Magnetic and Thermal Properties of Second-Kind Superconductors. II. Thermal Conductivity in the Mixed State*

L. DUBECK, P. LINDENFELD, E. A. LYNTON, and H. ROHRER†

Rutgers, The State University, New Brunswick, New Jersey

INTRODUCTION

In an earlier note[1] we reported on the magnetic field dependence of the thermal conductivity of second-kind superconductors. In the mixed state, with $H_{c1} \leqslant H \leqslant H_{c2}$, our results on a In–Bi alloy specimen as well as earlier observations by Sladek[2] on a sample of In–Tl showed a minimum in the conductivity which became more pronounced with decreasing temperature. We were able to explain this in terms of the Abrikosov vortex model,[3] according to which a specimen in the mixed state is still entirely superconducting, except along filaments of negligible volume. The material can therefore be described in terms of a mean square order parameter which Abrikosov shows to be proportional to the magnetization of the specimen, at least near H_{c2}. Gor'kov[4] demonstrated that the superconducting energy gap is proportional to the order parameter. We therefore characterized the superconductor of the second kind in the mixed state by an average energy gap which varies as the square root of the magnetization. This allowed us to calculate the field dependence of the electronic and the lattice conductivities in the mixed state, using the expressions of the theory of Bardeen, Rickayzen, and Tewordt[5] in a manner fully discussed elsewhere.[6] Adding the two contributions gave a total conductivity, whose field dependence agreed very well with the experimental results.

This suggested mechanism for the conductivity minimum in the mixed state is thus quite different from that which is responsible for the quite similar minimum often observed in the intermediate state.[7] It has been shown both for cases in which phonon conduction predominates[8] and for primarily electronic conduction[9] that the nonmonotonic variation of the heat conduction in the intermediate state is caused by extra scattering of the heat carriers by the boundaries between the normal and superconducting laminae.

In order to investigate the possibility of some scattering by the quite different structure of the mixed state,[3] we have extended our measurements by observing the field dependence of the thermal conductivity in the mixed state with the heat flow both parallel and at right angles to the direction of the applied field and hence to the direction of the current vortices. We find that for a given sample at a certain temperature the results for both directions are very nearly the same. There is an indication of a small further lowering of the conductivity minimum when the heat flow is transverse to the magnetic field.

EXPERIMENTAL DETAIL AND RESULTS

In order to investigate the thermal conductivity in the mixed state without the possible complication of an intermediate state, and yet to be able to have the heat flow either parallel or at right angles to the external field, we prepared samples in the shape of plates, 0.1 cm thick, 2.0 cm long, and 2.0 cm wide. Two carbon resistance thermometers were attached to thin copper bars which were soldered with Wood's metal across one face of the specimen, 1.0 cm apart. A Constantan heater was wound on one end of the sample, and the other was soldered to a special holder. This latter could be clamped to the bottom of the helium bath in one of two ways, so that the direction of heat flow through the sample was either vertical or horizontal. In either orientation one of the

* This work has been supported by the National Science Foundation and the Rutgers University Research Council.

† Present address: IBM Research Laboratories, Zurich, Switzerland.

1 L. Dubeck, P. Lindenfeld, E. A. Lynton, and H. Rohrer, Phys. Rev. Letters **10**, 98 (1963).

2 R. J. Sladek, Phys. Rev. **97**, 902 (1955).

3 A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. **32**, 1442 (1957) [English transl.: Soviet Phys.—JETP **5**, 1174 (1957)]; J. Phys. Chem. Solids **2**, 199 (1957).

4 L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. **36**, 1918 (1959) [English transl.: Soviet Phys.—JETP **9**, 1364 (1960)].

5 J. Bardeen, G. Rickayzen, and L. Tewordt, Phys. Rev. **113**, 982 (1959).

6 P. Lindenfeld and H. Rohrer (to be published).

7 See, for example, K. Mendelssohn, and C. A. Shiffman, Proc. Roy. Soc. (London) **A255**, 199 (1960).

8 F. H. J. Cornish and J. L. Olsen, Helv. Phys. Acta **26**, 369 (1953); S. J. Laredo and A. B. Pippard, Proc. Cambridge Phil. Soc. **51**, 368 (1959).

