In summary, the general qualitative ideas of flux creep theory appear to be soundly based and to allow for the qualitative and semiquantitative understanding of a wide range of phenomena. Many problems remain, both for detailed quantitative study and even for better qualitative understanding. To list a few of these:

(1) More detailed understanding of flux line interactions, in particular a sounder basis for the "bundle" concept and an understanding of B_0 and of the peak effect.

(2) The peculiar transient pulses observed by Kim *et al.*[8] These support the creep idea qualitatively, but are too large to be individual lines or bundles and too small to be instabilities. Are they avalanches?

(3) The nonlinear diffusion equation: when is it unstable?

(4) The viscous state. This is a completely new theoretical problem and I know of no obvious way even of approaching it from fundamental theory.

Resistive States of Hard Superconductors

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In earlier reports[1] we have shown that the critical currents in high-field superconductors are limited by the Lorentz force relation

$$\alpha = J(H + B_0) \leqslant \alpha_c , \qquad (1)$$

where α_c and B_0 are structure-sensitive constants of the material. This relation obtained for J perpendicular to H has been extended to other orientations by Cullen *et al.*[2] Although the relevance of the Lorentz force in hard superconductivity was pointed out first by Gorter,[3] a more definitive formulation of the problem was proposed by Anderson[4] in his flux creep theory. This theory basically assumes the GLAG-type superconductors, but the phenomena involving transport currents are described primarily in terms of the concept of thermally activated motion of flux structures. More recently, our measurements[5] have been extended into what we may call the "resistive state," far above the critical state specified by the relation (1). Here again, we find the prevalence of the Lorentz force parameter α. These results are briefly summarized in this paper.

According to the flux creep theory,[6] the flux structures in a GLAG-type superconductor move with a rate proportional to

$$R = \omega_0 \exp\left(-F_b/kT\right) ;$$

$$F_b = \frac{pH_c^2\xi_0^3}{8\pi} - \frac{JH\delta^2 l\xi_0}{c} = F_0 - q\alpha . \qquad (2)$$

In tube magnetization experiments,[1] for example, the critical state is attained when the above rate falls below a practically observable limit. The exponent in (2) then becomes a constant value and α therein is identified experimentally as α_c. For $\alpha < \alpha_c$, the logarithmic decay of J as predicted by (2) has been verified with a high degree of accuracy. For $\alpha > \alpha_c$, however, the rate (2) is inconveniently large and J is externally supplied to hold α at a desired level. In this situation, the flux creep generates an uncompensated emf proportional to (2).

Figure 1 shows typical voltages observed across a 3Nb–Zr wire sample—plotted as a function of H for different sets of constant J's ($\perp H$). Although the strong dependence of V on J and H is evident, the raw data do not display readily recognizable systematics. If, however, V is plotted as a function of $\alpha = J(H + B_0)$ with $B_0 = 0.5$ kG as determined from the data,[5] $V(H)$'s for different J's all coalesce to a single curve within the scatters shown by the horizontal flags. This indicates that at a given temperature V is a function of α only. According to (2), the slope

$$\partial \ln V/\partial \alpha = q/kT \qquad (3)$$

is expected to be constant. As α increases, however, the observed slope decreases until V is almost linear in α. In this region the prevailing process is visualized as "flux flow" rather than "flux creep."[6] Since the

[1] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. Letters 9, 306 (1962); Phys. Rev. 129, 528 (1963).
[2] G. Cullen, G. D. Cody, and J. P. McEvoy (to be published).
[3] J. C. Gorter, Phys. Letters 1, 69 (1962); 2, 26 (1962).
[4] P. W. Anderson, Phys. Rev. Letters 9, 309 (1963).
[5] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. 131, 2486 (1963).
[6] P. W. Anderson and Y. B. Kim, Rev. Mod. Phys. 36, 39 (1964).

observed slope of $\ln V(\alpha)$ changes in a gradual manner, these two processes seem to overlap over a wide range of the resistive state.

