centers. Their results suffice to show that the w term in $\mathfrak{F}$ is unchanged by the presence of impurities, as is the spin term, whereas w'' is replaced by a new function which may be written as

$$w'' \rightarrow w''_\tau \equiv \pi T \sum_{n=0}^{\infty} (\omega_n^2 + |\Delta|^2)^{-1} \times \left[(\omega_n^2 + |\Delta|^2)^{\frac{1}{2}} + \left(\frac{1}{2\tau_{\rm tr}}\right) \right]^{-1}, \quad (4)$$

where $\omega_n \equiv (2n+1)\pi T$, $\tau_{\rm tr}$ is the transport collision time, and w''_τ reduces to w'' as $\tau_{\rm tr} \rightarrow \infty$. How the w''' term in $\mathfrak{F}$ is modified cannot be determined without additional computation.

A comparison of the spin energy, Eq. (3), with the orbital magnetic energy of Eqs. (1) and (4) may now be made very crudely by approximating A by $B\delta$ (except for films of thickness $d \ll \delta$, in which case $A \sim Bd$). Then, for an ideal pure electron gas model, the orbital energy is seen to dominate by a factor $(p_F\delta)^2$, which might typically be of order 10^5. However, the transition metal compounds with very high T_c and H_{cu}, such as the V_3X compounds or Nb_3Sn, also all have high densities of states at the Fermi surface and very large effective masses, typically $\sim 10^2$. In addition, the electronic mean free path l in these materials as prepared in short compared to the coherence distance ξ_0 of the pure metal. In these more realistic circumstances, the ratio of orbital to spin energies is roughly $(mv_F\delta)^2(l/\xi_0)$, which, in many cases, could be of order unity. Thus a preliminary estimate confirms the importance of the spin energy for high critical field superconductors, and stresses the significance of extending Abrikosov's detailed calculations[11] of the negative surface energy phase to include the spin energy term, Eq. (3).

Finally, it is necessary to point out the limits of applicability of the local theory of superconductivity outlined above. The derivation leading to Eqs. (1)–(4) is an expansion resting crucially on the assumption that the coherence distance is short, and inspection shows that ξ is proportional to w''_τ. However, near a second-order critical point where $|\Delta|$ is small, roughly[17]

$$w''_\tau \backsim (\pi T)^{-1} \left[\pi T + \left(\frac{1}{2\tau_{\rm tr}}\right) \right]^{-1}.$$

As T tends toward zero, w''_τ approaches infinity and the expansion breaks down, despite the presence of a finite mean free path. Thus even though a superconductor may be local in weak fields, it becomes nonlocal at lower temperatures in fields sufficient to reduce the gap function substantially. Stated differently, as both T and Δ become small (compared to T_c), the coherence distance increases to the point where it no longer can be the shortest characteristic length entering the problem. As an example, the low temperature ($T \ll T_c$) magnetic transitions of a thin film ($d \ll \delta$), predicted to be of first order by Bardeen[5] from a theory resembling ours, must rather be discussed on the basis of the Gor'kov–Shapoval[18] nonlocal integral equations.

[17] Gor'kov (Ref. 3b) gives the exact expression.
[18] L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. **37**, 833 (1959) [English transl.: Soviet Phys.—JETP **10**, 593 (1960)]; E. A. Shapoval, Zh. Eksperim. i Teor. Fiz. **41**, 877 (1961) [English transl.: Soviet Phys.—JETP **14**, 628 (1962)].

Exact Solution of the Ginzburg–Landau Equations for Slabs in Tangential Magnetic Fields

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I. INTRODUCTION

The Ginzburg–Landau (GL) equations[1] for a slab of superconductor in a tangential external magnetic field may be taken as one dimensional in form, all quantities then being functions only of the transverse coordinate. The equations have usually been solved in the approximation that assumes the order to be constant or nearly constant across the slab,[2,3] which is adequate for films of moderate thickness for small kappa materials. However, the constant order approximation is not only quantitatively poor for thick films and large kappa materials, but completely fails to reveal the important high-field behavior and the

[1] V. L. Ginzburg and L. D. Landau, Zh. Eksperim. i Teor. Fiz. **20**, 1064 (1950).

