "Damascus" sword blade from the Wallace collection, London. The surface of the blade had been formed by cutting through the irregular laminar structure which originated in the crystallization of the high-carbon steel and had maintained its identity during the forging. Magnification  $\times 1.6$ .

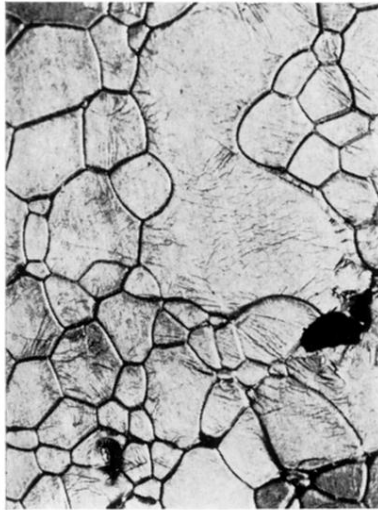


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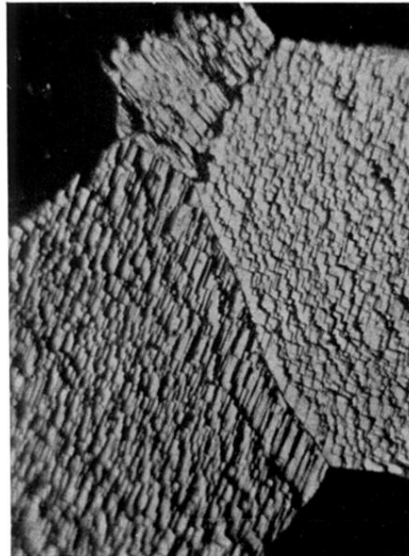


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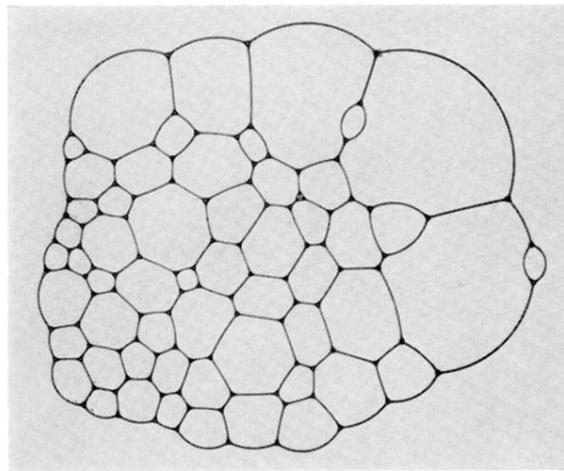


FIG. 4. Froth of irregular soap bubbles showing a cellular structure analogous to that of metals. These bubbles were blown between parallel glass plates and are essentially two dimensional. Magnification  $\times \frac{1}{2}$ .



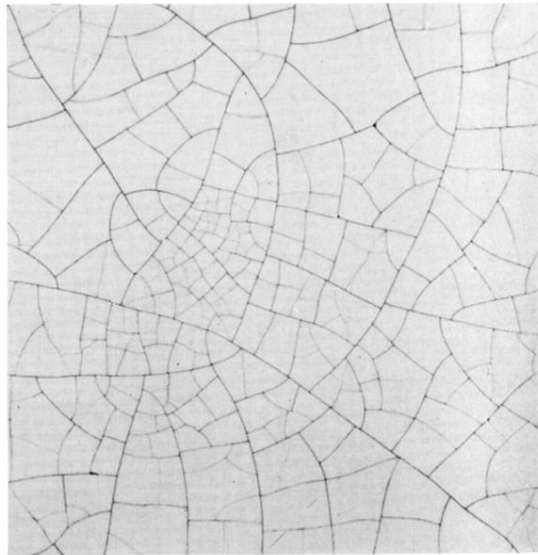


FIG. 5. Pattern of craze lines in the glaze on a ceramic surface. Magnification  $\times 0.6$ .

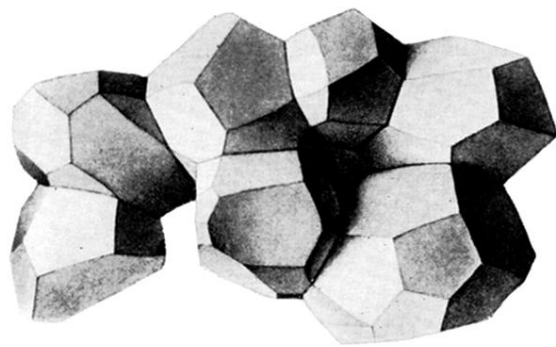


FIG. 6. The shape of cells in human fat tissue. Magnification  $\times 288$ . (F. T. Lewis, Courtesy American Academy of Arts and Sciences.)

