Now there are four possibilities: if reactions (9) are a direct-direct, simultaneous process (9a), or are delayed-direct (9d), they will have a bell-shaped energy distribution of the coincidence spectra and imply that τ_2 is zero and thus reaction (8) is direct. If reactions (9) are a sequential reaction of either the direct-delayed (9b) or delayed-delayed (9c) type, their coincidence energy distributions will have peaks, implying that $\tau_2 \neq 0$ for reaction (8) as well as (9). Thus a measure of the relative amounts of broad and peaked coincidence energy distributions for the three-body process (9) should allow one to conclude the relative importance of the direct versus delayed nature of reaction (8). To make this determination effective, however, it will be necessary to ascertain which particle is emitted first if the reaction is sequential. As was seen in Figs. 5–8, this appears to be a possible measurement in some circumstances.

An example of such a possible measurement might be to study the time delay of (d, p) measurements. These reactions are generally assumed to be of the direct type; yet they frequently have fluctuating cross sections. If the three-body reaction (He^3, $2p$) were studied, and if it were to be found to be of the direct-direct, or delayed-direct type, when the emitted protons have the proper energy to simulate the (d, p) reaction, then the direct nature of the (d, p) reaction would be independently established. Alternatively, if the reaction showed peaks in the angle-energy spectra, then the reaction is either delayed-delayed or direct-delayed, and in either case the (d, p) process could be presumed to proceed with some time delay. Finally, as discussed above, a comparison of the (d, p) cross-section fluctuations with the energy–angle diagrams of the corresponding (He^3, p, p) reaction will provide new information about the widths of the states.

ACKNOWLEDGMENTS

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Statistical Properties of Compound States*

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I. INTRODUCTION

The concept of an energy-averaged cross section has been relevant to the understanding of experimental data for at least as long as the concept of scattering experiments. However, it has been mainly in the analysis of nuclear scattering experiments that the understanding of this concept has been sharpened to the point where the new ideas that have evolved in the rather specialized sphere of nuclear reactions may have much broader implications in other areas of physics and chemistry.

The cross section $\sigma_{cc'}(E)$ for the reaction leading from channel c to the channel c' is a function not only of the energy E (or the driving angular frequency $\omega = E/\hbar$), but also of the parameters E_λ (energy positions) and $\gamma_{\lambda c}$ (reduced-width amplitudes) for the states λ that are excited by driving channel c. Thus we write

$$\begin{aligned}\sigma_{cc'} &= \sigma_{cc'}(E; \{E_\lambda\}, \{\gamma_{\lambda c}\}) \\ &= \sigma_{cc'}(E; H),\end{aligned} \tag{1}$$

where the sets of parameters $\{E_\lambda\}$ and $\{\gamma_{\lambda c}\}$ have been replaced symbolically by the Hamiltonian operator H which defines the eigenvalues E_λ and the eigenfunctions X_λ, the latter of which enter into the $\gamma_{\lambda c}$. Integrated cross sections of the form of (1) are additive with respect to symmetry (for example total angular momentum J and parity π):

$$\sigma_{cc'} = \sum_{J,\pi} \sigma_{cc'}{}^{J\pi}. \tag{2}$$

For one symmetry, the cross section can be written in terms of the scattering matrix element $U_{cc'}{}^{J\pi}$:

$$\sigma_{cc'}{}^{J\pi} = \pi \lambda\!\!\!^{-}{}_c{}^2 g_{J\pi} \mid \delta_{cc'} - U_{cc'}{}^{J\pi} \mid^2. \tag{3}$$

If we drop the $J\pi$ labels and set $g_{J\pi} = 1$ (s waves), we can more conveniently examine the notion of averaged cross section. To average the cross section over energy we introduce an energy resolution function $R_I(E-E')$ which we take to be a function of the energy difference $(E-E')$ between the energy E' at which the unaveraged cross section is specified and the energy E at which the averaged cross section is determined. In addition, the resolution function R_I depends on the interval I over which the average is carried out. The interval I is important since it defines the time $\hbar/I$ which the measurement takes. If the time $\hbar/I$ is small compared to the Poincaré recurrence time $2\pi\hbar/D$, where D is the mean distance between the fine

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† Deceased, 14 August 1964.

structure levels, then the conditions for a thermodynamically irreversible process are present; this is what a complex potential is designed to describe.

If only an energy average needed to be contemplated, then we would write the average cross section $\langle\sigma_{cc'}\rangle_{\text{Av}}$ as

$$\langle\sigma_{cc'}(E;I)\rangle_{\text{Av}}=\int dE'R_I(E-E')\sigma_{cc'}(E'). \quad (4)$$

