for a slab and I can state exactly how the order parameter varies in the high-field region. What I find, which was rather surprising when I first came upon it, is that the behavior does not look like the mixed state or the state in which the order parameter oscillates as you go across the slab—instead, the state has a superconducting core at the center of the slab surrounded by nearly normal material. For example, if I let kappa equal 2, I find the central core is about 7 or 8 penetration depths wide, nearly completely superconducting, and then it tails off to values less than 10^{-4} which can extend over macroscopic distances. This solution can be shown to be the one with the lowest free energy; that is, it can be shown rigorously to have a lower free energy than any of the oscillatory solutions in which the order parameter oscillates around zero in going across the slab. I might say that even though no flux "vortex" is present in this solution, the magnetic moment curve looks just like the type II curve with a first-order transition at a lower critical field and an abrupt drop in the magnetic moment. It falls off to zero at a field approximately equal to the usual high-field limit when H is equal to kappa. I'll show the free energy curves and other details of this in my discussion on Thursday—it will look quite similar to Professor Gorter's results.

Magnetization of High-Field Superconductors

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INTRODUCTION

Magnetization measurements have played a crucial role in the development of the understanding of superconductivity.[1] On the one hand, the discovery of the Meissner effect (complete flux expulsion in macroscopic specimens) gave the first clue to the present concept of the soft superconductor as a large scale quantum state. The relative lack of irreversibility in the transition to the normal state caused by an applied field gave assurance that thermodynamical reasoning could be applied with confidence in consideration of the various physical changes attendant to the transition. On the other hand, measurements on alloys[2–5] indicated a more complicated state of affairs. First of all, the flux penetrated the specimens over a broad range of field—extending the process to much higher fields than the critical fields of pure superconductors—and secondly, there was observed a large hysteresis in the magnetic behavior. After a brief flurry of experimental interest in the mid 1930's, the field lay dormant. It may be that the hysteresis noted above discouraged careful experiment and theory. The present understanding started with a remarkable paper in 1957 by A. A. Abrikosov[6] who, following a suggestion by Landau, developed a quasimicroscopic theory of magnetization of alloys in which he showed that the flux entered the specimen in quantized superconducting electron current vortices (variously called fluxons, flux lines, flux threads). He showed this penetration to start at a low field (H_{FP} or H_{c1}) and be complete at a higher field (H_N or H_{c2}). Between these regions of field, the specimen is presumed to possess a structure of fluxons—a mixed state. While, to date, there has been no direct experimental confirmation of this structure, indirect evidence[7] (mainly magnetization curves) points to the essential correctness of this picture. An important feature of this theory is that it calculates states of thermodynamic equilibrium. This predicted lack of hysteresis in homogeneous alloys has been well confirmed in recent years.[7,8]

This paper deals with the theory and experiment on hysteretic high-field superconductors. The ideas were prompted by an older model of high-field superconductors—the Mendelssohn sponge model[9]—wherein one assumes a multiply connected internal structure of high critical field material surrounding a matrix of material lower (or zero) critical field. The treatment, however, is phenomenological with only occasional contact with the microscopic or quasimicroscopic features of superconductivity. Surprisingly enough this simple theory appears to account for the magnetic properties of inhomogeneous mixed-state superconductors as well as the sponge model

[1] An excellent account of the development of this field is given by K. Mendelsson, Cryogenics **3**, 129 (1963)

[2] T. C. Keeley, K. Mendelssohn, and J. R. Moore, Nature **134**, 773 (1934).

[3] W. J. de Haas and J. M. Casimir-Jonker, Nature **135**, 30 (1935).

[4] F. G. A. Tarr and J. O. Wilhelm, Can. J. Res. **12**, 265 (1935).

[5] J. N. Rjabinin and L. V. Shubnikov, Physik. Z. Sovjietunion **7**, 122 (1935).

[6] A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. **32**, 1442 (1957) [English transl.: Soviet Phys.—JETP **5**, 1174 (1957)].

[7] For example, J. D. Livingston, Phys. Rev. **129**, 1943 (1963).

[8] A. Calverley and A. C. Rose-Innes, Proc. Roy. Soc. (London) **A255**, 267 (1960).

[9] K. Mendelssohn, Proc. Roy. Soc. (London) **A152**, 34 (1935).

for which it was developed. In fact, I think it is fair to say that the hysteresis of high-field superconductors is at least as well understood as that of ferromagnetic materials.

In detail, the program of this paper is to give an exposition of this theory of the static magnetization of hard superconductors, compare it to experiment, and lastly to discuss the response of hard superconductors to alternating fields with a superimposed steady field.