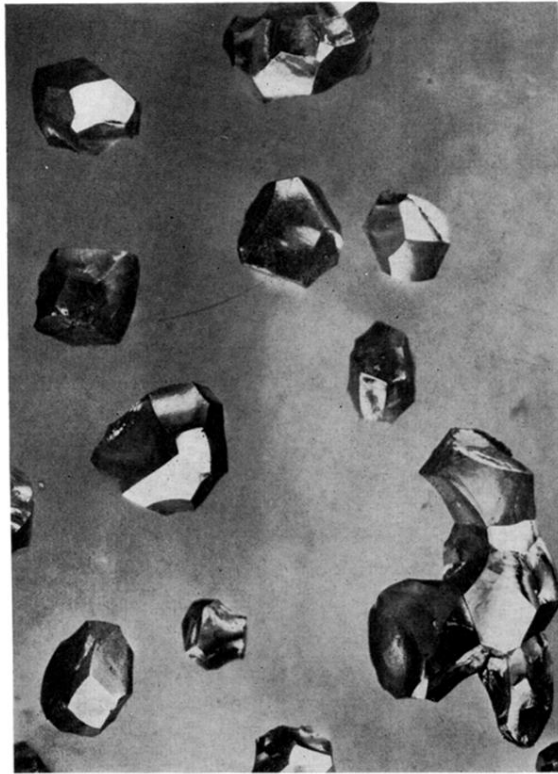


FIG. 7. Crystal grains separated from an aggregate (a coarse-grained piece of brass), showing the natural shape of crystals in random contact with each other. Note the frequency of pentagons and curved surfaces. Magnification  $\times 8$ .

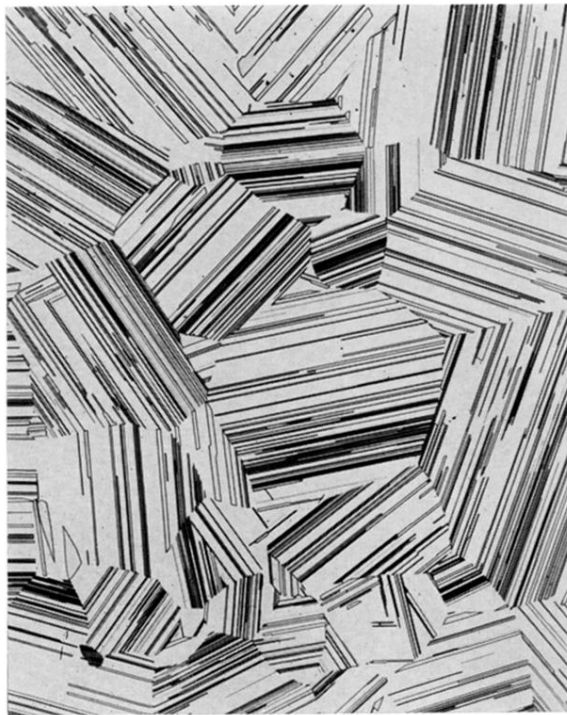


FIG. 8. Duplex crystals with bands of different composition in exact orientation relationship within one grain, but forming an overall foam structure like a pure metal, here seen in a copper-silicon alloy, worked and annealed. Magnification  $\times 120$ .



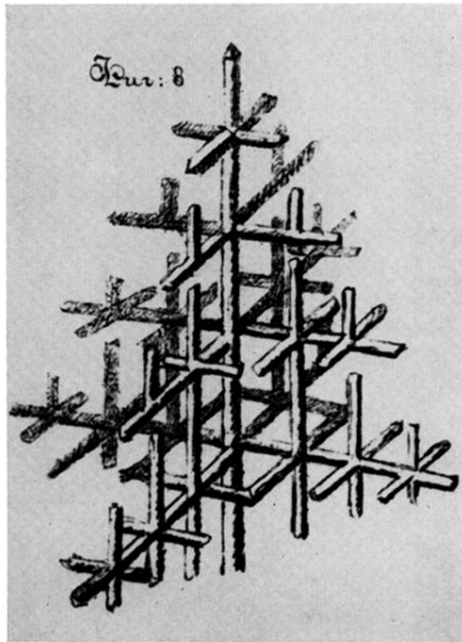


FIG. 9. Dendritic growth of iron crystal. Magnification about  $\times 300$ . (This historic drawing was made in 1876 by the Russian metallurgist D. K. Tschernoff.)