[2] V. L. Ginzburg, Zh. Eksperim. i Teor. Fiz. **34**, 113 (1958) [English Transl.: Soviet Phys.—JETP **34**, 78 (1958)].
[3] Paul M. Marcus, in *Proceedings of the Eighth International Conference on Low Temperature Physics, London, 1962* (Butterworths Scientific Publications, Ltd., London, 1962).

interesting new classes of solutions in which the order oscillates around zero.

In the present work we discuss the behavior of the exact solutions, obtaining the conditions for stable, metastable, and unstable states by study of the free energy. Typical type II behavior[4,5] will be shown to occur for large kappa materials, but the structure of the order at large fields will be shown to resemble the intermediate state more than the mixed state of Abrikosov or Goodman, i.e., there is a small strongly superconducting core surrounded by a thick outer layer of nearly normal material extending to the surface. No oscillations of order from superconducting to normal and back occur in this lowest stable state. However, a series of additional locally stable states exists in which the order oscillates one or more times about zero within the slab. Such states are always metastable with respect to the lowest, nonoscillatory state, but may be either stable or metastable with respect to the normal state. A systematic classification and discussion of these states will be given.

II. BASIC EQUATIONS AND PROPERTIES OF THE SOLUTIONS

The GL theory starts from an assumed form of the free energy of a superconductor with respect to the normal state.[6] In one dimension, in reduced variables, this free energy difference between normal and superconducting states per unit volume of a slab of thickness d in a tangential external field h_e is

$$g \equiv \frac{G_s - G_n}{(VH_{cb}^2/8\pi)} = \frac{1}{d}\int_{-\frac{1}{2}d}^{\frac{1}{2}d} d\eta \left[\phi^2(\phi^2 - 2) + \frac{2\phi'^2}{\kappa^2} + 2a^2\phi^2 + 2(a' - h_e)^2\right]. \quad (1)$$

In (1) G_s and G_n are the free energies of the total specimen, of volume V, in the superconducting and normal states, respectively, and H_{cb} is the bulk critical field; h_e and the vector potential a are expressed in the usual reduced units,[1] $h_e = H_e/\sqrt{2}\,H_{cb}$, $a \equiv A/\sqrt{2}\,H_{cb}\lambda_0$, where λ_0 is the weak-field penetration depth. Similarly d and the transverse coordinate η are given in units of λ_0. The reduced order ϕ is zero in the normal state and has magnitude unity in the equilibrium superconducting state in the absence of a field at the given temperature. The derivatives in (1) are defined by $\phi' = d\phi/d\eta$ and $a' = da/d\eta = h(\eta)$, the reduced magnetic field; κ is a dimensionless intrinsic parameter distinguishing different superconductors. The vector potential has been fixed by choosing the unique gauge in which $\phi(\eta)$ is real (although not necessarily positive). The density of the reduced intrinsic free energy difference has been chosen in the simple form assumed by GL, $\phi^2(\phi^2 - 2)$, which has a minimum at $\phi^2 = 1$ and vanishes at $\phi = 0$.

