ACKNOWLEDGMENTS

The author is indebted to Dr. Elliott Lieb of IBM Research for discussion of the nature of the solutions of (2) and for pointing out the nodeless character of the solution with lowest g, a result similar to that obtained by Lieb and Mattis, Phys. Rev. **125**, 164 (1962), in a discussion of the ground states of electronic systems. He is also indebted to Dr. John Slonczewski of IBM Research for his lively interest in this work which directed attention to a number of important points.

Discussion 47

D. DOUGLASS, *Institute for the Study of Metals:* A remark on Marcus' paper. I think your initial assumption can be criticized. Namely, that all the quantities of interest vary only across the slab, in the thin dimension. As you know, the Ginzburg–Landau equations are partial differential equations involving derivatives in the other two directions. Professor Blatt's remark concerning the two-dimensionality of this problem is, I think, relevant to your case. There is no valid basis for dropping the terms involving the derivatives in the other directions. I'm going to present circumstantial evidence this afternoon for the case that the energy gap and other parameters are spatially modulated in the direction of the width of a very thin film. If this indeed is true then your calculations would be only of academic interest.

MARCUS: I think your point is well taken. What I will state is that this is a solution of the Ginzburg–Landau equations even for the bulk; but it probably does not have a lower free energy than the Abrikosov solution which of course does have transverse variables. There may be circumstances under which a solution like this will be observed if you choose the dimensions suitably. That would still have to be established.

W. H. KLEINER, *Massachusetts Institute of Technology:* Stimulated by a conjecture made by Stanley Autler, Laura Roth and I have independently discovered a solution of the Ginzburg–Landau equations of the Abrikosov type just below the upper critical field. This solution has field maxima on a triangular lattice with each lattice point having six nearest neighbors. This solution has a lower free energy than Abrikosov's solution with field maxima on a square lattice. The triangular lattice solution has β equal to 1.16, whereas the square lattice solution has β equal to 1.18. (Lower values of the parameter β of Abrikosov's theory correspond to lower free energies.) The square lattice solution is unstable with respect to the triangular lattice solution not merely metastable.

GORTER: May I just ask what you (Marcus) would expect to happen if you took d/λ much larger than 10. Would you then think that the lowest free energy would correspond to a thin superconductive region in a very thick slab?

MARCUS: They will shift slightly but still have essentially the same structure with about the same thickness of superconducting core.

Critical Currents in Thin Planar Films*

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INTRODUCTION

Repeated efforts have been made to measure the critical value of the current density necessary to suppress superconductivity in thin films. Agreement between various experiments and with theory has often been poor.[1–7] Experimentally there are two main difficulties. For geometries in which the current is not uniformly distributed in the film, it is necessary to know the details of the distribution. Conveniently prepared flat rectangular film strips have this difficulty. The second trouble is due to warming of the film by Joule heating which can occur if small normally conducting regions are present, for example where electrical contact is made to the film. The problem is acute since the current densities exceed 10^6 A/cm^2.

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2 W. F. Brucksch, Jr., and W. T. Ziegler, Phys. Rev. **62**, 348 (1942).
3 L. A. Feigin and A. I. Shal'nikov, Dokl. Akad. Nauk SSSR **108**, 823 (1956) [English transl.: Soviet Phys.—Dokl. **1**, 377 (1956)].
4 N. E. Alekseevski and M. N. Mikheeva, Zh. Eksperim. i Teor. Fiz. **31**, 951 (1956) [English transl.: Soviet Phys.—JETP **4**, 810 (1957)].
5 R. E. Glover, III, in *Low Temperature Physics and Chemistry,* edited by J. R. Dillinger (University of Wisconsin Press, Madison, Wisconsin, 1958), p. 330.
6 E. C. Crittenden, Jr., J. N. Cooper, and F. W. Schmidlin, Space Technology Tech. Rept. 60–0000–NR 356, 1960.
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The first of the difficulties can be avoided by using a so-called compensated geometry for which the current density is uniform. Such measurements[8,9] made in the vicinity of the zero current transition temperature indicate a temperature dependence which agrees with the $J_c \propto (1 - t^2)^{\frac{3}{2}}$ form suggested by Bardeen.[10] [This reduces to the Ginzburg–Landau result, $(1 - t)^{\frac{3}{2}}$, for temperatures near $t = 1$. Mercereau,[11,12] Hunt and Crane find this dependence for the critical current in ring-shaped films. Their result is remarkable since it is not clear that a compensated geometry is used.] The plane rectangular films, for which measurements are reported here, are also found to show this dependence when the experiments are interpreted with regard for the nonuniform current distribution.