9 S. Strässler and P. Wyder, Phys. Rev. Letters **10**, 225 (1963).

long dimensions of the sample was parallel to the external field which was always vertical and supplied by a niobium coil wound on the calorimeter can containing the sample. Thus, measurements of heat conduction both parallel and perpendicular to the field could be made on a single specimen in successive runs with a minimum of disturbance. The general experimental method has been described before.[10]

We have thus far made measurements on well-annealed In–Bi alloys containing approximately 2 and 4 at.% Bi. Both fall within the range for which the interphase surface energy is negative, and on which magnetization measurements have been made at this laboratory.[11] We thus had magnetization curves from which we could calculate the expected field dependence of the average energy gap in the mixed state. Furthermore, alloys of similar concentration were among those used by two of us for a recent extensive study of thermal conductivity in the normal and superconducting phases.[6] This helped us in our present work to separate the measured conductivities into electronic and lattice contributions.

Although the quantitative details of the field dependence of the heat conduction vary both with the alloy concentration and with the temperature, the general features are always very similar to those displayed in Fig. 1. With increasing field the conductivity remains constant up to $H = H_{c1}$, drops sharply to a minimum value, and rises again more gradually to a limiting value at $H = H_{c2}$. The depth of the minimum depends on the ratios of the conductivities in the normal and superconducting phases, and becomes more pronounced at lower temperatures. Not shown on the figure is that when the field is again decreased, the behavior is closely reversible to $H \sim H_{c1}$, below which the conductivity rises to a value intermediate between those in the normal and in the superconducting phases. This indicates trapped flux, the amount of which can be analyzed in the usual way.[2] The apparent reversibility in the mixed state may be due in part to the rather small difference between the normal conductivity and that at the minimum in the mixed state.

In measuring the heat conductivity of a given sample in two directions, care was taken to repeat the same ambient temperatures and to duplicate as closely as possible the measuring power and other experimental parameters. Figure 1 displays our results with the 4 at.% bismuth specimen for two sets of measurements both at 1.95°K but in different orientations. For greater clarity we show only the values taken in increasing field. The absolute values of the thermal conductivities in the superconducting and normal phases differed by about 3% between the two runs. To facilitate comparison we therefore normalized our results to the normal phase conductivity K_N, and plotted K/K_N as a function of the magnetic field H. The superposition of the curves for the two orientations of heat flow show that the general features of the variation of the conductivity with magnetic field are essentially independent of orientation. This gives strong support to our previous interpretation of the conductivity minimum in terms of an average, field-dependent energy gap.[1] The effect of this on the conductivity should of course be the same for any direction of the heat flow with respect to the mixed state vortex structure. If the heat carriers are also scattered by direct interaction with the spatial modulation of the energy gap, the additional lowering of the conductivity due to this should be most pronounced when the heat flow is transverse to the structure. Indeed we find for all three temperatures between 2.96° and 1.96°K for which we have complete measurements of the field dependence of the conductivities in two directions that for transverse heat flow the minimum is lowered by about 2%, as shown in Fig. 1. Note that the vertical scale does not start at the origin. This effect is less than the variation in K_S or K_N at a given temperature from run to run, and we will try to improve our re-

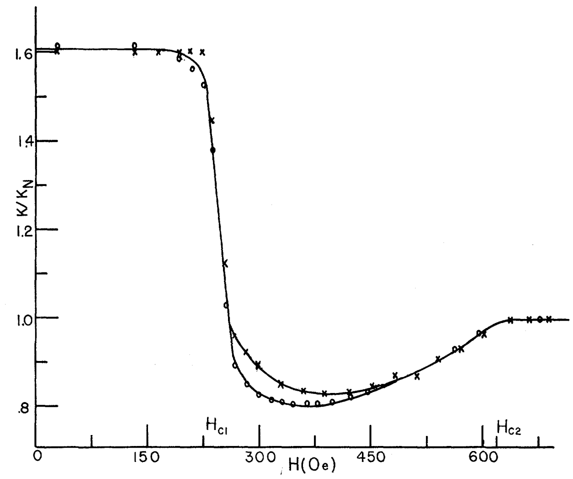

FIG. 1. The magnetic field dependence in increasing field of the thermal conductivity K of a In-4 at. % Bi specimen, normalized with respect to the normal state conductivity K_N. The crosses are the results with the heat flow parallel to the field; the circles with the heat flow transverse. H_{c1} and H_{c2} are the limiting fields of the mixed state.