The conditions that g, as given by (1), be stationary with respect to general first-order variations of the functions $a(\eta)$, $\phi(\eta)$ then lead to the GL differential equations (2a) and boundary conditions (2b) for ϕ and a,

$$\phi'' = \kappa^2\phi(\phi^2 - 1 + a^2)\,, \quad a'' = \phi^2 a\,, \quad (2a)$$

$$\phi_e' = 0\,, \quad a_e' = h_e\,, \quad (2b)$$

where ϕ_e, ϕ_e' and a_e, a_e' are the values of ϕ, ϕ' and a, a' at the edges of the slab ($\eta = \pm\frac{1}{2}d$). We are interested in the solutions of (2) which make g a minimum, and most interested in the lowest minimum for given values of the parameters. Also of interest is the magnetic moment of the slab in the field h_e. It is convenient to define a reduced moment μ by dividing by the magnetic moment the specimen would have in the bulk critical field if the field were completely excluded,[7] which gives

$$\mu = \sqrt{2}(|h_e| - 2|a_e|/d)\,. \quad (3)$$

The GL theory thus contains three intrinsic parameters—λ_0, H_{cb}, and κ—but these are reduced to one by using reduced variables. A physical problem (for slabs) is then specified by giving two problem parameters, d and h_e, which occur in the boundary conditions. The solution of (2) for given κ, d, and h_e leads to $\phi(\eta)$, $a(\eta)$ which may then be used to evaluate g from (1) and μ from (3). If several solutions are found, comparisons of g values in the different states obtained in this way will then lead to the stablest state.

The following general observations may be made about the solutions of (2): (1) The normal state is always a trivial solution of (2), $\phi(\eta) = 0$, $a(\eta) = h_e\eta$, $g = 0$. (2) The function $a(\eta)$ must have one node within the slab, and no minima or maxima.[8] (3) If

[4] A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. **32**, 1442 (1957) [English Transl.: Soviet Phys.—JETP **5**, 1174 (1957)].
[5] B. B. Goodman, Phys. Rev. Letters **6**, 597 (1961); IBM J. Res. Develop. **6**, 63 (1962).
[6] The appropriate free energy must be used which is a minimum at equilibrium in a given external field.
[7] This comparison moment treats the slab as if it were bulk material, and the penetration of the field could be neglected; thus for bulk material in the bulk critical field $h_e = 1/\sqrt{2}$, $a_e = 0$, and $\mu = 1$. Field penetration then always reduces the values of μ below 1.
[8] If there were no node, a'' would have only one sign, a' would be monotonic, and so could not have the same value at the two boundaries; if there were an extremum, from that position a would curve away from the axis in both directions and never approach the axis, hence would have no node, which is not possible.

$|\phi(\eta)|$ reaches the value unity from below, then $|\phi(\eta)|$ will increase and never return to the value unity (or zero).[9] (4) The solution $\phi(\eta)$ corresponding to the lowest g (for given values of d and h_e) has no nodes.[10] (5) Symmetric solutions of (2) exist [$a(\eta)$ odd, $\phi(\eta)$ even or odd about the center of the slab].

A general proof that all solutions are symmetric has not been found, but it seems likely to be true since the solutions are symmetric in two different limiting situations. In the limit of thin-enough films the order may be assumed constant across the film—then $\phi(\eta)$ is an even function, hence, $a(\eta)$ is odd. In the limit of weak fields when $h(\eta)$ and $a(\eta)$ vanish, the equation for ϕ has strictly periodic solutions (ϕ is an elliptic function of $\kappa\eta$[11]) and the solutions for $\phi(\eta)$ in the slab obtained by placing the boundaries at maxima or minima are all strictly even or odd (see Fig. 1). We assume symmetry, although the possibility remains open that nonsymmetric solutions exist.

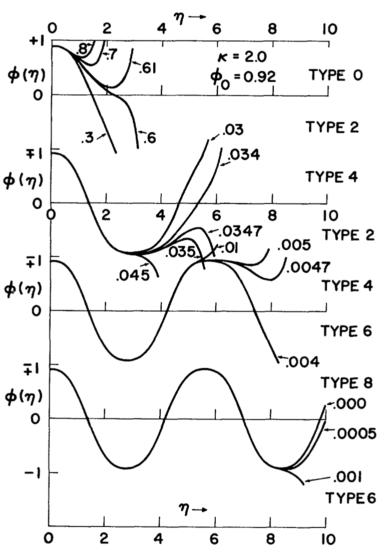

Fig. 1. Behavior of $\phi(\eta)$, $0 \leq \eta \leq 10$ at $\kappa = 2.0$ for $\phi_0 = 0.92$ as h_0 decreases from 0.8 to 0, showing the successive development of extrema and nodes. Solutions of (2) of type N have N nodes (in the slab, of which only the right half is plotted); slab solutions of type N–L place the slab boundaries at the Lth extremum from the center of the type N solution.