To discuss this situation, we introduce the notion of velocity of flux lines. In a GLAG-type superconductor, each flux line experiences the Lorentz force per unit length

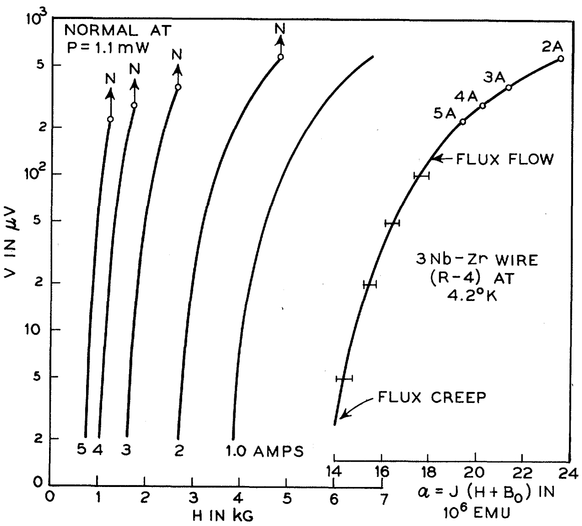

Fig. 1. Voltages observed across a 3Nb–Zr wire sample at 4.2°K. $V(H)$'s for different sets of constant J's and $V(\alpha)$ with $\alpha = J(H + B_0)$, $B_0 = 0.5$ kG.

$$F = J\Phi_0/c\,, \tag{4}$$

where Φ_0 is the flux unit $hc/2e$ and J is transverse to the direction of the flux line. In the absence of pinning, the flux line moves in the direction of the force. But, because of interactions of various sorts, the flux line will encounter a viscous resistance and may attain an equilibrium velocity. As a simple possibility, we assume that

$$\mathcal{V}_{\text{flow}} = \eta F = (\eta\Phi_0/c)J\,. \tag{5}$$

$\eta(H,T)$ is a parameter characterizing the bulk superconducting properties, and its dependence on H and T should be amenable to theoretical calculations. The power dissipation per unit volume arising from the viscous flow of flux lines is given by

$$P = nF\mathcal{V}_{\text{flow}} = (\eta\Phi_0/c^2)(n\Phi_0)J^2\,, \tag{6}$$

where n is the number of flux lines per unit area and $n\Phi_0 = B \simeq H$. Using the relations $P = VJ$ and $\alpha = JH/c$, we obtain

$$V = (\eta\Phi_0/c)\alpha\,. \tag{7}$$

If the H dependence of η is a mild one, the voltage in the flux flow region is expected to be approximately linear in α. In the creep region, the velocity of flux lines may be expressed as

$$\mathcal{V}_{\text{creep}} = (\omega_0/N)\exp\left[-(F_0 - q\alpha)/kT\right]\,, \tag{8}$$

where N is the number of pinning centers per unit length in the direction of flux flow. When $\mathcal{V}_{\text{creep}}$ becomes comparable to $\mathcal{V}_{\text{flow}}$, the velocity of flux motion is more correctly

$$\mathcal{V} = [1/\mathcal{V}_{\text{flow}} + 1/\mathcal{V}_{\text{creep}}]^{-1}\,, \tag{9}$$

which leads to a voltage expression

$$V = \mathcal{V}H = \frac{(H\omega_0/cN)\exp\left[-(F_0 - q\alpha)/kT\right]}{1 + (c\omega_0/\eta\Phi_0 NJ)\exp\left[-(F_0 - q\alpha)/kT\right]}\,. \tag{10}$$

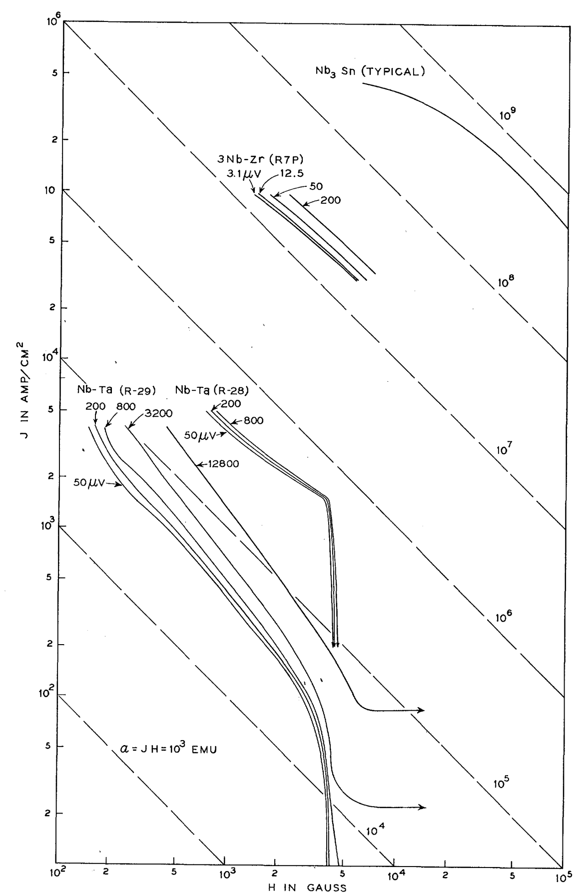

Fig. 2. Resistive states for various wire samples, all at 4.2°K. Each curve represents a set of J and H values that yield a constant voltage.