However, we have noted that the cross section depends upon the parameters E_λ and $\gamma_{\lambda c}$; in taking the average we actually average over a sample of these parameters in addition. Thus we write (suppressing indices)

$$\langle\sigma(E;I)\rangle_{\text{Av}}=\int\dot{H}P(H)\int dE'R_I(E-E')\sigma(E';H), \quad (5)$$

where the Hamiltonian matrix ensenble $P(H)=P(H_{11}, H_{12}, \cdots)$ and its associated volume element $\dot{H}=dH_{11}\,dH_{12}\cdots$ have been introduced. The best-known matrix ensemble is probably the Gaussian ensemble for which

$$P(H)=C\exp\left[-\text{ Tr }(H-E_0I)^2/4a^2\right]. \quad (6)$$

It is well known that the consequences of (6) are the independent eigenvalue and reduced-width amplitude distributions (with $E_0=0$)

$$P(\{E_\lambda\})=C'\exp\left[-\sum_\lambda E_\lambda^2/4a^2\right]\prod_{\mu<\nu}|E_\mu-E_\nu|,$$

$$P(\{\gamma_{\lambda c}\})=C''\prod_\lambda\exp\left[-\tfrac{1}{2}(\gamma_\lambda,\Sigma^{-1}\gamma_\lambda)\right], \quad (7)$$

in which a and the channel covariance matrix Σ are parameters. These results, at least in an extreme statistical view, provide in principle the theoretical machinery needed to calculate cross-section averages. In practice these calculations are not so easy.

From (3) we note that σ depends, in general, quadratically on the amplitude (scattering matrix element) for the process in question. Only the total cross section is an exception to this statement, i.e.,

$$\sigma_c(\text{tot})=\pi\lambda\!\!\!^{-}{}_c^2\sum_{J\pi}g_{J\pi}2\text{ Re }(1-U_{cc}^{J\pi}). \quad (8)$$

We know that if a typical neutron total cross section is viewed with higher and higher resolutions we see a sequence of cross sections that appear as shown in Fig. 1. (In particular $I\rightarrow\infty$ gives the classical geometrical cross section.) Figure 2 shows a recent collection of data[1] exhibiting fluctuations in neutron total cross sections.

We remark briefly that our comments apply strictly, without some editing, to a sharp-surface system only. Real systems have diffuse surfaces and hence, for example, the hard-sphere cross section is increased somewhat although it may be possible to speak of an effective hard sphere.

[1] D. G. Foster, Jr., and D. W. Glasgow, (private communication).

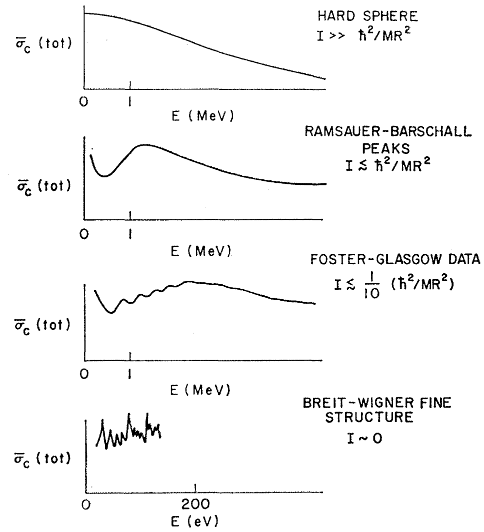

FIG. 1. Sketches of total cross sections observed with various energy resolutions.

Neither the large-I (hard-sphere to classical-single-number) nor the low-energy, small-I (fine-structure) cases now present special conceptual difficulties. The large-I cross section (or better, amplitude) serves as a background for the fluctuating phenomena found as I is decreased. Thus we are led to introduce a cross section based on an averaged amplitude. Since such a cross section usually depends on what we like to call the size or shape of the target (or compound) system, the cross section is referred to as a shape cross section (sometimes it is called a direct cross section—shape or size processes occur relatively rapidly). The average total cross section can be split up into shape contributions and fluctuation contributions as follows:

$$\langle\sigma_c(\text{tot})\rangle_{\text{Av}}=\sigma_c(\text{shape})+\sigma_c(\text{fluct}),$$

$$\sigma_c(\text{shape})=\sum_{c'}\sigma_{cc'}(\text{shape}),$$

$$\sigma_c(\text{fluct})=\sum_{c'}\sigma_{cc'}(\text{fluct}),$$

$$\sigma_{cc'}(\text{shape})=\pi\lambda\!\!\!^{-}{}_c^2\sum_{J\pi}g_{J\pi}|\delta_{cc'}-\langle U_{cc'}^{J\pi}\rangle_{\text{Av}}|^2,$$

$$\sigma_{cc'}(\text{fluct})=\pi\lambda\!\!\!^{-}{}_c^2\sum_{J\pi}g_{J\pi}\langle|\delta U_{cc'}^{J\pi}|^2\rangle_{\text{Av}}. \quad (9)$$

It is not possible, for instance, to isolate at this stage the compound nucleus events although it is reasonable to assume that they enter in a major way into the fluctuation cross section. It is, however, reasonable to define the compound cross section $\sigma_c(\text{comp})$ as

$$\sigma_c(\text{comp})=\langle\sigma_c(\text{tot})\rangle_{\text{Av}}-\sigma_{cc}(\text{shape}), \quad (10)$$

i.e., to subtract out the shape scattering in channel c from the total cross section and to consider the remaining scattering to involve a "compound" process of some sort. The definition (10) gives the compound