Symmetric solutions of (2a) are fixed by two parameters $\phi_0 \equiv \phi(0)$ and $h_0 \equiv a'(0)$ [for $\phi(\eta)$ even[12]] which provide a complete set of initial conditions at the center of the slab for these second-order equations, since $\phi'(0) = 0$, $a(0) = 0$; the solutions are readily generated by numerical integration for any given ϕ_0 and h_0. The behavior of the even solutions in the ϕ_0, h_0 representation is illustrated in Fig. 1, which plots $\phi(\eta)$ (for $0 \leqslant \eta \leqslant 10$) at $\phi_0 = 0.92$ and a succession of h_0 values, for $\kappa = 2.0$. The solutions may be classified according to the total number of nodes they show, hence as type 0, type 2, type 4, etc. As h_0 decreases, they approach the limiting periodic solution, especially near the origin where a is small, but as a increases going out from the origin, ϕ'' eventually changes sign and ϕ diverges to plus or minus infinity. Solutions in a slab are obtained by placing the boundaries at the successive extrema (symmetrically on the two sides of the origin), and may be labeled by the number of the extremum from the origin, thus, as type 0–1, type 2–1, type 2–2, etc. From the succession of curves of Fig. 1 we see that a solution of type 0–1 at the given ϕ_0 can be found for any d by choosing h_0. However type 2–1 can be found only within a certain range of d, at the upper limit of which a solution of type 2–2 takes over and then exists for all greater d out to infinity; similarly for type 4 and higher.

Solutions in the slab have been generated by systematic use of the ϕ_0, h_0 representation, with suitable interpolation on ϕ_0, h_0 to fit desired boundary conditions[13]; from these g and μ have been found as functions of the problem parameters d and h_e.

III. THE FREE ENERGY AND CRITICAL BEHAVIOR

The behavior of g as a function of h_e along the type 0 or nodeless solution curve for a succession of d values is shown in Fig. 2 at the low value of $\kappa = 0.2$ and in Fig. 3 at the high value $\kappa = 2.0$. Figure 2 shows that as d increases above 2, the solution rises above $g = 0$ to a maximum value at a maximum field,[14] where g shows a cusp, then decreases along the upper branch of the cusp to zero at a finite field h_{sc} (where it forms a second cusp with the normal state above h_{sc}). The lower branch is stable up to $g = 0$ (since it is the lowest solution up to there) and probably locally stable (hence metastable) between $g = 0$ and the maximum, while the upper branch is probably

[9] Since the equation for ϕ then becomes similar to the equation for a.

[10] If $\phi(\eta)$, $a(\eta)$ are solutions of (2) and $\phi(\eta)$ has nodes, then from (1) the functions $|\phi(\eta)|$, $a(\eta)$ give the same value of g. But $|\phi(\eta)|$ differs from $\phi(\eta)$ and is not a solution of (2), hence does not correspond to an extremum of g. Thus functions exist (close to $|\phi(\eta)|$, $a(\eta)$) which make g still smaller, hence $\phi(\eta)$, $a(\eta)$ do not give the lowest g.

[11] The relation is $\phi = \phi_0 \operatorname{sn} [(1 - \phi_0^2)^{\frac{1}{2}}\kappa\eta + C]$, an elliptic function of modulus $k = \phi_0/(2 - \phi_0^2)^{\frac{1}{2}}$, $\phi_0 =$ the maximum value of ϕ; C is determined by the choice of origin; the period in η is $4K(k)/\kappa(1 - \phi_0^2/2)^{\frac{1}{2}}$ where $K(k)$ is the complete elliptic integral of the first kind.