THEORY OF THE MAGNETIZATION OF HARD SUPERCONDUCTORS

The basic premise of this theory[10,11] is that there exists a limiting macroscopic superconducting current density $J_c(H)$ that a hard superconductor can carry; and further, that any electromotive force, however small, will induce this full current to flow into an imhomogeneous mixed-state superconductor.[13–15]

If we assume, as a starting point, that this current density is independent of field, the process of magnetization of a slab of thickness D in a field parallel to its surface is shown in Fig. 1. The field within the specimen decreases linearly with distance as a consequence of Ampere's law, curl $\mathbf{H} = 4\pi\mathbf{J}/10$. (In this equation and all that follow I employ practical units, i.e., oersteds, amperes/cm^2, and volts.) The configuration of fields are shown in Figs. 1(a) and 1(b). In the initial stages of magnetization, the current flows in superficial layers whose thickness Δ is just enough to reduce the internal local field to zero, i.e., $\Delta = 10H/4\pi J_c$. This field-dependent penetration depth is the central result of the assumptions made above and leads directly to size-dependent magnetization curves. At fields of $H \geqslant \pi J_c D/5 \equiv H^*$,

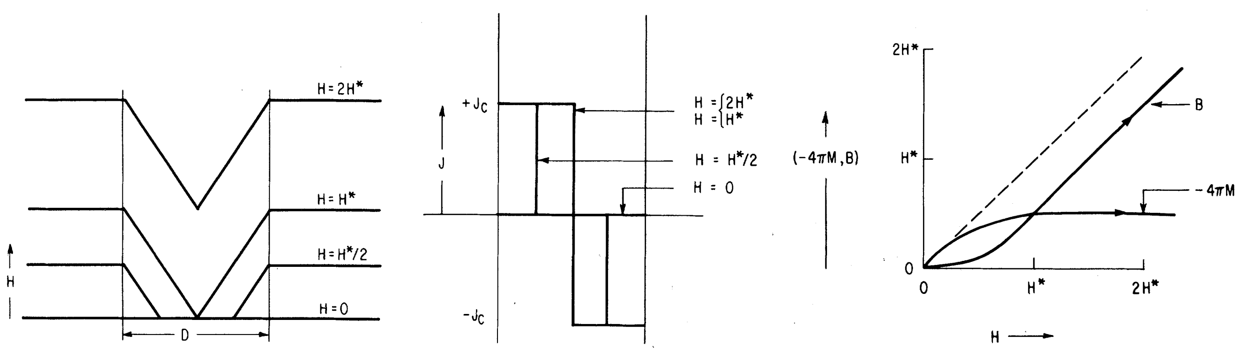

FIG. 1. A plot of local fields and current density, as well as magnetization curves, for fields O, $H^*/2$, H^*, and $2H^*$ applied parallel to the surface of a slab of thickness D. The critical current density J_c is assumed independent of field.

locally. On this picture only three states of current flow are possible with a given axis of magnetic field, zero current for those regions that have never felt the magnetic field and full current flow perpendicular to the field axis, the sense depending on the sense of the electromotive force that accompanied the last local change of field. The critical current may be, on the one hand, an intrinsic property[12] of the walls of the sponge in the Mendelssohn model or, on the other hand, a consequence (by Ampere's law) of the gradient of flux lines that exists as flux is driven currents flow through the entire volume of the specimen. To calculate the magnetization curves of the specimen, we have, by definition, that

$$\mathbf{B} = \int \mathbf{H} dv \Big/ \int dv$$

$$4\pi\mathbf{M} = \mathbf{B} - \mathbf{H} . \quad (1)$$

In other words, B is merely the volume average of the local field while $4\pi M$ is the average field created by the currents. By inspection of Fig. 1, one obtains

[10] C. P. Bean, Phys. Rev. Letters **8**, 250 (1962).
[11] C. P. Bean and M. V. Doyle, J. Appl. Phys. **33**, 3334 (1962).
[12] J. Bardeen, Rev. Mod. Phys. **34**, 667 (1962).
[13] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. Letters **9**, 306 (1962); P. W. Anderson, Phys. Rev. Letters **9**, 309 (1962).
[14] J. Friedel, P. G. DeGennes, and J. Matricon, Appl. Phys. Letters **2**, 119 (1963).
[15] J. Silcox and R. W. Rollins, Appl. Phys. Letters **2**, 231 (1963).

$$
\begin{aligned}
B &= H^2/2H^*\,, \\
&\qquad\qquad H \leqslant \pi J_c D/5 \equiv H^* \\
-4\pi M &= H - H^2/2H^*\,, \\
&\qquad\qquad\qquad\qquad \text{(slab)} \quad (2) \\
B &= H - H^*/2\,, \\
&\qquad\qquad H \geqslant \pi J_c D/5 \equiv H^* \\
-4\pi M &= H^*/2\,.
\end{aligned}
$$

These results are sketched in Fig. 1(c). For a cylinder of radius R, in an axial field the integrations indicated in Eq. (1) give

$$
\begin{aligned}
B &= H^2/H^* - H^3/3H^{*2}\,, \\
&\qquad\qquad H \leqslant 4\pi J_c R/10 \equiv H^* \\
-4\pi M &= H - H^2/H^* + H^3/3H^{*2}\,, \\
&\qquad\qquad\qquad\qquad \text{(cylinder)} \quad (3) \\
B &= H - H^*/3\,, \\
&\qquad\qquad H > 4\pi J_c R/10 \equiv H^* \\
-4\pi &= H^*/3\,.
\end{aligned}
$$

