Hysteresis in Hard Superconductors*

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We discuss in this paper a development[1] of the model put forward by Abrikosov[2] and Goodman[3] for the description of the magnetic properties of type II superconductors. The effect of pinning of fluxoid filaments due to defects (see, e.g., Anderson[4]) are incorporated and the magnetization curves show hysteresis and size dependence as observed in practice (see, e.g., Livingston,[5] Kim and others,[6] Swartz,[7] and Hauser[8] and discussed theoretically by Bean[9] and Kim and others[6] on the basis of a phenomenological model). Following Silcox and Rollins,[1] we determine the interfluxoid forces semi-empirically as follows. A fluxoid structure of the Abrikosov type is assumed with the flux threading the specimen quantized such that each fluxoid represents one quantum of flux ϕ_0.[10–13] The Gibbs free energy G can then be written:

$$G = n\epsilon + f(n) - n\phi_0 H/4\pi ,$$

where $B = n\phi_0$. The first term $n\epsilon$ in this expression represents the energy increase due to filaments at relatively large separations and the term $f(n)$ is identified with the increase in energy due to repulsive interactions between the fluxoids as they are forced together under increasing magnetic pressure. The equilibrium value of n (or B) can be determined by minimizing G and gives

$$df/dn = (\phi_0/4\pi)(H - H_{c1}) .$$

If an experimentally determined reversible curve of H as a function of B is available, then this relation can be integrated to give $f(n)$. If $f(n)$ is now identified with the repulsive energy associated with two-body interactions between the fluxoids and we further assume for simplicity that only nearest-neighbor contributions are important, we obtain $f(n) = nzU(a)/2$, where z is the coordination number of the lattice, a the nearest-neighbor distance, and $U(a)$ the interaction potential. The interfluxoid force F is given as $-(dU/da)$.

In order to incorporate pinning effects, we consider the equilibrium of one fluxoid under forces due to neighboring fluxoids and the forces F_P due to each one of a density ρ of defects per unit length of fluxoid. For a triangular lattice, considering again only nearest neighbors, this equation may be approximated by use of Taylor series expansions to give for the equation of equilibrium

$$3a(dF/da)(da/dx) = -\rho F_P ,$$

where a one-dimensional variation of the fluxoid lattice parameter with x the position coordinate has been considered (this is equivalent to a gradient of the fluxoid density). This can be solved for particular forces and for particular defects and defect distributions. Once this equation has been solved, it can then be integrated over the specimen area perpendicular to the applied field to find the mean flux threading the specimen and hence the magnetization. For the case of a linear B/H curve and ρF_P proportional to n and independent of x^1, the result is of the form

$$B_x^2 = B_0^2 - \beta x H_{c1}^2 , \qquad (1)$$

where B_x is the induction at x, B_0 the induction at the origin, and β is a parameter including F_P and the volume density of defects and represents the net pinning force due to the defects. It should perhaps be noted that as defined β^{-1} has the dimensions of length and could be described as a second penetration depth (see, for example, Bean's model[9]).

We present here a discussion of the way in which Eq. (1), or the analogous equation, is used in determining the hysteresis curve. We first consider the question of the boundary conditions. The origin is taken at the surface of the specimen. A question arises as to the interaction of interior fluxoids with the surface penetration layer. Fluxoids are nucleated or absorbed at this surface layer and the details of the interaction could be important. Fortunately within the range of magnetic field $H_{c1} < H < H_{c2}$ this question can be avoided by taking our boundary condition to be that, at the surface, the magnetic in-

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duction immediately within the specimen is the value for the reversible curve at the same applied field. Since the surface layer and the fluxoids in the specimen interior are in equilibrium in both cases, and also the range of interaction is small (of the order of the penetration depth) in comparison with the size of the specimen, this enables us to avoid the details of the interaction. This, of course, cannot be done in the region $-H_{c1} < H < +H_{c1}$ since we have essentially a situation with no counterpart in the reversible magnetization curve. We therefore confine ourselves at the present to a discussion of the hysteresis curves in fields higher than H_{c1}.