[12] For odd solutions, ϕ_0' and h_0 fix the solution. Discussion of odd solutions will be omitted here for brevity.

[13] A description of the rather complicated computational techniques must be omitted in this brief presentation.

[14] As $d \rightarrow \infty$, h_m approaches a finite limit whose value depends on κ and increases to infinity as $\kappa \rightarrow 0$ (see Ref. 2); this limit is then the maximum tangential superheating field that can occur at the surface of a bulk superconductor.

unstable[15]; the normal state (the $g = 0$ axis) is probably locally stable for fields down to h_{sc}.

The behavior of the thicker slabs in Fig. 3 shows a striking contrast to Fig. 2, with the cusp in g now forming below $g = 0$; the second (presumably unstable) branch now terminates at a second cusp at negative g, and gives rise there to a third superconducting branch going up to quite high fields. This third branch must be a second stable superconducting phase where it lies lowest, and is probably locally stable where it is higher than the other superconducting phase (the first branch). There is a similar but much smaller multiphase region on the $d = 5$ curve from $h_e = 0.844$ to $h_e = 0.850$, which does not show up on the scale of Fig. 3.

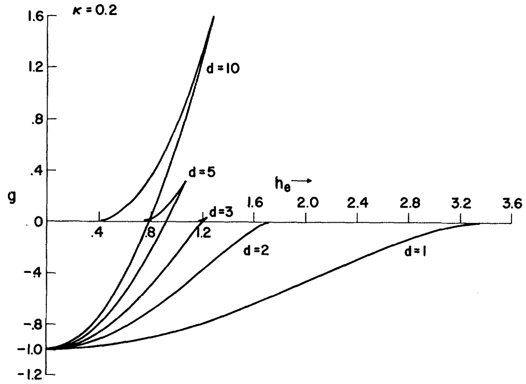

FIG. 2. The free energy difference g, defined in (1), vs the reduced external field h_e for slabs of reduced thickness $d = 1, 2, 3, 5, 10$ at $\kappa = 0.2$. A metastable range forms above $g = 0$ for thick slabs, ending in a cusp from which an unstable branch approaches the normal state from above and through decreasing fields.

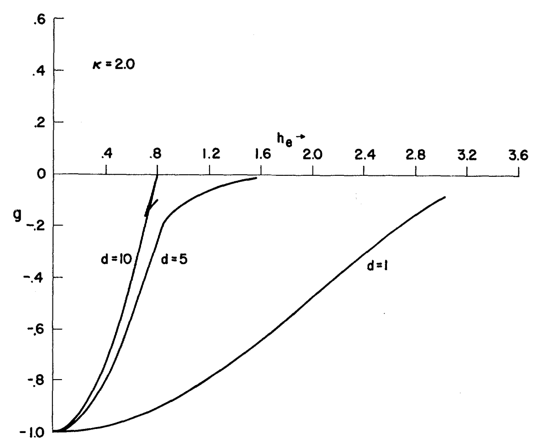

FIG. 3. g vs h_e for $d = 1, 5, 10$ at $\kappa = 2.0$. For thicker slabs, the first branch terminates in a cusp below $g = 0$, from which an unstable branch goes to still lower g, ending in a second cusp at which a new superconducting phase appears; the new phase approaches the normal state from below and through increasing fields to a critical value $\cong 2.0$.

[15] Namely, it corresponds to a local maximum of g. This identification of locally stable and unstable ranges is suggested by the calculations in the constant order approximation, as in Ref. 2, but has not been demonstrated in the general case.