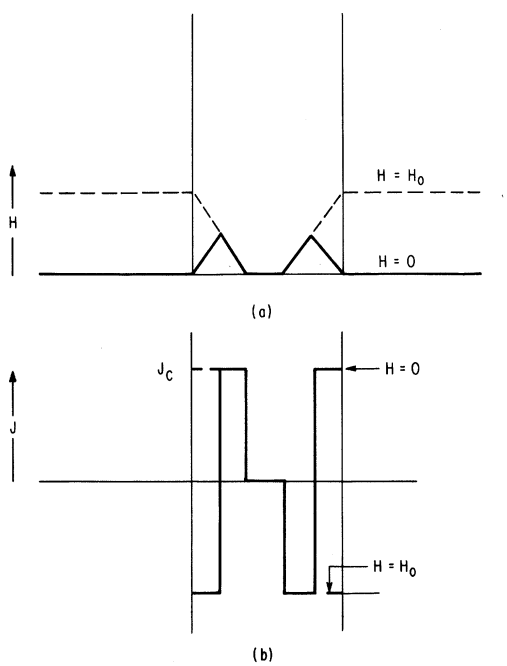

FIG. 2. A plot of local fields and current density in a slab after a field H_0 has been applied and removed.

Let us now consider what happens if a field H_0 is applied to a virgin slab and then removed. This state of affairs is indicated in Fig. 2. As the field is removed, the surface feels an emf oppositely directed to the one felt as the field was increasing—hence the surface currents reverse. At zero field, the current distribution is that shown in Fig. 2(b) wherein each surface is coated with two equal and oppositely directed current sheaths. The trapped flux or remanent flux density B_r is seen from Fig. 2(a) and Eq. (1) to be exactly half the flux penetration at H_0, if $H_0 \leqslant H^*$, therefore,

$$
\begin{aligned}
B_r &= H_0^2/4H^*\,, \quad H_0 \le H^* \\
&\qquad\qquad\qquad\qquad \text{(slab)} \\
B_r &= H^*/4\,, \quad H_0 \ge H^* \\
B_r &= H_0^2/2H^* - H_0^3/4H^{*2}\,, \quad H_0 \le H^* \qquad (4) \\
&\qquad\qquad\qquad\qquad \text{(cylinder)} \\
B_r &= H^*/4\,, \quad H_0 \ge H^*\,.
\end{aligned}
$$

In fact, the entire hysteresis loop may be calculated using the principles outlined above. The results are

$$
\begin{aligned}
B &= HH_0/2H^* \pm (H^2 - H_0^2)/4H^*, \quad H_0 \le H^* \quad \text{(slab)} \\
B &= HH_0/H^* \pm (H^2 - H_0^2)/2H^* \\
&\quad \pm (H_0^3 + HH_0^2 - H^2H_0 + H^3/3)/4H^{*2}\,, \\
&\qquad\qquad H_0 \le H^* \qquad \text{(cylinder)}\,. \qquad (5)
\end{aligned}
$$

The plus signs apply for the course from $-H_0$ to H_0 and the minus sign for the course from H_0 to $-H_0$. It is interesting to note that this loop is the precise diamagnetic equivalent of the Rayleigh hysteresis loop used in ferromagnetism. Lord Rayleigh[16] introduced this form as the simplest analytic expression to fit empirically the magnetization loop of weakly magnetized ferromagnets.

Since we have the hysteresis loop, the loss of energy per unit volume per cycle, W_v, may be immediately calculated from its area, i.e.,

$$W_v = (1/4\pi) \oint H dB\,, \qquad (6)$$

which gives

$$
\begin{aligned}
W_v &= H_0^3/6\pi H^*\,, \quad H_0 \le H^* \quad \text{(slab)} \\
W_v &= H_0^3/3\pi H^* - 5H_0^4/16\pi H^{*2}\,, \quad H_0 \le H^* \\
&\qquad\qquad\qquad \text{(cylinder)}\,. \qquad (7)
\end{aligned}
$$

This gives the Rayleigh cubic dependence of loss on maximum field. In the case of $H_0 \gg H^*$, the loss approaches a linear dependence. It is illuminating to consider the loss in small fields, $H_0 \ll H^*$, where the loss is a surface loss W_s and both Eqs. (7) become, after multiplying by the volume and dividing by the surface area,

$$W_s = 5H_0^3/12\pi^2 J_c \quad \text{ergs/cm}^2\text{/cycle}\,. \qquad (8)$$

[16] Lord Rayleigh, Phil. Mag. 5, 23 (1887).

A more microscopic picture of the losses may be had by considering that within the specimen local electric fields exist during the change of magnetization. The local Joule heating is the product of these fields and the local current density.

In the paragraphs above I have assumed the current density to be independent of field. The approach may be generalized by consideration of Fig. 3.

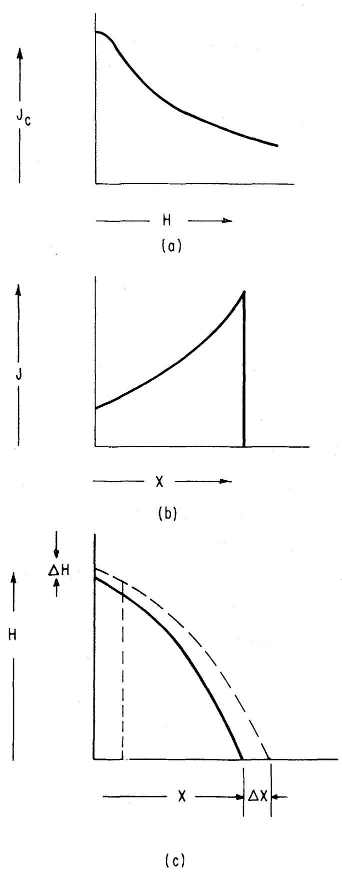

FIG. 3. A plot of the effects of a current density J_c that depends on field H. Figure 3(a) shows a schematic curve of this dependence. Figure 3(b) shows the currents at a plane surface for an applied field H (solid curve) and an applied field $H + \Delta H$ (dashed curve). Figure 3(c) shows the internal fields under the same conditions.