EXPERIMENTAL DETAILS

Films were prepared inside a vacuum cryostat and the measurements made *in situ*. Vacuum was maintained at all times.

The substrate used was a crystalline quartz plate bonded with Apiezon N vacuum grease to a copper block in thermal contact with the cooling liquid (N_2 or He). To try and further improve thermal contact the substrate was pressed against the copper block with a mask machined from a $\frac{1}{8}$-in. copper plate. Separation of the mask and the substrate was maintained by a 1-mil sheet of Mylar. The substrate was surrounded by a thermal shield held at 77°K. A shutter in the shield could be opened to permit preparation of the film. A tungsten ribbon oven located outside the shield and about 2 in. from the substrate was used to evaporate spectroscopically pure tin (Vulcan Detinning Company). The films were condensed in a vacuum of about 10^{-6} mm Hg over a period of 3 to 5 min onto substrates which were held at 77°K during condensation. The films were subsequently annealed at room temperature. Thicknesses ranged from 200 to 600 Å as determined from the temperature-dependent resistance of the annealed films.[13]

Initially efforts were made to reduce Joule heating of the films by using short current pulses. Attempts to get consistent results by extrapolating to pulses of zero length were, however, not successful.

Static measurements were then made with special care to eliminate sources of Joule heating. Superconducting lead contacts and lead wires were used. A special set of potential leads made it possible to check that the contacts were free of resistance. The current necessary to restore 10^{-3} Ω of resistance in the films was measured as a function of temperature and taken to be the critical current. It is actually the value associated with the region of the film able to carry the least amount and is therefore a lower limit. However, the region which first becomes normally conducting presumably shows the same temperature dependence as the rest of the film. By limiting the restored resistance to this small value, Joule heating could be held to a few μW. Preliminary experiments indicated that the increase in temperature of the substrate was negligible at this power level. With this small amount of heating and the relatively good thermal contact the amount of restored resistance was found to be a single-valued function of the applied current, indicating that there was no tendency for the normally conducting zone to propagate through the film.[14]

For the measurements to be characteristic of a particular region of the film, it is important that the same resistance be restored at each temperature. The sometimes used procedure of noting the current at which a small fixed voltage appears is undesirable as the voltage depends on the current as well as on the resistance of the film. In a typical experiment, the measured critical current varies by a factor of 10^3 between the lowest temperatures and those just below the transition temperature T_c. Using a fixed voltage criterion 10^3 more material would be driven into the normal state for the high-temperature measurements than for the low-temperature ones.

RESULTS AND DISCUSSION

Critical current measurements as a function of temperature and restored resistance are given in Fig. 1 for one of the films. The value obtained for the critical current is found to be relatively insensitive to the amount of restored resistance chosen as a criterion. The critical current is seen to approach the abscissa with a finite slope in seeming contradiction to the $(1 - t^2)^{\frac{3}{2}}$ dependence suggested by Bardeen for the critical current density. The average current density (current/cross section) for the film of Fig. 1 is 1.8×10^6 A/cm² at 2°K, considerably lower than

[8] N. E. Alekseevski and M. N. Mikheeva, Zh. Eksperim. i Teor. Fiz. **38**, 292 (1960) [English transl.: Soviet Phys.—JETP **11**, 211 (1960)].
[9] N. I. Ginzburg and A. I. Shal'nikov, Zh. Eksperim. i Teor. Fiz. **37**, 399 (1959) [English transl.: Soviet Phys.—JETP **10**, 285 (1960)].
[10] J. Bardeen, Rev. Mod. Phys. **34**, 667 (1962).
[11] J. E. Mercereau and T. K. Hunt, Phys. Rev. Letters **8**, 243 (1962).
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[13] J. Niebuhr, Z. Physik **132**, 468 (1952).
[14] J. W. Bremer and V. L. Newhouse, Phys. Rev. **116**, 309 (1959).

the theoretical estimates of the critical current density. This measured value is, however, necessarily lower than the true value since the current distribution in the film is known to be nonuniform.