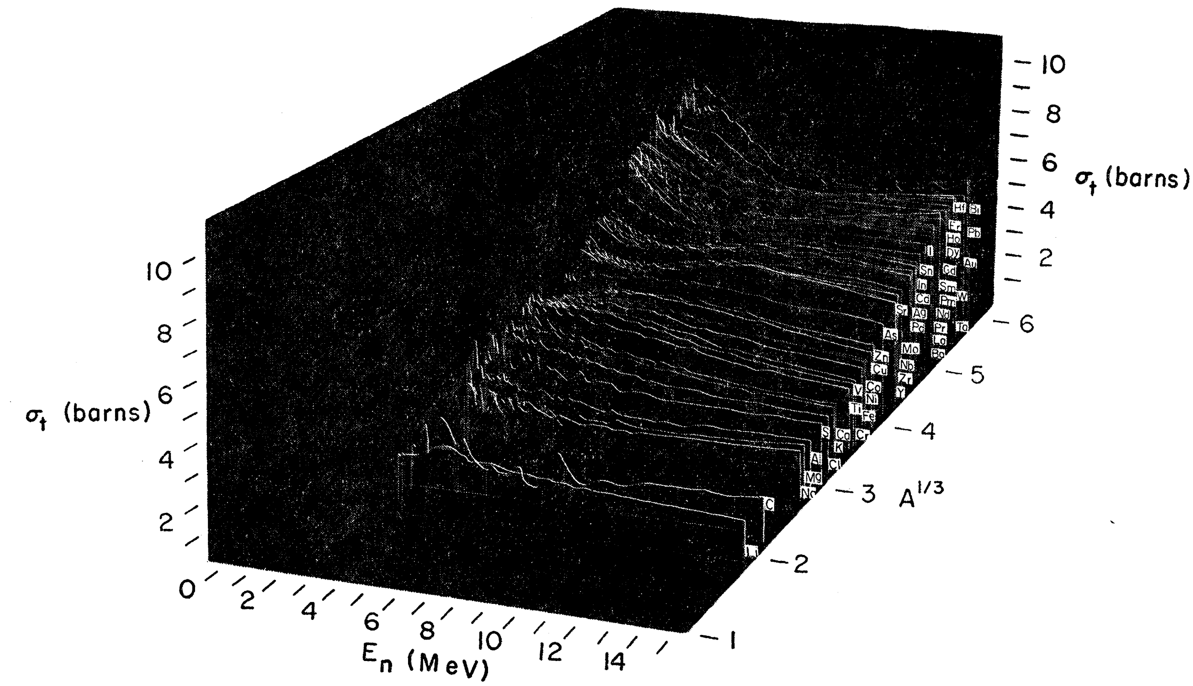

FIG. 2. Data of Foster and Glasgow[1] showing fluctuations in neutron total cross sections. Figure courtesy of Dr. Foster and Dr. Glasgow.

cross section the form of a nonelastic cross section obtained from the average scattering matrix. From (9) and (10) we find

$$\sigma_c(\text{comp}) = \sum_{c'\neq c} \sigma_{cc'}(\text{shape}) + \sum_{\text{all } c'} \sigma_{cc'}(\text{fluct}). \quad (11)$$

Thus the compound cross section may contain shape channel-transfer processess. In order to compute $\langle U_{cc'}\rangle_{\text{Av}}$ for $c'\neq c$ and $\langle | U_{cc'} |^2\rangle_{\text{Av}}$, more than the traditional entrance-channel optical model is needed since cross-channel strength functions $\langle \gamma_{\lambda c}\gamma_{\lambda c'}\rangle/D$ with $c\neq c'$ are not expected to vanish in general.

II. CROSS-SECTION PROBABILITY DISTRIBUTION

As soon as any sort of average quantity is introduced, it is immediately natural to ask what about fluctuations? In fact no statistical theory is complete until the distribution underlying the average is exhibited. In the case of cross sections, the distribution function is extremely important because it can now be measured and *may* be the only useful way to interpret the tremendous amount of data exhibiting fluctuations that is beginning to appear.

Since it is thought that the source of cross-section fluctuations is somehow related to the E_λ and $\gamma_{\lambda c}$ statistics, it is natural to extrapolate from (5) to obtain the distribution function of σ:

$$P(\sigma; E, I) = \int \dot{H} P(H) \int dE' R_I(E-E')\,\delta(\sigma-\sigma(E', H)). \quad (12)$$

Thus the probability distribution for σ depends on the energy E and the resolution interval I. Of course, any parameters arising from $\int \dot{H} P(H)$ also enter, viz., the level spacing D and the strength functions $\langle \gamma_{\lambda c}\gamma_{\lambda c'}\rangle/D$. Equation (12) is a well-defined recipe, but it is not so easy to carry out the integration indicated.

Clearly it is essential to use (12) or something like it in the resonance region of energy and probably also in the energy region in which levels are just beginning to overlap. However, once the levels begin to overlap, another point of view may be useful. This point of view has been developed especially by Ericson[2] and Brink and Stephen.[3] Following Ericson we write an angular distribution (neglecting spins) as

$$d\sigma_{cc'}(\theta, E)/d\Omega = | f_{cc'}(\theta) |^2. \quad (13)$$

Now $f_{cc'}$ can be written (suppressing indices)

$$f = \langle f\rangle_{\text{Av}} + \delta f. \quad (14)$$

Thus

$$d\sigma/d\Omega = | \langle f\rangle_{\text{Av}} |^2 + | \delta f |^2 + 2 \,\text{Re}\, \langle f\rangle_{\text{Av}}^* \delta f. \quad (15)$$

Clearly, we recover the shape-plus-fluctuation feature of the average cross section

$$\langle d\sigma/d\Omega\rangle_{\text{Av}} = | \langle f\rangle_{\text{Av}} |^2 + \langle | \delta f |^2\rangle_{\text{Av}} = (d\sigma/d\Omega)(\text{shape}) + (d\sigma/d\Omega)(\text{fluct}). \quad (16)$$

[2] T. A. Ericson, Phys. Letters 4, 258 (1963).
[3] D. M. Brink and R. O. Stephen, private communication via G. R. Satchler.

