Effect of Microwaves on Josephson Currents in Superconducting Tunneling*

SIDNEY SHAPIRO,† ANDRE R. JANUS, and SANDOR HOLLY
Arthur D. Little, Inc., Cambridge, Massachusetts

Josephson[1] predicted that paired electrons could tunnel from one superconductor through a thin dielectric barrier into a second superconductor. This paired electron tunneling current was shown to have two forms. A direct supercurrent could flow up to some maximum value without any voltage being developed across the tunneling sample. These zero-voltage currents have been reported by Anderson and Rowell[2] and by Rowell.[3] In addition, with a finite bias V across the sample the paired electron tunneling current would alternate at a frequency $2eV/h$. Josephson also argued that the alternating supercurrent would be frequency-modulated by an applied rf field and that this would lead to regions of zero slope in the dc (or low frequency) I–V characteristic at bias voltages given by $nhf/2e$, where f is the radio frequency and $n = 0,1,2,\cdots$.

One of us[4] has reported the observation of these zero-slope regions at two widely separated frequencies (9400 Mc/sec and 24 850 Mc/sec) and confirmed the quantum relation indicated above. It is the purpose of this paper to present results on the periodicity of the current associated with each zero-slope region and to show that this periodicity follows from the frequency modulation picture indicated by Josephson.

The sample preparation and instrumentation were similar to that used earlier.[4] Aluminum–aluminum oxide–tin samples were mounted in a microwave cavity resonant at 9380 Mc/sec and the tunneling current vs voltage characteristic was studied as a function of microwave power. Data were taken at 0.9°K and in the earth's magnetic field.

Figure 1 shows a typical tunneling characteristic in the absence of microwave power. The zero-voltage Josephson current is clearly visible. Figure 2 shows, on a greatly expanded scale, the portion of the I–V characteristic near the origin (the center of each trace) at increasing microwave power levels. The periodic variation in current amplitude with power level is indicated by the figure.

In view of the many steps to be observed (see Fig. 3 of Ref. 4) and the noise and instability associated with each step at certain microwave power levels, it proved advantageous to take moving pictures of the I–V characteristic as the microwave power was continuously varied. (During the presentation of this paper at the conference, a typical moving picture was shown of the effect of microwave power on the Josephson current. However, Figs. 1 and 2 of this paper together with the figures of Ref. 4 provide a reasonably accurate presentation of the nature of the modifications in the I–V characteristic encountered as a result of the application of microwave power to the tunneling sample.) Several schemes were tried to

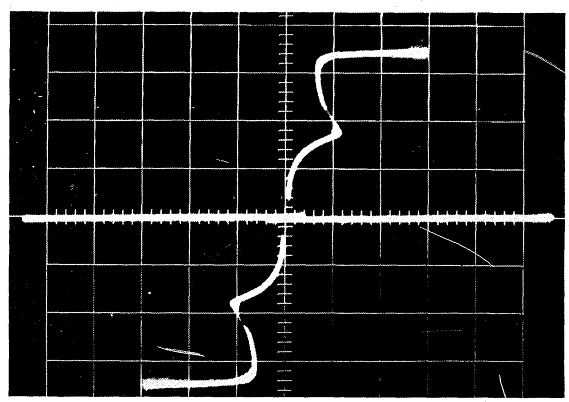

FIG. 1. Typical tunneling characteristic in the absence of microwave power. Scales: (V) 0.17 mV/cm, (H) 1.5 nA/cm.

correlate the microwave power level with the I–V curve in the moving picture. The most satisfactory was to photograph simultaneously both the I–V curve and the dial of a calibrated attenuator which was used to vary the power level. Data of the current amplitude in various zero-slope regions (corresponding to $n = 0,1,2$ etc. in Josephson's relation) were then obtained from the film by making measurements on selected frames with the aid of a viewer.

Figure 3 is a plot obtained in this manner of the

* Supported by the Electronic Technology Division, Aeronautical Systems Division, U. S. Air Force, Wright-Patterson Air Force Base, Ohio, under Contract No. AF33(657)-7915.