Bowers[15] has given equations for the current dis-

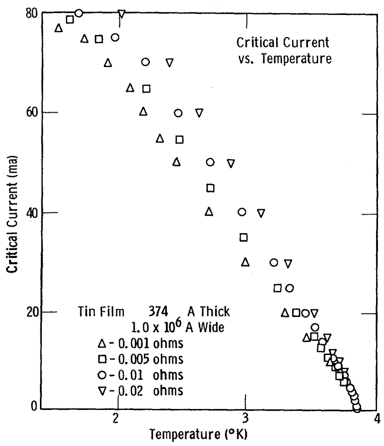

FIG. 1. Critical current necessary to restore resistance in a superconducting tin film as a function of temperature. The flat rectangular film was condensed at 77°K and annealed at 300°K. The various symbols correspond to different amounts of restored resistance.

tribution in a flat rectangular film subject to the London equations. It is assumed that the current density is constant through the film thickness $d(d < \lambda)$ and that $wd \gg \lambda^2$, where w is the film width and λ the penetration depth. In all but the extreme edge regions, the current density $J(x)$ is given by

$$J(x) = \frac{J(0)}{[1 - (2x/w)^2]^{\frac{1}{2}}} . \qquad (1)$$

The coordinate x is measured from the center of the film. Rhoderick and Wilson[16] measured the variation of magnetic field across a current-carrying superconducting film and found it consistent with this expression. For the edge regions of the film Bowers gives

$$J(x) = J(\tfrac{1}{2} w) \exp[-(\tfrac{1}{2} w - x)d/b\lambda^2] , \qquad (2)$$

where b is a constant on the order of unity. By matching these solutions at the point for which slopes and magnitudes are equal one obtains

$$J(\tfrac{1}{2} w) = \left[\frac{ewd}{2b}\right]^{\frac{1}{2}} \frac{J(0)}{\lambda} , \qquad (3)$$

[15] R. E. Glover, III, Annual Summary Report for U. S. Office of Naval Research Contract Nonr-855(08) NR 048-131 (University of North Carolina, Chapel Hill, North Carolina), 1959). Pertinent results are quoted in Ref. 16. See also discussion following paper by L. N. Cooper, in *Proceedings of the Seventh International Conference on Low Temperature Physics* (University of Toronto Press, Toronto, Canada, 1960), p. 416.

[16] E. H. Rhoderick and E. M. Wilson, Nature **194**, 1167 (1962).

where e is the base of the natural logarithms. Integrating over the width of the film the total current i is found to be

$$i = \pi w d \tfrac{1}{2} J(0) . \qquad (4)$$

The maximum current density occurs at the edge and the minimum at the center of the film. Since λ is temperature-dependent the peaking of the current at the edges will also change with temperature, becoming more pronounced as the temperature is lowered. Assuming $\lambda = \lambda_0/(1 - t^4)^{\frac{1}{2}}$ [where $t \equiv T/T_c$, $\lambda_0 \equiv \lambda(t = 0)$] for the temperature dependence of the penetration depth and eliminating $J(0)$ from (3) and (4) the current density at the edge of the film can be determined. When the total current is the critical current (the measured quantity) the current density at the edge is the critical current density

$$J_c(t) = (2e/bwd)^{\frac{1}{2}} i_c(t)(1 - t^4)^{\frac{1}{2}}/\pi\lambda_0 . \qquad (5)$$

For purposes of examining the temperature dependence it is convenient to divide this by the value at 0°K. This gives

$$J_c(t)/J_0 = i_c(t)/i_0(1 - t^4)^{\frac{1}{2}} , \qquad (6)$$

where $J_0 \equiv J_c(t = 0)$ and $i_0 \equiv i_c(t = 0)$. Values of this normalized critical current density obtained from measurements of i_c and t have been plotted in Fig. 2 as a function of $(1 - t^2)^{\frac{3}{2}}$. The constant i_0 was

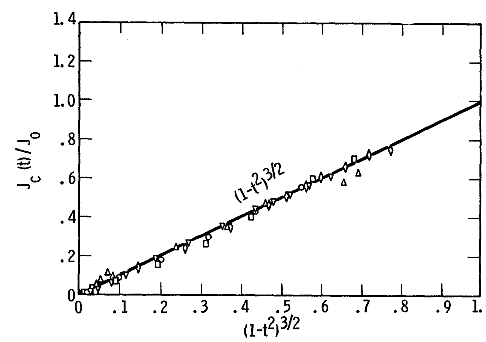

FIG. 2. Normalized critical current densities for five tin films as a function of $(1 - t^2)^{3/2}$ where t is the reduced temperature, $t \equiv T/T_c$. Values of $J_c(t)/J_0$ were obtained from critical current measurements by use of formula (6) which is intended to take account of the nonuniform current distribution in plane rectangular films. Values were normalized so that the points would fall on the straight line in the vicinity of $t = 0.6$.

