Flux Quantization and Persistent Currents

T. I. SMITH and H. E. RORSCHACH, JR.
Rice University, Houston, Texas

I. INTRODUCTION

The recent discovery of quantized flux[1] appeared to explain the extremely long lifetime of persistent currents in superconductors. The existence of the flux quantum indicated that the energy of a superconductor in a current carrying state is a local minimum with respect to current variations if the flux is an integral number ν of flux quanta $hc/2e$. Bohr and Mottelson[2] attributed this local minimum to the pairing energy 2Δ of electron pairs having identical nonzero center of mass momentum, where Δ is the width of the gap in the BCS theory.[3] Small changes in the current are thus energetically unfavorable, since it is necessary to break the pairing to change the center of mass momentum for a single pair. Stability at $T = 0$ is thus adequately explained. For $T > 0$, some phonons of energy sufficient to overcome the energetic barriers are present. This is especially true near T_c where Δ vanishes, and calculations[4] based on the BCS theory show that instability should set in at a temperature T_c' given by

$$T_c/T_c' = 0.57[\ln|\nu| + \tfrac{1}{2}\ln(32\pi\ln|\nu|) + \cdots],$$

where T_c is the critical temperature defined by the discontinuities in the thermal properties. For macroscopic size hollow cylinders and laboratory size fields, $\ln|\nu|$ can easily be as large as 10, leading to the prediction that the decay of persistent currents should occur for temperatures considerably less than the transition temperature. Stability in the charged Bose gas model for which there is no energy gap was investigated by Bloch and Rorschach.[5] The stability in this case is due to electrodynamic effects, and instability appears at a critical field H_c' given by

$$H_c'/H_c = \tanh[\ln(r_2/r_1)],$$

where r_1 and r_2 are the inner and outer radii of the hollow cylinder and H_c is the critical field for the Meissner effect. For cylinders with a wall thickness $r_2 - r_1 \ll r_1$, H_c' can be appreciably less than H_c.

The above limits on the stability region for persistent currents may be too severe. It may appear experimentally that persistent currents can be stable at higher temperatures and fields than those given above if lifetime effects are important[5] and the decay to the stable state is sufficiently slow.

Measurements have been made on the stability of persistent currents in a long hollow tin cylinder to test the validity of the above estimates. Preliminary results indicate that end effects must be considered, but the experimentally observed stability limits exceed the limits mentioned above. Several interesting lifetime effects associated with the current decay have also been observed. These results suggest that lifetime effects may be important and necessary for a complete understanding of persistent currents.

II. MEASURING METHOD

All measurements were made on a tin cylinder 12.5 cm long with an inner diameter of 1.5 cm and a wall thickness of 0.15 cm machined from Johnson–Matthey 99.999+% tin.

The magnetic field H_i inside the cylinder was measured with a fluxgate probe consisting of a small solenoid surrounding a sensitive element. Current flowing in a solenoid nulls the field to be measured. The magnetic field is proportional to the measured current in the solenoid. Since the probe is a null device, it is extremely stable, and its sensitivity is independent of temperature and other variables. The probe was 0.75 cm long with a diameter of 0.2 cm. The sensitivity was sufficient to measure fields with an absolute accuracy of ± 0.5 G, while changes of 0.2 G could be observed. Through a feedback circuit, the apparatus measured the field continuously with a time constant of less than 0.1 sec.

The external field H_e was produced by a long solenoid and was uniform to $\pm 0.04\%$ over a distance of 12 cm on the axis. The magnitude of the field was 17.55 ± 0.01 G/A, and all fields given in this paper in A must be referred to this constant. The current was electronically regulated to $\pm 0.05\%$, and could be held at a fixed value or swept smoothly and continuously at any rate from 0 A/sec to 10 A/sec.

The pressure of the helium bath was measured by

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[1] B. S. Deaver and W. M. Fairbank, Phys. Rev. Letters **7**, 43 (1961); R. Doll and M. Nabauer, *ibid.* **7**, 51 (1961).
[2] A. Bohr and B. R. Mottelson, Phys. Rev. **125**, 495 (1962).
[3] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
[4] J. Parikh, Phys. Rev. **128**, 1530 (1962); G. Wentzel (private communication).
[5] F. Bloch and H. E. Rorschach, Phys. Rev. **128**, 1697 (1962).

the angular deflection of a quartz bourdon tube pressure gauge (Texas Instruments Incorporated). The gauge has an external output giving a dc signal of magnitude and sign proportional to the deviation of the deflection from a reference angle. This signal was used to control the current through a resistor in the bath, and the temperature could be regulated to $\pm 0.0005°K$, although the absolute temperature was known only to $\pm 0.005°K$.

Measurements of internal field as a function of the external field could be made under several conditions. To obtain "equilibrium" values, the current in the external solenoid was varied in 1-A steps. This variation was accomplished in 30 sec. The external field was then held constant for approximately 2 min while the internal field decayed to its equilibrium value. Alternatively, the external field could be swept continuously from $+H_c$ to $-H_c$ at any desired rate, and a continuous recording made of H_e vs H_i on an $x - y$ plotter.