While this expression qualitatively accounts for the observed resistive behavior, including the gradual decrease in $\partial\ln V/\partial\alpha$, its application is rather limited at present since very little is known about the dependence of η and F_0 on J and H. It should also converge asymptotically to $V = \rho_n J$, the voltage due to the normal resistivity of the material.

As H is varied over a wide range, we find the representation of resistive states in the form of Fig. 1 becomes rather constrained. This trend is readily detectable in Fig. 2, where constant V lines are shown in a logarithmic procession in the log J–log H plane. In this representation, $V(\alpha) = \text{const}$ with $\alpha = J(H + B_0)$ will have a variable slope

$$d \ln J/d \ln H = -(1 + B_0/H)^{-1}, \qquad (11)$$

and the flow region ($V \propto \alpha$) will appear as equidistant parallel lines. The usual critical current densities for a Nb_3Sn wire sample are shown in the figure. For the 3Nb–Zr wire sample, constant V lines are shown up to $V = 200\ \mu V$. The region of higher power dissipation cannot be traced because of thermal instability.[5] These data are reducible in the form of $V(\alpha)$[5] and follow the pattern shown in Fig. 1. For the Nb–Ta wire sample, the upper critical field H_{c2} is only 4 kG and the entire region of the mixed state can readily be followed. Near H_{c2}, the resistive behavior is much like that of the 3Nb–Zr wire. As H decreases, however, the slopes of constant V lines rise, or the values of B_0 decrease. This trend becomes much more severe for the annealed Nb–Ta sample[7]—B_0 is now negative for the entire region of the resistive states. Thus, if the Lorentz force parameter is to be maintained in the form $\alpha = J(H + B_0)$, B_0 does not remain constant over a wide range of H variation. As for the dependence on microstructure, B_0 generally decreases as the pinning effects are reduced. In spite of these ramifications, however, Fig. 2 clearly points out the important fact—over 5 decades of $\alpha = JH$ values the resistive states are controlled primarily by the Lorentz force.

[7] The Nb–Ta (R-29) sample has been annealed thoroughly to remove most of the defects. Magnetization measurements showed, however, a considerable hysteresis, indicating that the sample is not defect-free.

Collective Modes of Vortex Lines in Superconductors of the Second Kind

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I. ONE SINGLE VORTEX LINE

We consider an Abrikosov vortex line[1] in a superconductor of the "extreme second kind": the penetration depth λ is much larger than the coherence length ξ. Then the line consists of a small hard core (cylinder of radius ξ) where the order parameter is seriously modified, plus a large "electromagnetic region" (radius $\sim \lambda$) where the distribution of fields and currents is ruled by a simple London equation (Fig. 1). We first recall how the line energy per unit length is computed in this case. To simplify the notation we consider only pure metals, with electrons of effective mass m^*, superfluid concentration n_s, and velocity v_s. (Note that for the superconductors of interest $m^* \sim 10$ to 100 electron masses.) The energy is

$$\mathfrak{J} = \int_{r>\xi} dx\, dy \left(\frac{h^2}{8\pi} + \tfrac{1}{2} n_s m^* v_s^2\right)$$

$$= \int_{r>\xi} dx\, dy\ (1/8\pi)\ (h^2 + \lambda^2 \operatorname{curl}^2 h),$$

$$\lambda^{-2} = 4n_s e^2/m^* c^2. \qquad (\text{I}.1)$$

The minimization of $\mathfrak{J}$ leads to the condition

$$h + \lambda^2 \operatorname{curl} \operatorname{curl} h = \varphi_0 \delta_2(r), \qquad (\text{I}.2)$$

where $\varphi_0 = ch/2e$ is the flux quantum and the δ function describes the singularity on the hard core. For $r \ll \lambda$ the solution of (I.2) is of the form

$$h = (\varphi_0/2\pi\lambda^2) \ln (\lambda/r), \qquad (\text{I}.3)$$

while for $r \gg \lambda h$ decreases exponentially. The energy $\mathfrak{J}$ may be derived from (I.1) by a partial integration

$$\mathfrak{J} = \frac{\lambda^2}{4} [rh|\operatorname{curl} h|]_{r=\xi} = \left(\frac{\varphi_0}{4\pi\lambda}\right)^2 \ln \frac{\lambda}{\xi}. \qquad (\text{I}.4)$$

The field H_{c1} above which the thermal equilibrium corresponds to a finite density of lines in the super-

[1] A. Abrikosov, Zh. Eksperim. i Teor. Fiz. **32**, 1442 (1957) [English transl.: Soviet Phys.—JETP **5**, 1174 (1957).