Under this procedure, the initial stages for a virgin (i.e., no flux penetration at zero field) specimen are represented in Figs. 1 and 2. The specimen remains

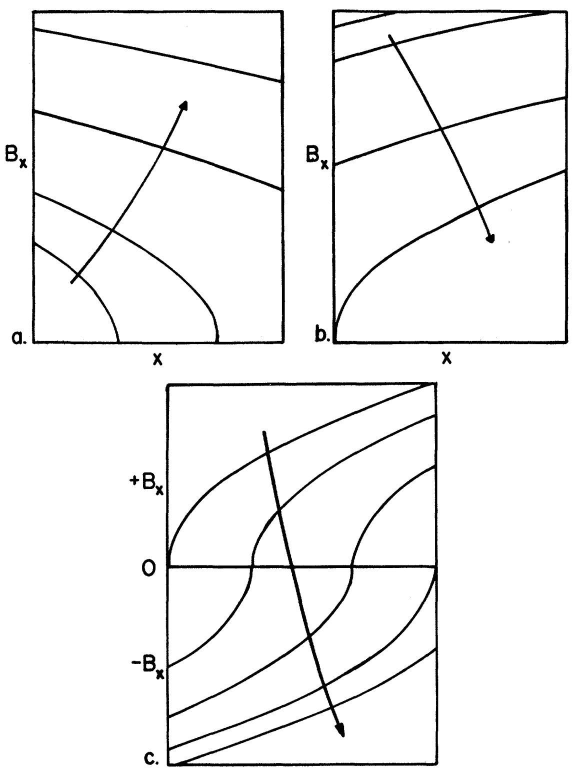

FIG. 1. Schematic indication of variation of fluxoid density (identical with B_x) with position at various stages of the magnetization curve. (a) Arrow indicates increasing H (virgin specimen). (b) Arrow indicates decreasing H. (c) Arrow indicates increasing negative H (cyclic condition).

diamagnetic until the field H_{c1} is reached. At this stage the field still cannot penetrate the bulk of the specimen since the initial fluxoids have to overcome the pinning forces. This results in diamagnetism until the magnetic pressure becomes sufficient to overcome the forces. This effect can be small, however, and for the purposes of this paper can be neglected. Figure 1(a) shows the flux as a function of position in the specimen at successive stages of the increasing magnetization curve. The resultant $\langle B\rangle_{av}$ is obtained

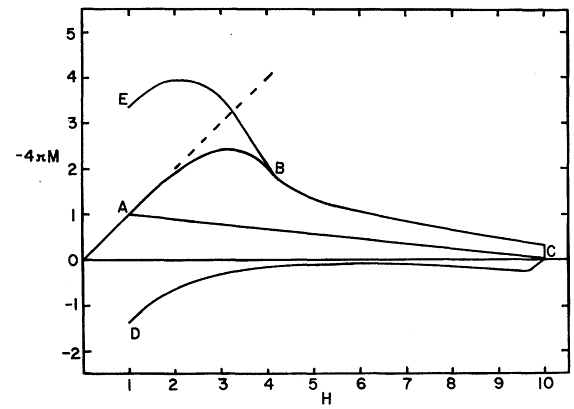

FIG. 2. Magnetization curve corresponding to $\beta t = 12.5$. Region ABC corresponds to Fig. 1(a), region CD to Fig. 1(b) and region EB to Fig. 1(c).

by finding the average of B_x across the specimen and gives rise to section ABC of the magnetization curve shown in Fig. 2. It is worth noting at this point that this curve may well be shape-dependent apart from demagnetizing effects since a thin plate, i.e., the one-dimensional symmetry used here, could give a different Eq. (1) and $\langle B\rangle_{av}$ from a cylindrical specimen. The calculations shown in Fig. 2 are for a plate, and a ratio $H_{c1}/H_{c2} = 0.1$ and $\beta t = 12.5$, where $2t$ is the thickness of the sample. The flux at the surface continues to rise until the applied field reaches H_{c2} at which point the specimen becomes normal in a catastrophic manner. In practice, this transition is likely to be blurred by demagnetizing effects. In reducing the field, similar reasoning gives the flux penetration effects shown in Fig. 1(b) and the corresponding part CD on the magnetization curve shown in Fig. 2.