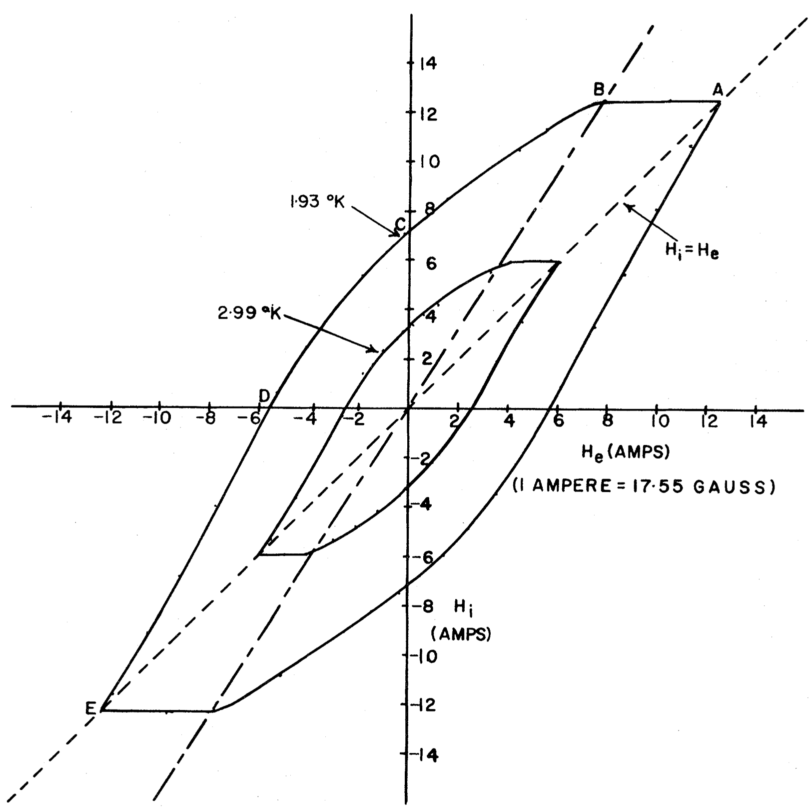

FIG. 1. "Equilibrium" values of the internal field H_i in a hollow superconducting cylinder as a function of the external field H_e.

III. RESULTS

"Equilibrium" values of the internal field as a function of external field are shown in Fig. 1. Curves for two temperatures are shown; at other temperatures the curves are geometrically similar. Each curve shows a region AB for which the full critical field is trapped. The region BCDE is obtained by waiting for equilibrium following successive small reductions in external field. Here the internal field is decaying, and times of the order of 2 min are required for stabilization. Interesting time effects were observed in these decays. Following a change in external field, the internal field remained constant for a time of the order of 5 sec. An exponential decay to the final value then began, with a time constant of approximately 20 sec. This suggests that these decays are associated with end effects, the 5-sec delay being the time necessary for the disturbance to reach the field detector at the center of the cylinder. A second time effect was found to be associated with the rate of change of the external field dH_e/dt. The final "equilibrium" value of the internal field, for example, at point C depends on dH_e/dt *during the time the field is changing,* independent of the length of time allowed for the decay at each point. For large value of dH_e/dt, the trapped field is appreciably larger than for small values.

Table I shows the field trapped at point C at a temperature of 2.08°K as a function of the time T in seconds required to continuously reduce the external field from H_c to zero.

The trapped fields were found to be quite stable if the temperature is held constant. We could detect

TABLE I. The trapped field H_i at point C is given for various values of the time T required to uniformly reduce the external field from H_c to zero.

T (sec)	H_i at C (in A)
0.5	7.67
13	7.58
33	7.47
113	7.04
340	6.80
3245	6.54

no change greater than 0.1% over times as long as 15 min. The effect of a temperature change depended on the region of the equilibrium curve investigated. An increase in temperature always led to a decrease in internal field, consistent with the equilibrium curve at the higher temperature. On section BCDE, a decrease in temperature did not affect the field. Along section AB, however, a decrease in temperature led always to an *increase* in internal field. In some cases the field could be increased as much as 25% by decreasing the temperature. The internal field was always equal to the critical field at the reduced temperature, as long as the field was changing with temperature. The original field could be restored by returning to the initial temperature. Each repetition of the temperature cycle resulted in a reduced maximum increase in the internal field.

The equilibrium curves of Fig. 1 display an interesting symmetry. The stability region is bounded by fields produced by the cylinder equally above and below the line $H_i = H_e$. In Sec. IV, we show that this result is probably associated with end effects.

Another interesting time effect occurred in the neighborhood of point B (Fig. 1) as the external field was uniformly decreased at various rates from above the critical field. Figure 2 shows a continuous record of the internal field as a function of the external field for various sweep rates. The unusual feature is the sudden rapid decrease in internal field near point B for the more rapid sweep rates. In some cases the internal field for one sweep rate is far below that for a slower sweep, which contradicts the expectation that slower sweeps should lead to fields nearer the "equilibrium" value.