Figure 3(a) shows an arbitrary dependence of current density on field. The consequent profiles of current density and field are shown in Figs. 3(b) and 3(c). Consider that a field H has been applied, giving the field and current distributions shown as solid curves. If an increment of field ΔH is applied, the curves are displaced inward by an amount $\Delta x = 10\,\Delta H/4\pi J_c(H)$, where $J_c(H)$ is the current density appropriate to the field at the surface. An amount of flux enters that is proportional to H and Δx. For a slab geometry, $\Delta B = 2H\,\Delta x/D$.

By the definition of Δx above, $\Delta B = 20H\,\Delta H/4\pi J_c(H)D$. If we define a new penetration field $H^*(H) = \pi J_c(H)D/5$ and pass to the differential limit

$$dB/dH = H/H^*(H)\,, \quad H \ll H^*(H) \quad \text{(slab)}\,.$$

In the limit that H^* is independent of field, we obtain the parabolic relationship of Eq. (2). If, for instance, $J_c(H)$ were assumed to be inversely proportional to field[13] then the flux density would increase with the cube of the applied field. Conversely, $H^*(H)$ could be obtained experimentally by measuring the slope of the magnetization curve. In practice this technique is somewhat difficult to employ since it requires very accurate measurement of the B,H curve to obtain accurate derivatives.

Before leaving this section several comments are in order concerning the validity of the assumptions that underlie this development. First, the assumption of an infinitely sharp front on the curve of current density against distance is never completely valid. There is always an exponentially decreasing tail to the curve. In the case of the inhomogeneous mixed state this is a London penetration depth in which the field is reduced from H_{FP} to 0. In the case of the filamentary superconductor it is a quasi-London depth but one which is longer than that of the bulk material.[17] The assumption employed above will be asymptotically valid as the field-dependent penetration depth Δ greatly exceeds the London penetration depth. A second point concerns the tacit assumption that the critical current density is independent of time. Owing to the possibility of thermal activation of flux lines through filaments or across barriers in the inhomogeneous type II superconductor[13] this is not a universally valid assumption. However, many processes in solid-state physics have this character—for instance the movement of dislocations or the reorientation of ferromagnetic domains. But depending on the temperature and the nature of the barriers one can often describe the properties over a wide frequency range with the assumption of a time-independent yield stress or coercive force. Whether this be true or not for high-field superconductors is a subject for experimental investigation.

EXPERIMENTAL MAGNETIZATION CURVES

The static magnetization measurements reported here are made by flipping the specimen, generally

[17] C. P. Bean (to be published).

0.125 in. in diameter and an inch or so in length, in and out of a 2000-turn coil mounted axially in an electromagnet. The integrated voltage signal, proportional to the magnetization, is displayed on a General Electric fluxmeter. The calibration is made by the assumption of perfect diamagnetism at very low fields.

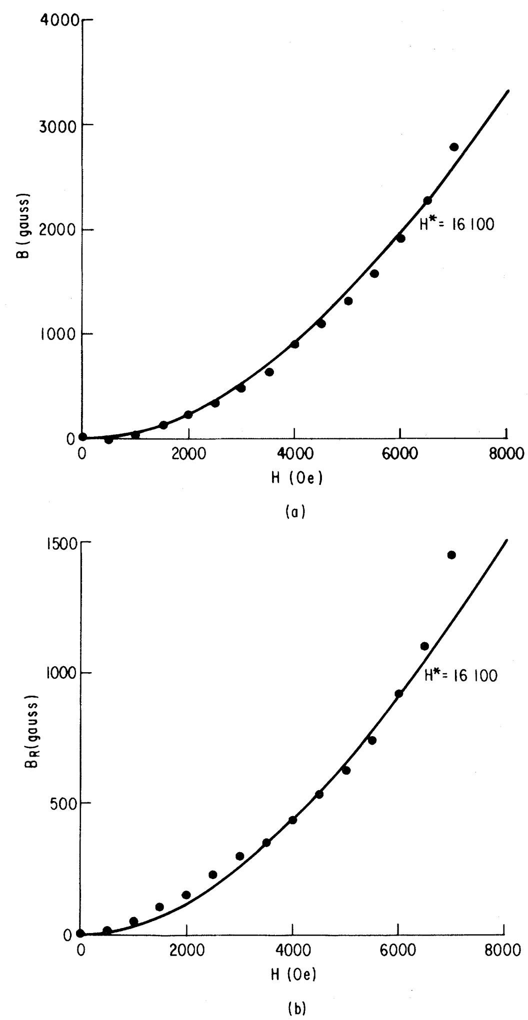

FIG. 4. Flux penetration and flux retention in a sample of sintered V_3Ga. Figure 4(a) shows the flux penetration at 4.2°K in a sample 0.38 cm in diameter. The solid curve assumes a current density of 6.6×10^4 A/cm². Figure 4(b) shows the remanent flux density after exposure to various fields *H*. The solid curve is theoretical.