† Present address: Bell Telephone Laboratories, Murray Hill, New Jersey.

[1] B. D. Josephson, Phys. Letters 1, 251 (1962).
[2] P. W. Anderson and J. M. Rowell, Phys. Rev. Letters 10, 230 (1963).
[3] J. M. Rowell, Phys. Rev. Letters 11, 200 (1963).
[4] S. Shapiro, Phys. Rev. Letters 11, 80 (1963).

variation with microwave power level in the current at zero voltage and at the first three steps ($n = 0,1,2,$ and 3). The points represent experimental data, the solid curves represent the magnitude of the Bessel function, $J_n(2ev/hf)$, where v is the amplitude of rf voltage generated across the tunneling sample. A fit

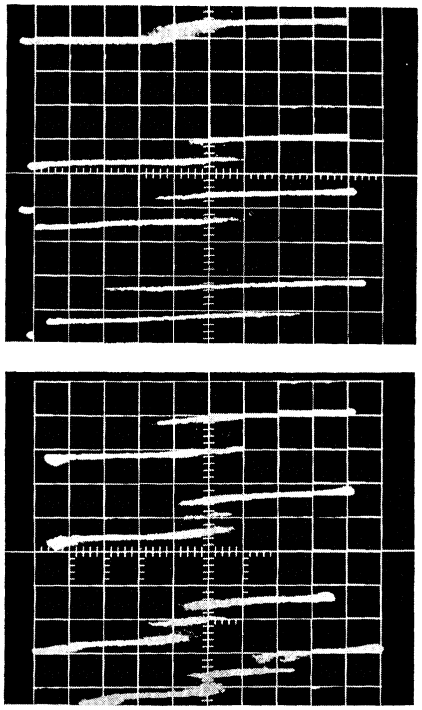

Fig. 2. The portion of an I–V characteristic near the origin ($I = 0$, $V = 0$ is at the center of each trace) showing the effect of microwave power at 9380 Mc/sec. The periodic change in current at each zero-slope region (here, $n = 0$ and 1) is indicated. Scales: (V) 42.5 mV/cm, (H) 3.75 nA/cm.

was made at one point only, namely, that point where the curve for $n = 1$ has its first maximum, which occurs at a voltage level (arbitrary units) of 10. Within experimental uncertainty each zero-slope current follows the pertinent Bessel function in its general variation but not in amplitude variation, particularly at higher rf voltages. The deviation at the higher rf voltages is most likely associated with current contributions from the background single-particle tunneling.

That the zero-slope currents should behave as Bessel functions was indicated by Josephson in his original paper. Consider that there is a dc bias voltage V maintained across the sample. Then, in general, the supercurrent is given by

$$I_s = I_0 \cos\theta\,, \tag{1}$$

where, again in general,

$$\theta = \int_0^t \omega_i dt \tag{2}$$

with ω_i the instantaneous angular frequency.