[10] P. Lindenfeld and W. B. Pennebaker, Phys. Rev. **127**, 1881 (1962).

[11] T. Kinsel, E. A. Lynton, and B. Serin, Phys. Letters **3**, 30 (1962); and Rev. Mod. Phys. **36**, 109 (1964), preceding paper.

producibility to become more certain about this possible added scattering. In the 2 at.% bismuth sample, in which the electronic contribution to the total conductivity is larger, the effect seems to be more pronounced at the one temperature for which we have complete data. According to the Abrikosov model,[3] the spacing of the current vortices in the mixed state, at least near H_{c2}, is of the order of the superconducting coherence length, ξ. In our short mean free path alloys, $\xi \approx (\xi_0 l)^{\frac{1}{2}}$.[12] For indium, $\xi_0 \approx 2600$ Å[13] and for the 4 at.% Bi specimen the mean free path $l \approx 150$ Å, so that $\xi \approx 600$ Å.

To obtain an upper limit on the possible effect on the electronic conductivity we used the analysis of Strässler and Wyder[9] to calculate the difference between the conductivity parallel to and transverse to the vortex lines at the field corresponding to the empirical conductivity minimum, substituting in their expression the appropriate average field dependent energy gap. This predicted for the 4 at.% bismuth sample a difference in the total conductivity between the two orientations of about 12%. Using this method overestimates any possible effect, both because the electronic mean free path is in our case much smaller than the scale of the structure, and because the spatial modulation of the energy gap accompanying the vortex structure is probably not as effective a scatterer as an actual phase boundary. An effect of about 2% is thus not unreasonable, but we cannot be certain of its presence without further measurements with an improved technique.

[12] L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. 37, 1407 (1959) [English transl.: Soviet Phys.—JETP 10, 998 (1960)].
[13] A. M. Toxen, Phys. Rev. 127, 382 (1962).

ACKNOWLEDGMENTS

We are grateful to L. Del Vecchio for assistance in performing the experiments. One of us (H. R.) thanks Rutgers University and his colleagues for their hospitality during his most enjoyable and interesting stay.

Thermal and Magnetic Properties of Second Kind Superconductors. III. Specific Heat of Nb in a Magnetic Field*

T. MC CONVILLE and B. SERIN

Rutgers, The State University, New Brunswick, New Jersey

INTRODUCTION

Morin *et al.*[1] recently observed that V_3Ga in a magnetic field showed a specific heat discontinuity very similar to the discontinuity observed in most superconductors at the transition temperature in the absence of a field. Similar results for a specimen of Ta were obtained some years ago by Keesom and Désirant.[2] It was suggested by Goodman[3] that this behavior could be understood on the basis of the theory of Abrikosov.[4] According to this theory, in superconductors of the second kind the phase transition from the mixed state to the normal state at the upper critical field H_{c2} is a second-order transition. Both the entropy and the magnetization are continuous at the transition, but their derivatives are discontinuous. The discontinuity in the derivative of entropy gives rise to a specific heat discontinuity.

In this article we present observations on the specific heat of pure niobium in a magnetic field. Recently, on the basis of magnetic and resistance measurements,[5,6] this element has been identified as a superconductor of the second kind. The specific heat data which we have obtained do not seem to fit in a consistent way into the theory of Abrikosov.[4] Because of the great current interest in this field we pre-

* Research supported by the National Science Foundation and the Rutgers University Research Council.
[1] F. J. Morin, J. P. Maita, H. J. Williams, R. Sherwood, J. H. Wernick, and J. E. Kunzler, Phys. Rev. Letters 8, 275 (1962).
[2] W. H. Keesom and M. Désirant, Physica 8, 273 (1941).
[3] B. B. Goodman, Phys. Letters 1, 215 (1962).
[4] A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. 32, 1442 (1957) [English transl.: Soviet Phys.—JETP 5, 1174 (1957)]; J. Phys. Chem. Solids 2, 199 (1957).
[5] T. F. Stromberg and C. A. Swenson, Phys. Rev. Letters 9, 370 (1962); cf. also S. H. Goedemoed, A. van der Giessen, D. de Klerk, and C. J. Gorter, Phys. Letters 3, 250 (1963).
[6] S. H. Autler, E. S. Rosenblum, and K. H. Gooen, Phys. Rev. Letters 9, 489 (1962).