Now as h_e increases from zero, g rises from -1 along the first branch to the intersection with the third branch. A first-order phase transition to the third branch is then thermodynamically favored above this intersection, but may not occur until the stability limit at the first cusp is reached; one or the other of these fields is then the lower critical field h_{c1} (for definiteness, we take the stability limit). Above h_{c1}, g rises along the third branch to zero and the normal state at an upper critical field $h_{c2} \cong \kappa$, where a second-order transition takes place.[16] This

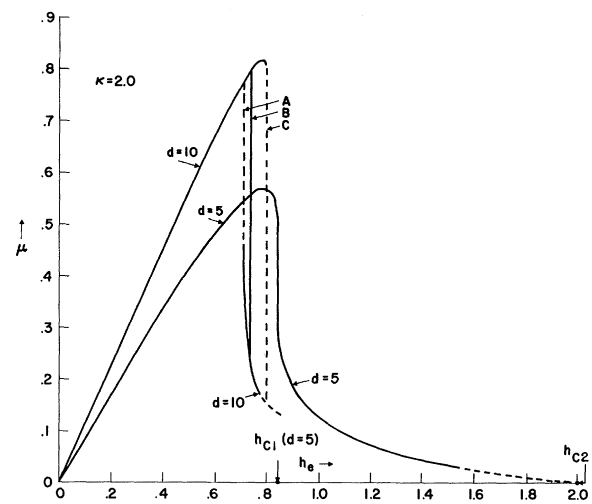

FIG. 4. The reduced magnetic moment μ defined in (3) and footnote 7, vs h_e at $\kappa = 2.0$ for $d = 5$ and 10 showing type II behavior, with a first-order transition at lower critical field h_{c1} (indicated for $d = 5$) and a second-order transition at an upper critical field $h_{c2} \cong 2.0$. On the $d = 10$ curve, A indicates the maximum supercooling field h_{sc}, B the field for thermodynamic equilibrium of the two phases and C the stability limit field h_m.

[16] The second-order nature of the transition is shown by the gradual decrease of $\phi(\eta)$ to zero everywhere in the slab as h_e increases; near the critical field then, the small ϕ approximation can be made {see Ref. 1 and A. Abrikosov, Zh. Eksperim. i Teor. Fiz. **32**, 1442 (1957) [English transl.: Soviet Phys.—JETP **5**, 1174 (1957)]} in which the linearized equations become $\phi'' + \kappa^2(1 - h_e^2\eta^2)\phi = 0$. This has solutions for given d only for discrete values of h_e which have a maximum; for large d these eigenvalues may be obtained from the harmonic oscillator problem, $h_e = \kappa/(2n + 1)$, $n = 0, 1, 2\cdots$ with largest value $h_e = \kappa$, above which there are no solutions. Thus the limit of the g curves on the normal axis is $h_e \cong 2.0$ at $\kappa = 2.0$ and $h_e \cong 0.2$ at $\kappa = 0.2$ as is consistent with Figs. 3 and 2 and indicated by more detailed calculation.

behavior of the state of the high-kappa specimen as h_e increases is directly illustrated by the variation of its magnetic moment plotted as a function of h_e in Fig. 4. Typical type II behavior is exhibited,[4,5] with an initial nearly linear rise, a discontinuous drop to a new state at h_{c1} (which is distinctly lower than the transition field of the low kappa slab of the same thickness), and then the long, slowly decreasing tail to the second-order transition to the normal state at $h_{c2} \cong 2.0$.

Insight into the character of the state of the specimen on the third branch of the type 0 solution curve (the high-field tail) is given by Fig. 5 which plots ϕ in the slab $d = 10$ ($\kappa = 2.0$) at $h_e = 0.736$ on the third branch and at $h_e = 0.746$ on the first branch. This shows that along the high-field tail there is a strongly superconducting core several penetration depths wide surrounded by a region of nearly normal material in which ϕ is nearly zero and h nearly constant. The small variation in h_0 (in the eighth significant figure) changes the ϕ curve from turning up (curves A and B) to turning down (curve C) hence brackets solutions in which the minimum of ϕ (and the surface of the specimen) can be at any distance.[17] By contrast, $\phi(\eta)$ on the first branch, at nearly the same problem parameters, does not show any nearly normal region. The corresponding $a(\eta)$ curves show that the field in the slab for the third branch is much greater than the field for the first branch solution.