IV. DISCUSSION AND CONCLUSIONS

The form of the curves limiting the stability region can be understood in terms of end effects without invoking the arguments of Refs. 4 and 5. We designate the field at the detector produced by the currents I flowing in the cylinder by H_I. At the ends of the cylinder, close to the wall, the field lines due to H_I are bending rapidly, and the field is increased by a factor $\alpha > 1$, whose value depends on the wall thickness. The field will be largest adjacent to the wall where the field lines are perpendicular to the cylinder axis. The lines of H_I bend around to the outside wall, and H_I decreases as the distance from the end increases. Stability is determined by the condition that the maximum total field on the surface is equal to H_c. Three equations define the stability region:

$$\alpha^2 H_I^2 + H_e^2 \leqslant H_c^2$$
$$|H_e \pm H_I| \leqslant H_c\,.$$

For given external field, each of these three equations determines a maximum value of H_I. The smallest of these determines the boundary curve and leads to the symmetry mentioned in Sec. III. Further evidence that end effects are responsible for the re-

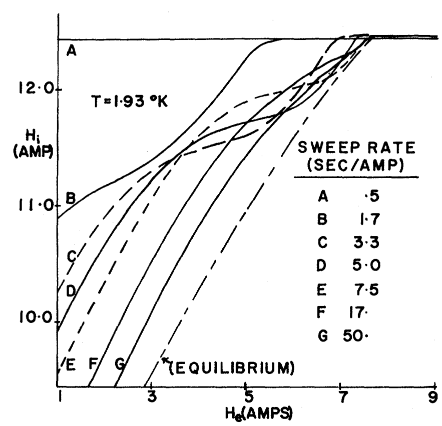

FIG. 2. Time effects associated with a uniformly changing external field. A continuous recording is given in this figure of the internal field H_i as a function of the external field H_e for various values of the rate of change of H_e.

duction in the trapped field was obtained at Oak Ridge National Laboratories by Coffey and Gauster.[6] Guard rings of niobium were added at the ends of a

[6] D. L. Coffey and W. F. Gauster (private communication).

lead cylinder. The trapped fields were thereby increased nearly to the critical field.

We do not yet understand the increase of the trapped field with decrease in temperature on line AB of Fig. 1 or the unusual time effects of Fig. 2.

The above results indicate that end effects limit the fields that can be trapped in finite length cylinders, but that these fields are still larger than the stability limits set by present theories. This suggests either that lifetime effects are extremely important or that an essential element is still lacking in our understanding of persistent currents.

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Discussion 43

Bardeen: I think there are two questions in regard to the stability of persistent currents. One is stability against quasi-particle scattering; in this case the superfluid is stable all the way up to the critical temperature. You made reference to Wentzel in that, if you accelerate everything to a given velocity and then let the quasi-particles decay to local equilibrium, you still have a supercurrent flowing. In other words, you do have quasi-particle equilibrium and still have a current flow. In addition, if it's stable against quasi-particle scattering it still may be unstable against the motion of quantized vortex lines or normal domains forming or something of that sort. These are two separate questions. It may be metastable in that there's no local energy minimum along the curve and it still may have a very long lifetime because the only way for it to decay is to introduce quantized vortex lines, or to decay some other way. It can't decay by quasi-particle scattering.

Rorschach: Would you interpret Wentzel's results in that below this lower transition temperature that the whole set of quasi-particles would not be able to decay?

Bardeen: Well, they will decay to a local equilibrium and that's what we call the supercurrent.

Does Superconductivity Imply Flux Quantization?*

BRIAN B. SCHWARTZ† and LEON N. COOPER‡

Department of Physics, Brown University, Providence, Rhode Island

The occurrence of flux quantization in multiply connected superconductors[1] can be understood if it is assumed that the ground state of the superconductor is composed of strongly correlated electron pairs with identical angular momentum.[2,3] If such pairs exist, as they do in the BCS theory,[4] then, given the structure of quantum electrodynamics, the quantization of the fluxoid (or, if a complete Meissner effect occurs, the flux) in units of $ch/2e$ is implied. It is possible however, to construct generalizations of the theory of superconductivity in which the ground-state pairs do not all have identical momentum. This is true in particular for nonuniform samples in which the electron–electron interaction is not invariant under translations or rotations.[5] Under these circumstances the logical connection between superconductivity and flux quantization seems to disappear.

In the usual formulation of the theory of superconductivity, the ground state is constructed by correlating zero momentum, singlet electron pairs $(k\uparrow, -k\downarrow)$, and neglecting other electron–electron correlations which are not already present in the normal metal. This pairing reduces the many-body problem to a soluble problem which retains the qualitative features necessary for a description of superconductivity. The choice of zero momentum pairs is natural, if one assumes momentum is conserved in electron–electron scattering. However, this choice incorporates a dynamical assumption (that pairing gives a correct description of the system), as

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† Brown University Physics Fellow, National Science Foundation Summer Fellow. Present address: Rutgers University, New Brunswick, New Jersey.

‡ Alfred P. Sloan Research Fellow.

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2 N. Byers and C. N. Yang, Phys. Rev. Letters **7**, 46 (1961).

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