Ericson argues that (ignoring thresholds) the real and imaginary parts of δf represent the summation of a number of Breit–Wigner amplitudes with large widths ($\Gamma \gg D$). Thus the major fluctuation arises from the numerators which are assumed to be independent random variables. The conclusion from the central limit theorem is that the real and imaginary parts of δf follow Gaussian distributions with the same dispersion. Letting

$$\begin{aligned} x &= d\sigma/d\sigma(\text{fluct}), \\ x_s &= d\sigma(\text{shape})/d\sigma(\text{fluct}), \\ \langle f \rangle_{\text{Av}} &= a+ib, \\ \delta f &= u+iv, \end{aligned} \tag{17}$$

we have

$$\begin{aligned} x &= \frac{a^2+b^2+u^2+v^2+2(au+bv)}{\langle u^2\rangle_{\text{Av}}+\langle v^2\rangle_{\text{Av}}} \\ &= \frac{(u+a)^2+(v+b)^2}{2\langle u^2\rangle_{\text{Av}}}, \end{aligned} \tag{18}$$

and therefore

$$P(x) = \int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} dv \times \frac{\exp[-(u^2+v^2)/2\langle u^2\rangle_{\text{Av}}]}{2\pi\langle u^2\rangle_{\text{Av}}} \delta\left(x - \frac{(u+a)^2+(v+b)^2}{2\langle u^2\rangle_{\text{Av}}}\right). \tag{19}$$

Let $s=(u+a)/(2\langle u^2\rangle_{\text{Av}})^{\frac{1}{2}}$ and $t=(v+b)/(2\langle u^2\rangle_{\text{Av}})^{\frac{1}{2}}$. This gives

$$\begin{aligned} P(x) &= \frac{1}{\pi}\int_{-\infty}^{+\infty} ds \int_{-\infty}^{+\infty} dt \\ &\times \exp -\left[\left(s - \frac{a}{(2\langle u^2\rangle_{\text{Av}})^{\frac{1}{2}}}\right)^2 + \left(t - \frac{b}{(2\langle u^2\rangle_{\text{Av}})^{\frac{1}{2}}}\right)^2\right] \delta(x-s^2-t^2) \\ &= \frac{1}{\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} d\rho\rho\delta(x-\rho^2) \\ &\times \exp\left[-\left(\rho\cos\varphi - \frac{a}{(2\langle u^2\rangle_{\text{Av}})^{\frac{1}{2}}}\right)^2 - \left(\rho\sin\varphi - \frac{b}{(2\langle u^2\rangle_{\text{Av}})^{\frac{1}{2}}}\right)\right] \\ &= \frac{1}{2\pi}\int_0^{2\pi} d\varphi \\ &\times \exp\left[-\left(x^{\frac{1}{2}}\cos\varphi - \frac{a}{(2\langle u^2\rangle_{\text{Av}})^{\frac{1}{2}}}\right) - \left(x^{\frac{1}{2}}\sin\varphi - \frac{b}{(2\langle u^2\rangle_{\text{Av}})^{\frac{1}{2}}}\right)\right] \\ &= \exp[-(x+x_s)]\frac{1}{2\pi}\int_0^{2\pi} d\varphi \\ &\times \exp\left[\left(\frac{2x}{\langle u^2\rangle_{\text{Av}}}\right)^{\frac{1}{2}}(a\cos\varphi + b\sin\varphi)\right]. \end{aligned} \tag{20}$$

Next we let

$$\begin{aligned} \sin\alpha &= a/(a^2+b^2)^{\frac{1}{2}}, \\ \cos\alpha &= b/(a^2+b^2)^{\frac{1}{2}}, \end{aligned} \tag{21}$$

so that

$$\begin{aligned} P(x) &= \exp[-(x+x_s)]\frac{1}{2\pi}\int_0^{2\pi} d\varphi \\ &\times \exp\left[\left(\frac{2x(a^2+b^2)}{\langle u^2\rangle_{\text{Av}}}\right)^{\frac{1}{2}} \sin(\varphi+\alpha)\right] \\ &= \exp[-(x+x_s)] I_0[2(xx_s)^{\frac{1}{2}}], \end{aligned} \tag{22}$$

where I_0 is a standard Bessel function. A check of this distribution has been made by Good, Kim, Moak, and Rayburn[4] for the reaction Mg(α,α)Mg among others. Their results are shown in Figs. 3 and 4. Note that data from different angles has been combined. The ground state of the target is of spin 0 and positive parity, while the first excited state has spin 2 and positive parity. Fig. 3 shows the elastic-scattering results, while Fig. 4 shows the inelastic-scattering results. The latter have $2\times 2+1=5$ degrees of freedom, while the former have only one degree of freedom. The appropriate Bessel function is indicated in the figures. The functional form of the distribution seems to coincide with the data for a value of $d\sigma(\text{shape})/d\sigma(\text{fluct})=3$.