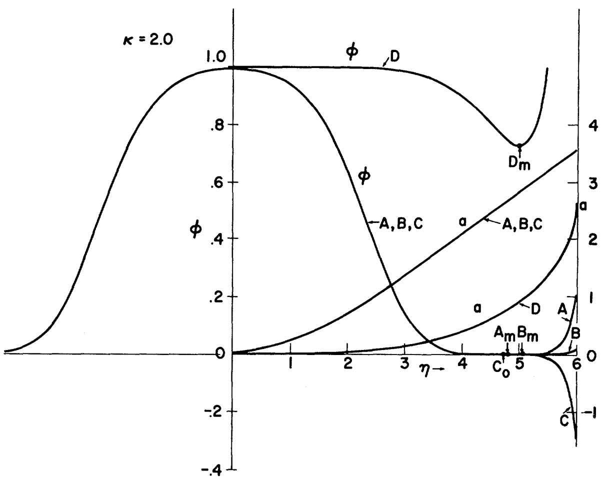

FIG. 5. $\phi(\eta)$ vs η and $a(\eta)$ vs η (ordinates on the right) at $\kappa = 2.0$ for four sets of values of ϕ_0 and h_0. Curves A, B, C have $\phi_0 = 0.99$ and the very closely spaced values $h_0 =$ 0.20455726, 0.20455724, 0.20455722, respectively. They correspond to an $h_e = 0.736$ (at $\eta = 5.0$), respectively, and bracket solutions with $\phi_0 = 0.99$ and minima at any η greater than 4.8 (A_m and B_m indicate the minima of A and B, C_0 the zero of C). Curve D, with $\phi_0 = 0.99995$ and $h_0 = 0.01429$, has a minimum at $\eta = 5.0$ (D_m), and an h_e (at $\eta = 5.0$) of 0.746. The solution with a minimum at $\eta = 5.0$ (between curves A and B) is on the third branch of the g curve of Fig. 6 (marked point A); curve D has nearly the same parameters, but is on the first branch of the g curve (marked D) and does not show the normal layer surrounding a superconducting core, characteristic of the high field phase.

In Fig. 6 the behavior of the type 0 (nodeless) solution is compared with the type 2 solution (two nodes) at $d = 10$, $\kappa = 2$. As noted in point (4) of the general observations, $g(h_e)$ for type 2 must lie above $g(h_e)$ for type 0 at every h_e. The type 2 curve does not show a high-field tail, but, like the solution at $\kappa = 0.2$, it rises above $g = 0$, and the unstable branch coming from the cusp heads toward the normal axis from above.[18] Another interesting feature is the phase transition at $h_e \cong 0.10$ when the solution switches from type 2–1 to type 2–2 and the first minimum moves inside the slab. Finally we note that between the type 0 and type 2 curves must lie $g(h_e)$ for the type 1 solution, the odd solution, with one node at $\eta = 0$.

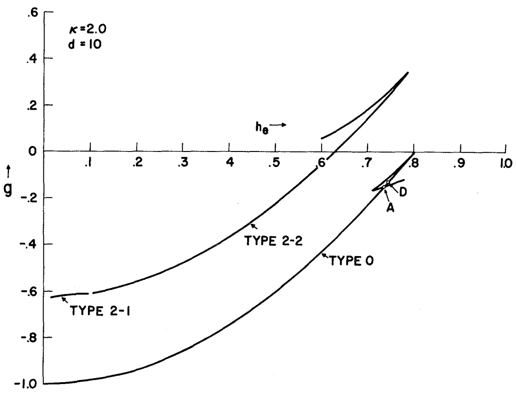

FIG. 6. g vs h_e at $\kappa = 2.0$ for $d = 10$ showing both the type 0 solution and the type 2 solution. The latter lies everywhere higher and does not show a high-field phase; it does show a phase transition from type 2-1 (slab boundaries at the first extremum, a minimum) to type 2-2 (slab boundaries at the second extremum, a maximum). The points marked A and D correspond to the curves of Fig. 5.