Figure 4(a) shows the penetration of flux into a sintered sample of V_3Ga.[18] The specimen was prepared by pressing a stoichiometric mixture at 50 000 psi and firing it in argon at 1150°C for 3 h. The expanded cylinder was then crushed, repressed, and given a final firing for 1 h at 1500°C in a vacuum of 10^{-6} mm Hg. The experimental points for both the flux penetration and flux retention are seen to be fit quite closely by Eqs. (3) and (4) with the assumption of a current density of 660 000 A/cm². A more complete hysteresis loop is given elsewhere.[19] The substantial agreement with theory implies that the theory may be valid for inhomogeneous type II superconductors of which V_3Ga is apparently an example.[20,21] The main disagreement with theory is seen in the high field point whereat a small amount of flux entered the specimen discontinuously. These flux jumps[22–24] which may be almost complete, in contrast to the small one shown here, are the principal limitation to the employment of this technique over a wide field range with thick, high current density specimens.

An experiment on a synthetic filamentary[25] material is shown in Fig. 5. The sample was prepared by R. J. Charles[26] and W. G. Schmidt who pressed lead into porous Vycor glass at 355°C and 44 000 psi. The filling was incomplete in contrast to that of a specimen reported later in this paper. The figure shows the initial magnetization and two hysteresis loops. The solid lines are derived from Eqs. (3) and (5) with the single assumption of a current density of 2.7×10^4 A/cm². The agreement is seen to be substantial. The lack of detailed agreement may be mainly a consequence of a surface layer, discussed later in this paper, as well as the inaccuracy, in detail, of the assumption that the current density is independent of field.

Kim, Hempstead, and Strnad[24] have reported an important series of experiments on the static magnetization behavior of Nb–Zr, Nb_3Sn, and V_3Ga. Usually they employed a thin wall, cylindrical shell geometry to minimize the difficulty of analysis owing to the dependence of critical current on field. In most cases their measurements were made under conditions of current flow through the entire cross section of the sample—a condition they term the "critical state." They found that the magnetization over

[18] J. E. Kunzler, Rev. Mod. Phys. **33**, 501 (1961).

[19] C. P. Bean and R. W. Schmitt, Science **140**, 26 (1963).
[20] P. W. Swartz, Phys. Rev. Letters **9**, 448 (1962).
[21] B. B. Goodman, Phys. Letters **1**, 215 (1962).
[22] R. M. Bozorth, A. J. Williams, and P. D. Davis, Phys. Rev. Letters **5**, 148 (1960).
[23] C. P. Bean and M. V. Doyle, J. Appl. Phys. **33**, 3334 (1962).
[24] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **129**, 528 (1963).
[25] C. P. Bean, M. V. Doyle, and A. G. Pincus, Phys. Rev. Letters **9**, 93 (1962).
[26] R. J. Charles and W. A. Harrison, Phys. Rev. Letters **11**, 75 (1963).

a wide range of field H could be derived from a current density, $J(H) = \alpha/(H^+ + H)$. In this expression α and H^+ are constants. The fact that H^+ was found to be ≈ 5 kOe accounts for the good fit we obtain in our low-field measurements using the assumption that the current density is independent of field.

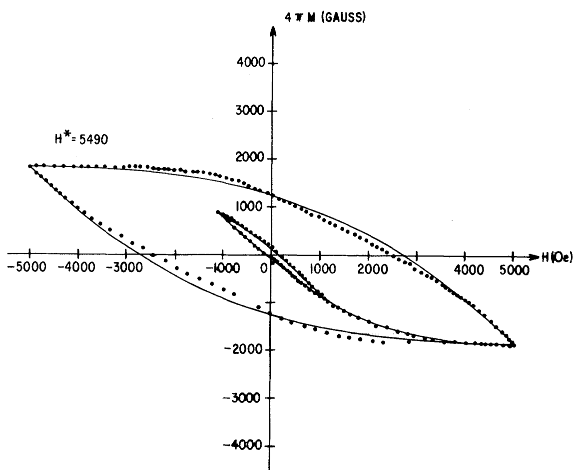

FIG. 5. Magnetization curve and hysteresis loops for lead in porous glass at 4.2°K. The points are experimental while the solid curves are theoretical with an assumed current density of 2.7×10^4 A/cm^2.

THE RESPONSE OF HIGH-FIELD SUPERCONDUCTORS TO ALTERNATING MAGNETIC FIELDS

The limitations of static magnetization measurements may be circumvented by a technique that measures the response of the superconductor to small alternating fields that are superimposed on a collinear steady field. In this case superconducting currents are induced to flow in a thin layer (the thickness is proportional to the amplitude of the alternating field) at the surface of the sample. Furthermore, these currents are flowing in fields that are identical to the static applied field within an error of plus and minus the amplitude of the alternating field. Lastly, a flux jump will allow flux to enter the bulk of the specimen but, hopefully, will not affect the response to alternating fields once the catastrophic voltage pulse has occurred.