It is of interest to ask if Ericson's arguments are applicable to the entrance channel and in particular to total cross sections. Neglecting spin again, we have

$$\begin{aligned} \sigma(\text{tot}) &= 4\pi\lambda\!\!\!^{-}\, \text{Im} f(0) \\ &= 4\pi\lambda\!\!\!^{-}\, \text{Im}(\langle f\rangle_{\text{Av}}+\delta f), \end{aligned} \tag{23}$$

where now we evaluate $\langle f\rangle_{\text{Av}}$ and δf at zero angle. In the notation we have already introduced,

$$\sigma(\text{tot}) = 4\pi\lambda\!\!\!^{-}(b+v). \tag{24}$$

In particular

$$\sigma(\text{tot}) - \langle\sigma(\text{tot})\rangle_{\text{Av}} = 4\pi\lambda\!\!\!^{-} v, \tag{25}$$

which means that

$$\begin{aligned} \langle v^2\rangle_{\text{Av}} &= \langle[\delta\sigma(\text{tot})]^2\rangle_{\text{Av}}/(4\pi\lambda\!\!\!^{-2})^2, \\ \langle[\delta\sigma(\text{tot})]^2\rangle_{\text{Av}} &= 8\pi\lambda\!\!\!^{-2}\sigma(\text{fluct}). \end{aligned} \tag{26}$$

Thus, the dispersion of a cross section is proportional to a cross section. Since Ericson's argument implies that v is Gaussian, we have

$$P(v) = \exp(-v^2/2\langle v^2\rangle_{\text{Av}})/(2\pi\langle v^2\rangle_{\text{Av}})^{\frac{1}{2}}, \tag{27}$$

where now

$$v = [\sigma(\text{tot}) - \langle\sigma(\text{tot})\rangle_{\text{Av}}]/4\pi\lambda\!\!\!^{-}. \tag{28}$$

The data of Glasgow and Foster has not been compared to (27) to see to what extent (27) is correct and

[4] Good, Kim, C. D. Moak, and Rayburn, private communication via G. R. Satchler. A recent letter by G. Temmer, Phys. Rev. Letters **12**, 330 (1964), shows more evidence along this line.

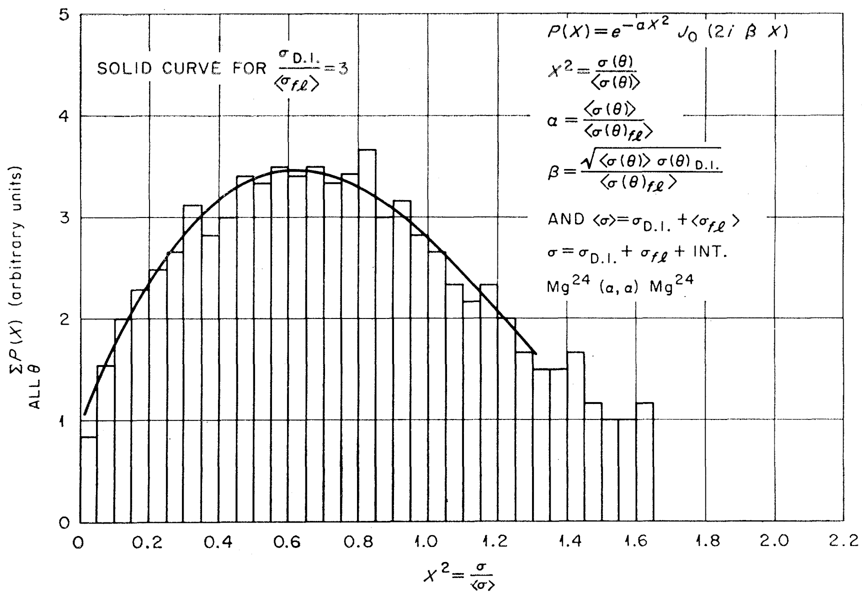

Fig. 3. Analysis of data of Good, Kim, Moak, and Rayburn. Figure courtesy of Dr. H. J. Kim. This figure shows the cross-section distribution for the elastic scattering of alpha particles on Mg^{24}. Data for all angles are combined.

therefore how accurate the arguments leading to (22) are in this case. The particularly special characteristic of (27) is the zero slope at $v=0$; there is no peak of (27) for $v^2>0$.

The arguments leading to (22) are essentially independent of those leading to (12) except that the distributions underlying (12) must exist and must be sufficiently uncorrelated that the central limit theorem apply. For this reason cross checks on the total cross section are of tremendous importance.