In considering the magnetization curve for a specimen in the cyclic state (i.e., one which has been magnetized and demagnetized at least once), we run into the difficulty mentioned above, that effects due to the surface are not clearly understood in the region $-H_{c1} < H < H_{c1}$. However, we can discuss some of the effects if we assume that the surface effects in this region are virtually negligible. In these circumstances, the flux penetrating the specimen at H_{c1} is equal to that penetrating at $-H_{c1}$. Upon increasing the field above H_{c1}, the specimen admits fluxoids with the magnetic field penetrating through the specimen in the reverse direction to the fluxoids already trapped. The interaction between fluxoids with fields in opposite directions is attractive (this arises from the fact that the origin of the interaction between fluxoids is due mainly to the need to minimize the magnetic energy). If we assume that the attractive interaction occurs only between nearest neighbors we arrive at a picture something like Fig. 1(c) in which there is a "point of annihilation" or "no man's land" of width roughly the range of the interfluxoid forces lying between the two regions of flux. As the applied field is increased, the point moves further into the specimen until it reaches the center and all the reverse flux has been annihilated. The appropriate sequence *EB* of the magnetization curve is shown in Fig. 2. It seems likely that this section of the curve would be one of great instability particularly if ρF_P is not relatively uniform but shows nonuniformities at distances greater than the interfluxoid distance, and may correspond to the region of maximum probability of flux jumps.[6]

These curves shown in Fig. 2 show a considerable similarity to the curves observed in practice. Also, the curves shown in Fig. 1 show a similarity to equivalent curves shown by Kim and others.[6] A number of features, however, should be noted in relating these curves to practical curves. Two points have already been mentioned—the shape-dependence of the curve and the problem of the surface–fluxoid interaction between $+H_{c1}$ and $-H_{c1}$. A further point is that so far only nearest-neighbor contributions have been considered. It seems feasible to include further neighbors at the expense of greater complexity. Such an analysis would be possible with a digital computer and the effects may be important at fields close to H_{c2} where the fluxoids are closest together.

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Discussion 3

GOODMAN: These diagrams which we have just seen indicating the local flux density as a function of position during the course of magnetization cycles reminds one very much of the rather elegant films which Dr. DeSorbo showed at the Toronto meeting a few years ago in which one saw (I have in mind the film on niobium which we now know to be a London superconductor), as the external fields increased, a sharp front progressing into the superconductor. This rather sharp front is presumably connected with the vertical tangents in the magnetization curve which we have seen on the screen; the pinning down of the flux lines in certain places was indicated by the concavity of the front towards the outside. We also saw the flux jumps in that film and then finally, after the specimen had been through a number of hysteresis loops, Dr. DeSorbo pointed out to us that one could distinguish sometimes as many as two, or three, or perhaps more, different local values of the magnetic field corresponding to successive additions of flux lines of different signs. I think that perhaps some further experiments of this kind would be extremely valuable.

Defects and Magnetic Hysteresis in Type II Superconductors

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INTRODUCTION

Defects in type II superconductors produce magnetic hysteresis by interacting with, and thereby obstructing the motion of, the flux threads[1] of the mixed state. To move flux threads it then becomes necessary to build up a gradient in flux density, and a resultant magnetic driving force, strong enough to overcome the resisting force produced by the defects. A flux density gradient $\partial B/\partial x$ gives a driving force[2,3] per unit length on an individual flux thread of $(\alpha\phi_0\partial B/\partial x)/4\pi$, where $\alpha(B)$ is $dH(B)/dB$ for the ideal, reversible material, and ϕ_0 is the flux quantum.

Through this balance of forces the defect–flux thread interaction establishes a "critical" internal flux gradient or, equivalently, a "critical" internal

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