If at some steady field H a collinear alternating field of amplitude h_0 is applied, the magnetization traverses a minor hysteresis loop. If the alternating field were a pure sine wave, the dependence of B on time would consequently be nonsinusoidal. A secondary coil wound on the sample would have a nonsinusoidal voltage induced across it and thus would have components of the voltage at harmonics of the impressed frequency. Observation of these harmonics can, in principle, give a measure of the minor hysteresis loop and hence of the critical current density. We have tacitly assumed that these critical currents are independent of the velocity of the motion of flux—an assumption that is unjustified for the entire frequency range of the electromagnetic spectrum. The frequency dependence of these voltages will indicate to what extent this assumption is valid.

To derive the magnitude of the harmonics of voltage we must perform a Fourier analysis of the voltage waveform. First we note that the instantaneous flux density, from Eq. (5), is

$$B = B(H) + hh_0/H^*(H) \pm (h^2 - h_0^2)/2H^*(H) \quad \text{(cylinder) } h_0 \ll H^* \tag{9}$$

with the plus sign for the course from $-h_0$ to $+h_0$. We assume that $h = h_0 \cos \omega t = h_0 \cos (2\pi f t)$, which gives

$$B = B(H) + [h_0^2/2H^*(H)][(2\cos\omega t \pm \sin^2 \omega t)] \,. \tag{10}$$

If we assume an expansion of the form

$$B = B(H) + \sum_{n=1}^{\infty} \alpha_n \sin(n\omega t) + \sum_{n=1}^{\infty} \beta_n \cos(n\omega t) \,, \tag{11}$$

we may evaluate the coefficients α_n and β_n by the usual technique of multiplying B by $\sin(m\omega t)$ or $\cos(m\omega t)$ and integrating from $\omega t = 0$ to $\omega t = 2\pi$. The results are[27]

$$\begin{aligned} &\alpha_n = 0 \,, \quad (n \text{ even}) \,; \\ &\alpha_n = -[h_0^2/H^*(H)][4/(n-2)(n)(n+2)] \,, \quad (n \text{ odd}) \\ &\beta_1 = h_0^2/H^*(H) \,; \quad \beta_n = 0 \quad (n > 1) \,. \end{aligned} \tag{12}$$

The voltage output for a secondary coil of N turns is

$$V = \pi R^2 N \dot{B} \times 10^{-8} = V_{1\mathrm{st}} \cos(\omega t - \gamma) + V_{3\mathrm{rd}} \cos(3\omega t) + V_{5\mathrm{th}} \cos(5\omega t) + \cdots \,, \tag{13}$$

which gives in conjunction with the definition of $H^*(H)$ for a cylindrical specimen

$$\begin{aligned} V_{1\mathrm{st}} &= 1.088\, V_{3\mathrm{rd}} \\ V_{3\mathrm{rd}} &= -[4h_0^2 f N R/J_c(H)]10^{-8}\ \text{volts} \\ V_{5\mathrm{th}} &= 0.238\, V_{3\mathrm{rd}} \\ V_{7\mathrm{th}} &= 0.111\, V_{3\mathrm{rd}} \\ V_{n\mathrm{th}} &= [5/(n-2)(n+2)]V_{3\mathrm{rd}} \,. \end{aligned} \tag{14}$$

By application of Eqs. (14), $J_c(H)$ may be de-

[27] D. B. DeHaan, *Nouvelles tables d'integrales definies* (Leide, Paris, 1897), p. 97.

termined or alternately this technique may be used to generate harmonics.

EXPERIMENTAL ARRANGEMENT

The steady field is produced by a $Nb_{0.75}Zr_{0.25}$ superconducting coil that is powered by Trygon power supply model M36–25-ov. The alternating field is produced by a primary coil of 541 turns of 8-mil wire wound on a plastic form of 0.135-in. diameter. The primary coil is powered by a 60-W McIntosh audio amplifier driven by an audio oscillator. The alternating field is monitored by a Ballantine vacuum tube voltmeter that senses the voltage across a 10-Ω noninductive resistor that is in series with the primary circuit. The secondary circuit consists of 101 turns of 4-mil wire that is wound directly on the 0.125-in. rod of porous glass impregnated with lead. The output of this coil is measured by a Hewlett–Packard harmonic wave analyzer, model 300A that has a Keithley model 102A decade preamplifier connected before it in the circuit. A series LC filter is used in the primary circuit to remove extraneous harmonics and a small mutual inductor is connected between the primary and secondary circuits to limit the output to the wave analyzer of voltage of the fundamental frequency.

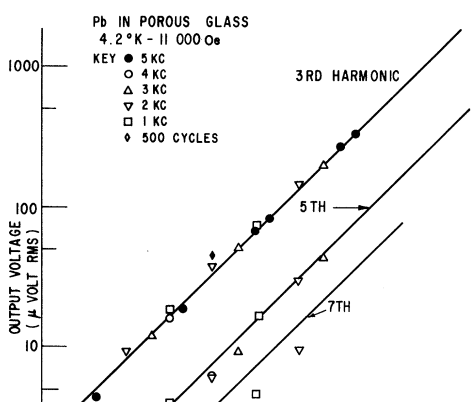

FIG. 6. Harmonic output from a sample of lead in porous glass in an alternating field. The ordinate indicates the voltage output of a 101-turn coil wound on the 0.32-cm-diam specimen. The abcissa is the product of the square of the amplitude of the alternating field h_0 and the frequency. The experimental points plot the output at odd harmonics of the fundamental frequency. The solid lines show the theoretical predictions for a current density of 2.1×10^5 A/cm².