In addition to one-exit-channel cross-section distributions at a given energy, it is possible to contemplate also a joint cross-section distribution for many channels [we adhere to a binary channel language even though this may not be completely realistic, for example, for the $(n, 2n)$ reaction]

$$P=P(\sigma_{cc}, \sigma_{cc'}, \cdots, \sigma_{c'c'}, \cdots). \tag{29}$$

A derivation of (29) along the lines of Ericson's arguments leading to (22) does not seem to be so easy; it is quite likely that it will be necessary to base a joint distribution like (29) more directly on the underlying $P(H)$ or equivalently $P(\{E_\lambda\})$ and $P(\{\gamma_{\lambda c}\})$; particularly $P(\{\gamma_{\lambda c}\})$ is important since it contains the

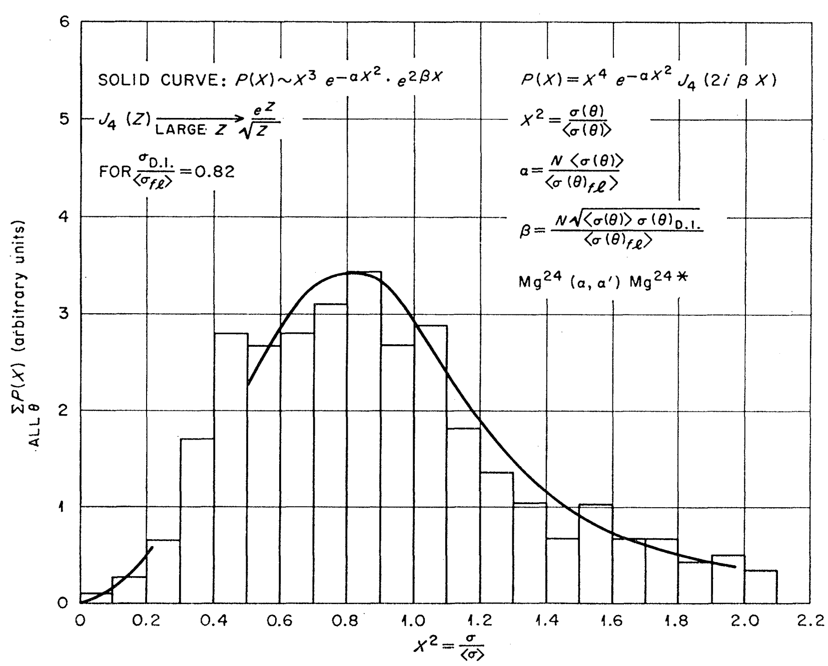

Fig. 4. Same as Fig. 3 except that the data are for inelastic scattering from the first excited state.

channel correlation information in the covariance matrix $\Sigma_{cc'} = \langle \gamma_{\lambda c}\gamma_{\lambda c'}\rangle_{\text{Av}}$. Thus Σ should enter as a parameter in (29). It may be that rather generalized arguments of Ericson's sort, not too intimately related to the second of equations (7),[5] could be developed to yield the functional form of (29) and its appropriate generalization to include angular distributions; however, at the moment, these arguments do not seem to be in sight.

In analogy to Σ, it is possible to define a four-index cross-section covariance matrix:

$$\Upsilon_{cc';c''c'''} = \langle \delta\sigma_{cc'}\delta\sigma_{c''c'''}\rangle_{\text{Av}}. \qquad (30)$$

From (30), we see that the cross-section correlation coefficient $C_{cc';c''c'''}$ is given by

$$C_{cc';c''c'''} = \Upsilon_{cc';c''c'''}/[\Upsilon_{cc';cc'}\Upsilon_{c''c''';c''c'''}]^{\frac{1}{2}},$$
$$-1 \leq C_{cc';c''c'''} \leq 1. \qquad (31)$$

Of course, it is not obvious that the parameters of (29) are all contained in the matrix Υ or what the connection between Υ and Σ is. On the other hand, quantities like (31) should probably be especially useful for analyzing data even though the ultimate desire is to know the functional form of (29).

The concept of cross-section energy-displaced correlation function was introduced some time ago by Egelstaff[6] and has been particularly exploited by Ericson.[7] The idea is to generalize (30) to contemplate the fluctuations $\delta\sigma_{cc'}$ and $\delta\sigma_{c''c'''}$ at displaced energies E and $E+\epsilon$. This leads to a correlation coefficient $K_{cc';c''c'''}(\epsilon)$ given by the symmetrized expression[8]

$$K_{cc';c''c'''}(\epsilon) = \frac{1}{2}\frac{\langle \delta\sigma_{cc'}(E)\delta\sigma_{c''c'''}(E+\epsilon)\rangle_{\text{Av}} + \langle \delta\sigma_{cc'}(E+\epsilon)\delta\sigma_{c''c'''}(E)\rangle_{\text{Av}}}{[\langle(\delta\sigma_{cc'})^2\rangle_{\text{Av}}\langle(\delta\sigma_{c''c'''})^2\rangle_{\text{Av}}]^{\frac{1}{2}}}, \qquad (32)$$

which reduces to $C_{cc';c''c'''}$ when $\epsilon \to 0$. Even more complicated (nonbinary channelwise) expressions than (32) could be contemplated, for example, in the $(n, n'f)$ reaction. Note that the combination $\delta\sigma_{cc'}(E)\delta\sigma_{c''c'''}(E+\epsilon)$ is a random variable which is a function of E and ϵ and also the underlying Hamiltonian H. It is possible in principle to construct the distribution $P(Q)$ of any quantity Q of this sort via the recipe

$$P(Q; E, I, \epsilon, \cdots) = \int \dot{H} P(H) \int dE' R_I(E-E')\,\delta[Q - Q(E', \epsilon, \cdots; H)]. \qquad (33)$$

[5] T. J. Krieger and C. E. Porter, J. Math. Phys. **4**, 1272 (1963).
[6] P. A. Egelstaff, J. Nucl. Energy **7**, 35 (1958).
[7] T. A. Ericson, Phys. Rev. Letters **5**, 430 (1960); Advan. Phys. **9**, 425 (1960); Ann. Phys. (N. Y.) **23**, 390 (1963).
[8] M. L. Halbert, F. E. Durham, C. D. Moak, and A. Zucker (private communication). We have generalized their expression slightly here to admit more than one entrance channel.