picked for each film so as to put the measured points on the straight line in the vicinity of $t = 0.6$. The points are seen, however, to lie close to the line throughout the entire interval. The critical current density for the plane rectangular films is therefore described by

$$J_c(t) = J_0(1 - t^2)^{\frac{3}{2}}. \qquad (7)$$

This is to be compared with Bardeen's expression[12]

$$J_c(t) = \frac{1}{2} H_0 \left[\frac{\Delta(0)\sigma}{\hbar}\right]^{\frac{1}{2}} (1 - t^2)^{\frac{3}{2}}, \qquad (8)$$

$H_0 \equiv$ critical magnetic field at 0°K,

$\Delta(0) \equiv \frac{1}{2}$ gap energy at 0°K,

$\sigma \equiv$ residual normal state conductivity,

derived for the case that the electron mean free path is short compared with the coherence length $[\xi_0 = \hbar v_F/\pi\Delta(0)]$, a condition fulfilled for the films studied here. The temperature dependences in (7) and (8) are the same. It is interesting to also compare the magnitudes.

From (6) and (7)

$$i_c(t) = i_0 \frac{1 - t^2}{(1 + t^2)^{\frac{1}{2}}}. \qquad (9)$$

This is the temperature dependence of the directly measured critical currents for the rectangular plane films. With its help the measurements can be extrapolated to 0°K to obtain i_0. Finally setting $t = 0$ in (5)

$$J_0 = \left[\frac{2e}{bwd}\right]^{\frac{1}{2}} \frac{i_0}{\pi\lambda_0}. \qquad (10)$$

To obtain J_0 it is necessary to know λ_0, a quantity which varies from film to film because of electron mean free path effects. Values were calculated as suggested by Tinkham[17] and are listed for five films in Table I. The values of J_0 obtained from the measurements using Eqs. (9) and (10) are listed in the third column of the table. For purposes of the estimate the constant b in Eq. (10) was assumed equal to unity. For comparison, theoretical estimates of J_0 are listed in column four of the table. It was assumed that $\Delta(0) = 1.75kT_c$ and measured values of T_c and σ were used. H_0 was taken to be 304 G.

[17] M. Tinkham, Phys. Rev. **110**, 26 (1958).

TABLE I. Comparison of experimental and theoretical estimates of the 0°K critical current density of tin films.

Film thickness[a] (Å)	λ_0 (Å)	J_0 (Exptl) ($\times 10^7$A/cm²)	J_0 (Theory) ($\times 10^7$A/cm²)
260	1580	1.7	2.29
340	1520	2.0	2.39
340	1600	2.5	2.30
370	1520	2.5	2.37
520	1550	3.2	2.70

[a] Films were 100 ± 5 μ wide.

CONCLUSIONS

The current necessary to restore resistance to plane rectangular films of superconducting tin was measured as a function of temperature. The films were thin compared to the 0°K penetration depth. Values of the maximum current density present in the films were calculated assuming the current distribution proposed for this case by Bowers. The temperature dependence of these critical current densities was found to be $(1 - t^2)^{\frac{3}{2}}$ in agreement with theory. Measurements on the films indicated a 0°K critical current density on the order of 2×10^7A/cm². This agrees in order of magnitude with theoretically predicted values.

Temperature Dependence of the Critical Current of Tantalum and Indium-Tin Alloy Films

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The critical current density for thin superconducting films has been derived by H. London[1] from the London and London theory, by Gor'kov[2] from the Ginzburg–Landau (GL) theory and by Bardeen[3] from the Bardeen–Cooper–Schrieffer (BCS) theory. These theories yield nearly the same result for the temperature dependence of the critical current density for temperatures near the critical temperature (T_c) but differ in their prediction of the magnitude of the critical current.

We have investigated the temperature dependence of the critical current of pure tin films near T_c and have found agreement with the theoretical predictions. However, our tantalum thin films and indium-

[1] H. London, Proc. Roy. Soc. (London) **A152**, 650 (1935).
[2] L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. **36**, 1918 (1959); **37**, 833 (1959) [English transl.: Soviet Phys.—JETP **9**, 1364 (1959); **10**, 593 (1960)].
[3] J. Bardeen, Rev. Mod. Phys. **34**, 667 (1962).