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[17] However, there is no significant change in g in pushing the minimum out to any larger value of d since the additional nearly normal layer has nearly zero free energy. Thus it is not necessary to fix h_0 with enormous precision to fix the values of g and μ.

[18] In fact, the limiting field on the normal state axis is approximately the third discrete state of the small ϕ approximation (Ref. 16), $n = 2$ and $h_e = 0.4$ (the state $n = 1$, $h_e = 0.67$ must be the limit of the type 1 solution which, as noted below, lies between type 0 and type 2).

ACKNOWLEDGMENTS

The author is indebted to Dr. Elliott Lieb of IBM Research for discussion of the nature of the solutions of (2) and for pointing out the nodeless character of the solution with lowest g, a result similar to that obtained by Lieb and Mattis, Phys. Rev. **125**, 164 (1962), in a discussion of the ground states of electronic systems. He is also indebted to Dr. John Slonczewski of IBM Research for his lively interest in this work which directed attention to a number of important points.

Discussion 47

D. DOUGLASS, *Institute for the Study of Metals:* A remark on Marcus' paper. I think your initial assumption can be criticized. Namely, that all the quantities of interest vary only across the slab, in the thin dimension. As you know, the Ginzburg–Landau equations are partial differential equations involving derivatives in the other two directions. Professor Blatt's remark concerning the two-dimensionality of this problem is, I think, relevant to your case. There is no valid basis for dropping the terms involving the derivatives in the other directions. I'm going to present circumstantial evidence this afternoon for the case that the energy gap and other parameters are spatially modulated in the direction of the width of a very thin film. If this indeed is true then your calculations would be only of academic interest.

MARCUS: I think your point is well taken. What I will state is that this is a solution of the Ginzburg–Landau equations even for the bulk; but it probably does not have a lower free energy than the Abrikosov solution which of course does have transverse variables. There may be circumstances under which a solution like this will be observed if you choose the dimensions suitably. That would still have to be established.

W. H. KLEINER, *Massachusetts Institute of Technology:* Stimulated by a conjecture made by Stanley Autler, Laura Roth and I have independently discovered a solution of the Ginzburg–Landau equations of the Abrikosov type just below the upper critical field. This solution has field maxima on a triangular lattice with each lattice point having six nearest neighbors. This solution has a lower free energy than Abrikosov's solution with field maxima on a square lattice. The triangular lattice solution has β equal to 1.16, whereas the square lattice solution has β equal to 1.18. (Lower values of the parameter β of Abrikosov's theory correspond to lower free energies.) The square lattice solution is unstable with respect to the triangular lattice solution not merely metastable.

GORTER: May I just ask what you (Marcus) would expect to happen if you took d/λ much larger than 10. Would you then think that the lowest free energy would correspond to a thin superconductive region in a very thick slab?

MARCUS: They will shift slightly but still have essentially the same structure with about the same thickness of superconducting core.

Critical Currents in Thin Planar Films*

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INTRODUCTION

Repeated efforts have been made to measure the critical value of the current density necessary to suppress superconductivity in thin films. Agreement between various experiments and with theory has often been poor.[1–7] Experimentally there are two main difficulties. For geometries in which the current is not uniformly distributed in the film, it is necessary to know the details of the distribution. Conveniently prepared flat rectangular film strips have this difficulty. The second trouble is due to warming of the film by Joule heating which can occur if small normally conducting regions are present, for example where electrical contact is made to the film. The problem is acute since the current densities exceed 10^6 A/cm^2.

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