Even joint distributions of more than one random variable can be generated by the appropriate generalization of (33).

In particular for $c = c''$ and $c' = c'''$, Ericson finds[7] (neglecting spin effects)

$$\langle \delta\sigma_{cc'}(E+\epsilon)\delta\sigma_{cc'}(E)\rangle_{\text{Av}} = [\Gamma^2/(\Gamma^2+\epsilon^2)][(\langle\sigma_{cc'}\rangle_{\text{Av}})^2 - (\sigma_{cc'}(\text{shape}))^2], \qquad (34)$$

where Γ is the mean total width of the underlying fine structure levels. In particular,

$$\langle(\delta\sigma_{cc'})^2\rangle_{\text{Av}}/(\langle\sigma_{cc'}\rangle_{\text{Av}})^2 = 1 - [\sigma_{cc'}(\text{shape})/\langle\sigma_{cc'}\rangle_{\text{Av}}]^2. \qquad (35)$$

Using (34), values[8] of Γ have been found for the $C^{12}(O^{16}, \alpha)Mg^{24}$ reaction and also from (35) an estimate of $\sigma_{cc'}(\text{shape})/\langle\sigma_{cc'}\rangle_{\text{Av}}$ is obtained. The cross-channel correlation was also examined by computing essentially (32) for $c = c''$ and was found to be relatively small.

Expression (34) agrees with the results obtained from (22), as it must. It can be shown from (22) that

$$\langle x^n\rangle_{\text{Av}} = \exp(-x_s)(q^2\, d/dq)^n q \exp(qx_s)\,|_{q=1}, \qquad (36)$$

so that in particular

$$\langle x\rangle_{\text{Av}} = 1 + x_s,$$
$$\langle x^2\rangle_{\text{Av}} = 2 + 4x_s + x_s^2. \qquad (37)$$

This implies

$$\langle(\delta x)^2\rangle_{\text{Av}}/(\langle x\rangle_{\text{Av}})^2 = 1 - [x_s/(1+x_s)]^2,$$
$$\langle(\delta\sigma)^2\rangle_{\text{Av}}/(\langle\sigma\rangle_{\text{Av}})^2 = [1 - (\sigma(\text{shape})/\langle\sigma\rangle_{\text{Av}})^2]. \qquad (38)$$

It is possible to examine polarization fluctuations as well as more conventional scattering results. The usual formula for polarization of spin-$\frac{1}{2}$ particles for an elastic process is

$$P = 2\,\text{Re}\,(f^*g)/(|f|^2 + |g|^2), \qquad (39)$$

where f is the spin-nonflip and g is the spin-flip amplitude. Arguments analogous to those above can be used to predict the form of the distribution function for P [or better $(d\sigma/d\Omega)P$] under various assumptions concerning the correlation between f and g. Data on neutron polarization exhibiting fluctuations is now available.[9]

III. REMARKS CONCERNING THE RANDOM-PHASE HYPOTHESIS

In general the notion behind all of the fluctuations is a random-phase hypothesis. An orthogonal canonical transformation O can be written

$$O = e^{\Phi}, \qquad (40)$$

[9] A. J. Elwyn, J. E. Monahan, R. O. Lane, and A. Langsdorf, Jr. private communication).

where $\Phi=-\tilde{\Phi}=\Phi^*$. Now the mn matrix element $(m\mid O\mid n)$ of O is related to the relevant wave functions in the coordinate representation via (this was called to my attention by Professor A. Klein)

$$(m\mid O\mid n)=\int(m\mid O\mid x)\,dx(x\mid O\mid n), \quad (41)$$

or

$$(m\mid n)=\int(m\mid x)\,dx(x\mid n). \quad (42)$$

But the usual way of writing $(m\mid x)$ is

$$(m\mid x)=\exp\left[iS_m(x)/\hbar\right], \quad (43)$$

where S is the action. Hence

$$(m\mid n)=\int dx\exp\left[i(S_m-S_n)/\hbar\right], \quad (44)$$

i.e.,

$$(m\mid e^{\Phi}\mid n)=\int dx\exp\left[i(S_m-S_n)/\hbar\right],$$

which means that the randomness of the phase operator Φ can be thought of as arising from random action *differences,* a rather attractive concept which may lead to a useful correspondence characterization of the various matrix ensembles.

The complementary notion of "explaining" the fluctuations as particle phenomena may be essentially what is being pursued by Feshbach and his co-workers.

Time Delay Measurements

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The title of this paper is a good example of poetic icense since I actually describe a proposed technique for measuring *scattering amplitudes.* In fact, as indicated by Austern in an earlier paper, there are difficulties in giving an acceptable operational definition of *time delay.* Nevertheless, the qualitative idea of time delay can provide a useful point of view, and is certainly related to what I describe here. So I ask you to let me use the idea of time delay in the beginning of this paper—towards the end I will become more sophisticated.