In the absence of rf,

$$\omega_i = \omega = 2eV/\hbar\,. \tag{3}$$

In the presence of rf, however, the instantaneous voltage V_i is

$$V_i = V + v\cos pt\,, \tag{4}$$

where $p = 2\pi f$, and so

$$\omega_i = 2eV_i/\hbar = \omega[1 + (v/V)\cos pt]\,. \tag{5}$$

Thus,

$$\theta = \omega t + (\omega v/pV)\sin pt\,, \tag{6}$$

and so

$$I_s/I_0 = \cos[\omega t + (\omega v/pV)\sin pt]\,. \tag{7}$$

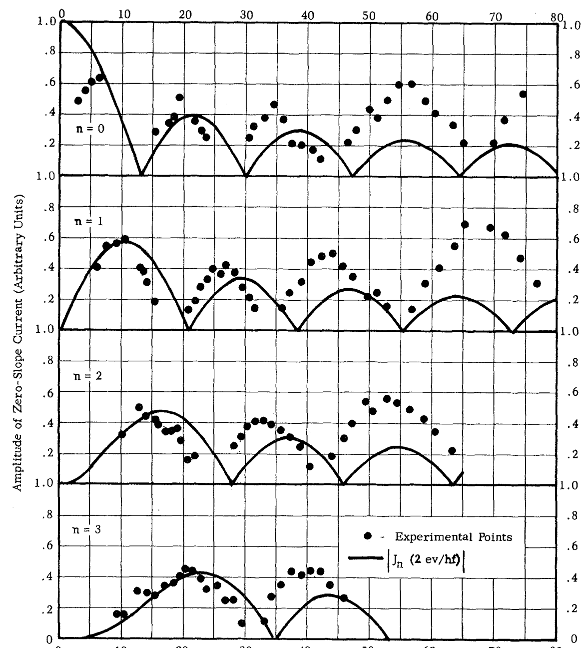

Fig. 3. The amplitude of zero-slope current at zero voltage and at the first three steps in voltage ($n = 0,1,2,$ and 3) plotted as a function of rf voltage amplitude across the tunneling sample. Also shown (solid line) is the magnitude of the corresponding Bessel function, $J_n\,(2ev/hf)$.

This relationship, familiar in frequency modulation analysis, can be written[5]

$$I_s/I_0 = \sum_{n=0}^{\infty} J_n(2ev/hf) \times [\cos(\omega + np)t + (-1)^n \cos(\omega - np)t]\,, \tag{8}$$

[5] L. B. Arguimbau, *Vacuum Tube Circuits and Transistors* (John Wiley & Sons, Inc., New York, 1956), p. 507.

where use has been made of Eq. (3). Equation (8) exhibits the desired relation; namely, at a bias V where $\omega = 2eV/h = np$, the supercurrent has a dc component (a zero-slope region) whose amplitude varies as the magnitude of $J_n(2ev/hf)$ with rf voltage.

The rf voltage across the sample is proportional to the square root of the microwave power coupled into the cavity. The proportionality constant is fixed for all the data by fitting at one point only. Thus the argument of $J_1(2ev/hf)$ at which the first maximum occurs is 1.84 and so v is taken to be equal to $1.84(hf/2e)$ or about 35 μV at that point. From the known position of the sample in the cavity, the known mode pattern, and measurements of cavity Q, frequency and coupling, it is possible to estimate the rf voltage across the sample. The value estimated in this way is two orders of magnitude below the value obtained above from the fitting procedure. This discrepancy is not understood. It is similar to that obtaining between the data of Dayem and Martin[6] and the theory of Tien and Gordon[7] on the effect of microwaves on single-particle tunneling.

---

[6] A. H. Dayem and R. J. Martin, Phys. Rev. Letters **8**, 245 (1962).

[7] P. K. Tien and J. P. Gordon, Phys. Rev. **129**, 647 (1963).

Discussion 34

KLEINMAN: Using the phonon assisted tunneling Hamiltonian that I discussed this morning and using the general techniques of Josephson, I've derived a phonon assisted tunneling of ground state pairs. To conserve momentum, the processes must involve the spontaneous omission of two phonons of equal and opposite wave vector. Thus to conserve energy we have $\hbar\omega = eV$ rather than $2eV$ as Josephson claims for photons. I think that's right because k is essentially zero for photons. The result I obtained is $J = J_0\alpha eV/2\Delta$, where J_0 is the discontinuity in the current at $eV = 2\Delta$ and α for lead is 0.58. This linear temperature independent leakage current has been observed by Rowell in lead where he finds values of alpha everywhere from 1 down to 0 and in tin by Taylor and aluminum by Giaever where alpha is less than 0.001. The experimental variation of alpha is at present a mystery. Because of the randomness of the phase of the spontaneously omitted phonons, the phase effects discussed by Josephson do not occur. The large value of alpha implies that if phonons a fixed phase are injected into the junctions the results may be even more spectacular than the microwave results we've just seen.

PIPPARD: I would like to ask Dr. Fiske if he has made any calculations of the resonant frequencies of the electromagnetic waves propagated between the plates. It seems to me that they ought to be at about $\frac{1}{10}$ the gap voltage, allowing for the very heavy inductive loading of these plates. I wonder in fact if these are not maser excited oscillations that he's observing, of the sort I think that Josephson's calculations would indicate ought to occur.