The sample was of porous Vycor glass that had been filled with 10% hydrofluoric acid and left under mineral oil for 108 h. Subsequently, reagent-grade lead was forced in at 355°C and 60 000 psi filling completely the 31% void space of the glass. Electron micrographs show the filled pores to 110 Å ± 20 Å in diameter while the pore junctions can be 200 Å or greater. For later reference we note that the pore

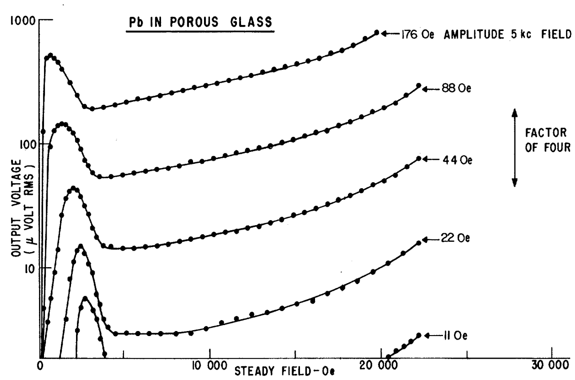

FIG. 7. Third harmonic output from a sample of lead in porous glass exposed to a 5-kc/sec alternating field of various amplitudes in a steady field *H*. The sample is that of Fig. 6.

structure of a thin surface layer of the porous Vycor is coarser than that of the bulk.

EXPERIMENTAL OBSERVATIONS

To test the equations developed above, we measured the harmonic output at 4.2°K in a steady field of 11 000 Oe. The results are shown in Fig. 6. The observations cover a range of 500 cps to 5 kc/sec. (We are limited on the upper end by the 16-kc/sec limit of our analyzer.) The alternating fields ranged from 10 to almost 200 Oe in amplitude. The voltages were proportional to frequency over the entire frequency range—indicating no measurable viscosity in flux motion. In another experiment with a 100-kc/sec tuned amplifier we extended this conclusion to 33.3 kc/sec. The voltages are closely proportional to the square of the alternating field at larger values of the field with a noticeable fall off at lower fields as exemplified by the points at the lower left-hand corner of the figure. The solid line marked "3rd harmonic" is that calculated from Eq. (14) with a critical current density of 2.1×10^5 A/cm² referred to the total volume of the specimen. The solid lines marked 5th and 7th are derived from Eqs. (14) using the same current density. While those for the fifth harmonic fall on the theoretical curve, the two measured seventh harmonic points are 50% low. Figure 7 shows the variation of the third harmonic of a 5-kc/sec fundamental as the steady field is swept over a wide range. The amplitude of the alternating field is changed by a factor of two for each run. It is seen that the upper three curves are exactly a factor of four apart, as required by the theory, above about 4000 Oe while the lower curves progressively deviate in the sense of exhibiting a smaller voltage than that predicted by the theory. The even harmonic voltages are buried in the noise in this range above 4000 Oe, while the noise itself is a strong function of steady field and amplitude of oscillating field. Apparently, the noise, which is quite flat with frequency, arises from the fact that the hysteresis loop is not, in detail, a smooth curve as we envisioned it but rather is a series of tiny flux jumps as the flux breaks through the individual threads of the matrix. If this be so it is a rather close analog to the Barkhausen noise in ferromagnets that arises from discontinuous motion of domain walls. Lastly, as was anticipated, gross flux jumps have no effect on the voltages other than to give a momentary blast over the entire spectrum.

The lower end of Fig. 7 with its peak in voltage appears, at first sight, to be very interesting. Initially I thought that this indicated a nonmonotonic course of critical current as a function of steady field. This is not true inasmuch as the scaling relationships do not apply in this region. These peaks are insensitive to frequency and broaden at lower temperatures. Their cause is, I believe, simply the coarse surface layer noted earlier. If this layer were presumed to have a critical field of 3000 or 4000 Oe and a thickness of 20 μ or so, the main elements of this behavior would be simulated.[28]

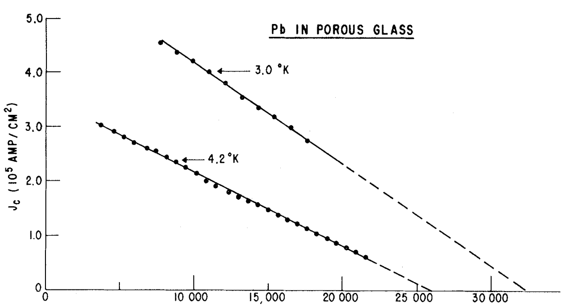

FIG. 8. Macroscopic critical current density J_c plotted as a function of the applied field H. The points are deduced from the data of Fig. 7.

Figure 8 shows the current densities derived from the data of Fig. 7 and another run at 3.0°K. The data are plotted only for those regions of steady field where Eqs. (14) are satisfied. The 3.0°K run terminates at 17.5 kOe because the $Nb_{0.75}Zr_{0.25}$ coil carries smaller superconducting currents at lower temperatures. The current density is seen to decrease linearly with increasing field. Another run, not shown here, carried the 4.2°K run to 28 kOe. The points followed the extrapolated line very closely with a slight concavity at high fields. The extrapolated values are $H_N(4.2°\text{K}) = 26.0$ kOe and $H_N(3.0°\text{K}) = 32.4$ kOe while the current densities extrapolated to zero field are $J_0(4.2°\text{K}) = 3.5 \times 10^5$ A/cm^2 and $J_0(3.0°\text{K}) = 6.0 \times 10^5$ A/cm^2.

