Observational Basis of Mach's Principle*

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During the last few years there have been several theoretical discussions of Mach's principle and its relation, or lack of relation, to the general theory of relativity and to cosmology. References 1 through 5 are a sample of these discussions. Very little, however, has been written about the quantitative observational basis of Mach's principle. The purpose of the present paper is to quote the only pertinent statement of which the writer is aware, and to review briefly the evidence on one particular aspect of Mach's principle in the hope of stimulating a more expert study of the matter.

The general concept embodied in Mach's principle is that the inertial properties of matter on the local scene derive in some way from the existence of the distant masses of the universe and their distribution in space. Inertia may be defined in terms of Newton's second law of motion, and this in turn implies the existence of a coordinate system with respect to which undisturbed objects move uniformly in straight lines. Thus if Mach's principle is to have any meaning, this inertial coordinate system must be one in which some average of the motions of the distant masses is uniform.

While it is difficult to be quantitative in regard to a possible small linear acceleration of one coordinate system with respect to another, it has been pointed out[6] that the value of g at the surface of the earth is a very large acceleration on an astronomical scale, since (without regard for the restrictions imposed by special relativity) it is almost exactly what is required to produce a velocity change of c in one year. Thus an average linear acceleration of the earth, sun, and galaxy that would be difficult or impossible to detect through a violation of Newton's laws would, in the course of astronomical time intervals, lead to an anisotropy of nebular red shifts which is not observed.

Our main point, however, has to do with the specification of the local inertial system with respect to rotation. Here there is the clear possibility of precise measurement of the angular velocity of rotation of the local inertial coordinate system with respect to the coordinate system in which the distant extragalactic nebulae are at rest. Bondi,[1] Sciama,[2] and Hoyle and Narlikar[5] remark that these coordinate systems agree within the experimental error, without however indicating the nature of the experiments or specifying their accuracy. On the other hand, in a different connection not related to a discussion of Mach's principle, Clemence states[7]: "It turns out experimentally that if there is any rotation of one [frame of reference] with respect to the other, the integrated effect during a century is less than 0.1 second of arc." He also describes in a general way the observations on which this conclusion is based.

The situation here may perhaps prove to be similar to that with regard to the equality of gravitational and inertial mass. The assumption of true equality (which can never be established experimentally) is one of the bases of the general theory of relativity, and hence of profound theoretical significance. But even the experimental demonstration of approximate equality, within some stated margin of accuracy, can be of importance. It could not have been foreseen by Eötvös and his collaborators that the precision of their experiments[8] would be such as to make possible, half a century later, a demonstration that antiparticles possess normal gravity.[9,10] Quite apart from this example, it is always of importance to know the uncertainty associated with an experimental or observational result, a null result as well as any

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† I first met Professor J. R. Oppenheimer when I went to Berkeley in the fall of 1937 after receiving my Doctor's degree. My next three years of association with him proved to be extraordinarily stimulating. Among other important influences which this period had on my subsequent scientific career was my introduction to general relativity and cosmology. It is a privilege for me to contribute to this issue of the *Reviews of Modern Physics*, and a pleasure to join with others in wishing Professor Oppenheimer a most happy sixtieth birthday.

[1] H. Bondi, *Cosmology* (Cambridge University Press, New York, 1960), 2nd ed.

[2] D. W. Sciama, *The Unity of the Universe* (Faber and Faber, London, 1959).

[3] P. W. Bridgman, Am. J. Phys. **29**, 32 (1961).

[4] O. Klein, in *Recent Developments in General Relativity* (Pergamon Press, Ltd., London, 1962), p. 293.

[5] F. Hoyle and J. V. Narlikar, Proc. Roy. Soc. (London) **A273**, 1 (1963).

[6] E. M. McMillan, Science **126**, 381 (1957); W. Rindler, Phys. Rev. **119**, 2082 (1960).

[7] G. M. Clemence, Rev. Mod. Phys. **29**, 2 (1957).

[8] For a summary of these experiments, which occupied the period 1890 to 1922, see R. V. Eötvös, D. Pekár, and E. Fekete, Ann. Phys. **68**, 11 (1922).

[9] P. Morrison, Am. J. Phys. **26**, 358 (1958).

[10] L. I. Schiff, Proc. Natl. Acad. Sci. U. S. **45**, 69 (1959).

other. And there is always the exciting possibility that a non-null effect can be established.

The local inertial coordinate system can in principle be defined with respect to rotation by a terrestrial experiment with a Foucault pendulum or a gyroscope. The errors incurred in this way are, however, far greater than those inherent in astronomical observation of nearby celestial objects. From a correlation of the motions of the inner planets of the solar system, it is possible to find a unique coordinate system (with respect to rotation) in which Newton's laws with no centrifugal or Coriolis terms, but including general relativity corrections, are satisfied. The uncertainty in the rate of rotation of this system is of the order of 0.4 second of arc per century.[11] Another dynamical method for establishing a local inertial system is based on the motions of stars in the neighborhood of the sun. The uncertainties in these motions[12] suggest that the rotation rate of this inertial system cannot be known to better than about 1 second/century. Although there is no disagreement between the two systems, the second is evidently not as useful as the first.

In astronomical discussions of the coordinate system in which the "fixed stars" are at rest, it is not customary to distinguish between the coordinate system derived from observations of the more distant stars of our galaxy, and that derived from observations of the distant extragalactic nebulae. In neither case would proper motions be expected to be significant. The dispersion in proper motion of a star would correspond to a tangential velocity dispersion of about 20 km/sec at a distance of about 2 kiloparsecs, or 0.2 second/century. For a galaxy with the random nebular velocity of about 100 km/sec at a distance of 20 megaparsecs, the angular velocity dispersion would be 10^{-4} second/century, which is far too small to observe. However, the angular sizes of these galaxies, which have linear diameters of the order of 10 kiloparsecs, would be a few seconds of arc even at the greatest distances, and so might be expected to limit the accuracy of specification of a coordinate system based on them. This limitation is actually not a severe one, since many galaxies have a well-defined nucleus of starlike appearance, and since only a change in the position of a galaxy, rather than the position itself, is significant. The distant stars and galaxies between them define a coordinate system with an uncertainty in rotation rate of about 0.1 second/century, and this system agrees with the inertial coordinate system discussed in the preceding paragraph.[13]

A few very distant galaxies, recently discovered in conjunction with strong radio sources, have angular diameters that are less than 1 second.[14] A large number of these objects will however have to be identified, and observed over a period of years, in order to improve on the precision of the "fixed-stars" coordinate system. Thus it appears at present that the observational basis for Mach's principle rests on a comparison between the rotation rates of the inner-planet inertial system and the "fixed-stars" coordinate system, and that the two systems agree within an uncertainty of about 0.4 second/century.

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[11] G. M. Clemence, Rev. Mod. Phys. **19**, 361 (1947). This figure is probably more reliable than the smaller figure quoted above from Ref. 7.

[12] R. v. d. R. Woolley, in *Vistas in Astronomy* (Pergamon Press, Ltd., London, 1960), Vol. 3, p. 3.

[13] The writer is indebted to Dr. G. M. Clemence (private communication) for an evaluation of the pertinent astronomical observations.

[14] M. Schmidt, Nature **197**, 1040 (1963); J. L. Greenstein and T. A. Matthews, Nature **197**, 1041 (1963).