M. D. FISKE, *General Electric Research Laboratory:* I have not made such calculations. I think your suggestion of maser-type oscillations would be a very interesting one to consider.

Boundary Effects in Superconductors

P. G. DE GENNES

Faculté des Sciences, Orsay (S & O) France

I. INTRODUCTION

The present paper is concerned with the properties of layered structures of superconducting (and nonsuperconducting) materials, and with the related boundary problems. The basic experimental facts in this field are the following:

(a) for small superconducting samples surrounded by a nonmetallic substrate (e.g., films evaporated on glass), the transition temperature is very close to the bulk value. Tunneling experiments also show that the energy gap is close to the bulk value (derived for instance by ultrasonic attenuation).

(b) for "NS sandwiches" (thin superconducting film S deposited on a normal metal substrate N) the transition temperature may be significantly lower than in the bulk S material—or even vanish completely. Such effects were observed in the **Pb** Ag and **Sn** Ag systems.[1,2] However, as pointed out by Rose-Innes and Serin,[3] some of these experiments cannot be trusted entirely because of spurious atomic migration effects. To circumvent this difficulty one must deposit the films and keep them constantly at

---

[1] D. Smith, S. Shapiro, J. L. Miles, and J. Nicol, Phys. Rev. Letters **6**, 686 (1961).

[2] W. A. Simmons and D. H. Douglass, Phys. Rev. Letters **9**, 153 (1962).

[3] A. C. Rose-Innes and B. Serin, Phys. Rev. Letters **7**, 278 (1961).

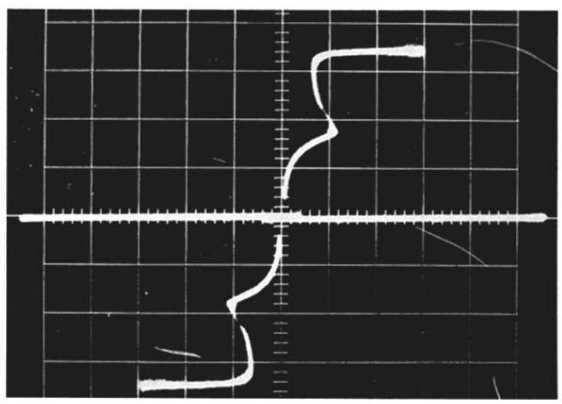

FIG. 1. Typical tunneling characteristic in the absence of microwave power. Scales: (V) 0.17 mV/cm, (H) 1.5 nA/cm.

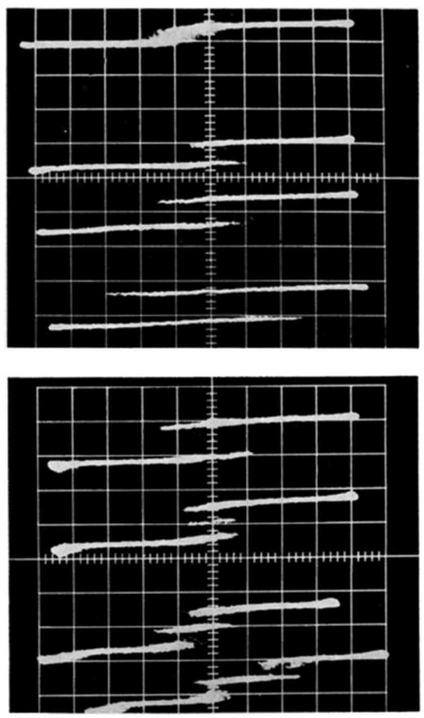

FIG. 2. The portion of an I–V characteristic near the origin ($I = 0$, $V = 0$ is at the center of each trace) showing the effect of microwave power at 9380 Mc/sec. The periodic change in current at each zero-slope region (here, $n = 0$ and 1) is indicated. Scales: (V) 42.5 mV/cm, (H) 3.75 nA/cm.