With these qualifications, it is evident that a fundamental distinction between the compound nucleus and direct interaction processes is that the compound nucleus process involves a time delay. Unfortunately, the time delay cannot, at present, be measured by any direct (e.g. electronic) technique—it is just too short. Of course no problem arises in the case of an isolated compound nucleus resonance since, as was first pointed out by Bohr, the uncertainty principle directly relates the time delay to the measured width of the resonance. But when the compound nucleus is excited to its continuum, it is necessary to be more subtle. Several years ago Ericson developed an analysis of the fluctuations in excitation functions which leads to the possibility of measuring time delays in the continuum. At about the same time a proposal for measuring time delays in the continuum, by studying the bremsstrahlung emitted in the nuclear reaction, was made in papers by Yennie, Wilkinson and Eisberg,[1] and by Feshbach and Yennie.[2] Ericson's proposal has received considerable experimental attention, but, as far as I know, the experimental work on our proposal has been done entirely by Hansen.[3] In this paper, I will review our proposal, and then describe very briefly the work of Dr. Hansen.

The process of bremsstrahlung emission in nuclear reactions seems to be one of those fortunate cases that can be understood, within limits, from classical considerations. Therefore, I will first describe it classically and then indicate the quantal modifications to the description.

To start, let us consider qualitatively a charged particle entering a nucleus and, after a time τ, a product particle being emitted elastically at some angle. In this process there will be a certain net amplitude for the emission of bremsstrahlung. One component of this net amplitude arises from the cessation of the current associated with the incident particle, and the other component arises from the initiation of the current associated with the product particle. The net amplitude depends on the charge and velocity of the particles, on the scattering angle, on the angle of emission of the bremsstrahlung, and also on the time delay τ. The time delay comes in through the circumstance that it leads to a phase difference $\exp(i\omega\tau)$ between the two components, where ω is the angular frequency of the emitted bremsstrahlung. For small values of $\omega\tau$, the two components interfere coherently (either constructively or destructively depending on the

[1] R. Eisberg, D. Yennie, and D. Wilkinson, Nucl. Phys. **18**, 338 (1960).

[2] H. Feshbach and D. Yennie, Nucl. Phys. **37**, 150 (1962).

[3] L. Hansen, *Proceedings of the International Symposium on Direct Interactions and Nuclear Reaction Mechanisms, Padua,* edited by E. Clementel and C. Villi (Gordon and Breach Publishers, Inc., New York, 1963).

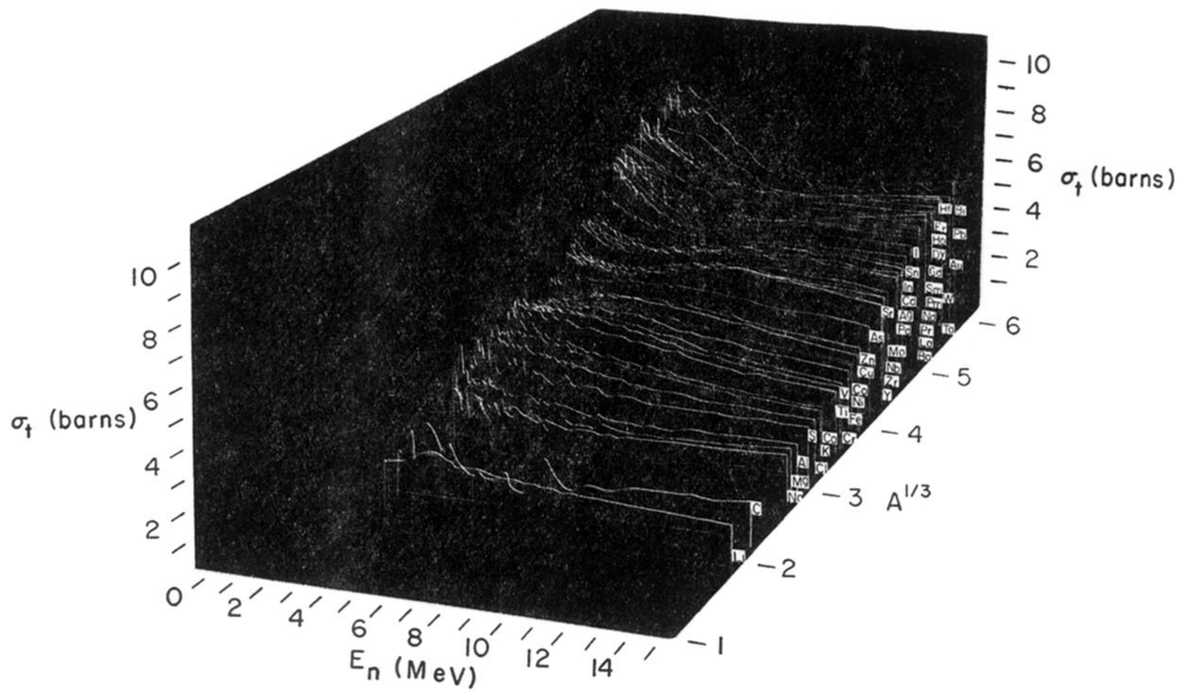

FIG. 2. Data of Foster and Glasgow[1] showing fluctuations in neutron total cross sections. Figure courtesy of Dr. Foster and Dr. Glasgow.