DISCUSSION

While it is somewhat risky to discuss the experimental results on one specimen, several points can be made. First, the fact that for low driving fields the harmonics are not present to the expected amount may be an indication that for low enough driving fields the magnetization becomes reversible, i.e., the field merely varies with the quasi-London penetration mentioned earlier. Figure 7 suggests that the critical field for this process is a few oersteds. A second point is the surprisingly simple dependence of critical current on field—a dependence that differs sharply from the substantially hyperbolic dependence observed in many high-field superconductors.[13] The observed dependence suggests a very simple model for the course of the current density as a function of field. We assume that resistance appears whenever the local current density in a filament at any point rises to a critical value. The current density is the vectorial sum of two components, the usual London currents and the circulating currents induced by motion of flux through the filament. If the circulating current is perpendicular to the applied field then these currents add and the total critical current is equal to the sum of the field-induced current and the circulating current. This simple picture is presented with some diffidence inasmuch as the question of critical currents and fields is not a simple one.[12] The actual value of the critical current density within the lead filaments is not measured precisely by these experiments but a lower bound may be estimated. At 3.0°K and zero field, the macroscopic current density is inferred to be 6×10^5 A/cm^2. Since the lead forms 31% of the volume and, for a random array, the current resolved in one direction is half that flowing in the individual filaments, a lower bound for the microscopic critical current density is about 4×10^6 A/cm^2. The actual value is probably closer to 10^7 A/cm^2 owing to the limiting effect of constrictions. This value is of the same order of magnitude as that predicted theoretically but the ob-

[28] We have abraded 0.006 in. from the diameter of the specimen and retested it. As predicted, the low-field voltage peaks disappeared but, in addition, the voltages at no field follow precisely the predicted h_0 dependence. This may indicate substantial damage in depth by the abrasion.

served temperature dependence of the critical current is about twice that which has been predicted.[12]

CONCLUSION

The main conclusion of this work is that the magnetization of hysteretic high field superconductors can be well understood in terms of one phenomenological parameter, the macroscopic critical current density. A main problem remaining is the determination of the relationship between this parameter and the microstructure of hard superconductors.

ACKNOWLEDGMENTS

G. E. Nichols carried out the experiments reported in this paper with great skill and energy. We wish to thank W. DeSorbo, P. E. Lawrence, D. S. Rodbell, and C. H. Rosner for loaning us various pieces of experimental equipment. R. J. Charles and W. D. Schmidt prepared the porous glass specimens and C. E. Van Buren made the sample of V_3Ga. Lastly, I wish to acknowledge many helpful conversations with T. Buchhold, H. R. Hart, Jr., J. D. Livingston, and P. S. Swartz on the subjects covered in this paper.

Hard Superconductivity: Theory of the Motion of Abrikosov Flux Lines

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The central concept in the theory of what one might call the "critical phenomena" of hard superconductors—critical currents, critical fields, decay of persistent currents, "excess" voltages, etc.—is clearly Abrikosov's notion of the quantized flux line.[1] This is made almost obvious by the remark that, because of the now universally accepted validity of the quantization of flux through superconductors, the smallest possible breakdown of superconductivity is the motion of a single quantum of magnetic flux through the wire or ring. Thus, in all cases so far conceived, the lowest activation energy for any critical breakdown is that for the motion—and creation, if necessary—of single Abrikosov flux lines. This statement is independent of whether the mechanism for hard superconductivity is the GLAG one or the Mendelssohn sponge theory, although we assume the former to be valid in most cases. Even the decay of currents in true soft superconductors under α-particle bombardment[2] is probably best explained by the threading of Abrikosov lines through normal holes punched by the α particles.[3]

The purpose of this paper is to see how many of the phenomena of hard superconductivity we can understand qualitatively in terms of the thermally activated motion of Abrikosov lines past pinning centers, without going into unnecessary detail on the nature of the pinning centers—whether they are dislocations, cavities, precipitates, etc.—or the precise internal structure of the superconductor. Our task, then, is to study the process—presumably thermally activated barrier penetration—by which flux lines move.

Let us then suppose that we have a superconductor penetrated by a magnetic field H and carrying a bulk current, for simplicity $\perp H$, $J = c\nabla \times \mathbf{H}/4\pi$. The magnetic field will penetrate in the form of Abrikosov lines; their density is clearly not uniform because of J, and we expect their arrangement is to some extent irregular. The magnetic energy per unit volume is $H^2/8\pi$; we can think of this as a magnetic pressure exerted by the flux lines on each other, and in the absence of pinning centers this pressure would have to be equalized by a rearrangement of the lines, leading to $J = 0$. Examination of Abrikosov's theory shows that actually at all but low fields the internal and external fields are nearly the same, so that we usually assume $B = H$, a minor simplification of which Friedel *et al.* have considered the errors.[4]

In finding the rate of the activation process we need to know two things: the driving force exerted by the magnetic pressure, and the nature of the barriers. The former is more available to us theoret-

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[2] P. de Feo and G. Sacerdoti, Phys. Letters **2**, 264 (1962).
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