The Cluster Model, Time Delay, and Three-Body Breakup*

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I. INTRODUCTION

The discovery of resonant compound nuclear effects for nuclear reactions and the interpretation of these effects by Bohr,[1] and Breit and Wigner[2] in terms of a time-delayed nuclear reaction are well-known. Bohr's original idea, in his paper in *Nature*, was that strong interactions of the whole system throughout a nuclear volume could produce a time delay in nuclear reactions which would be manifest by resonances. This original idea was very simple and physical; since that time many changes have occurred in the interpretation and development of these ideas. Two of these changes of interpretation deserve special comment: Firstly, Bohr's original ideas do not require, and in fact oppose, a statistical interpretation of compound nuclear states. Bohr's idea was that *definite* physical processes take a *definite* time which will be manifest by a resonance of cross section versus energy. However, statistical interpretations of the compound-nucleus model have been strongly emphasized—especially for highly excited nuclear states. Secondly, the idea of resonances has been frequently specialized to include only very narrow, long-lived resonances. It does not appear that this was Bohr's intention. More to the point, it does not seem these interpretations are the fact: many nuclear experiments fit neither the "statistical" nor the "very narrow" compound-nucleus theory.

Thus, one is faced with an overwhelming mass of nuclear data that is neither interpretable as "statistical compound-nucleus" data nor as very narrow, long-lived resonances. One, of course, has other interpretations available. In particular, direct-interaction theories in many cases seem applicable. These theories, assuming that no significant time delays occur in nuclear reactions, have had many successes but also have failed to explain many features of nuclear reactions such as resonances and fluctuating cross sections.

It seems that a resolution of these problems can only come with an experimental and theoretical study of the time delay in nuclear reactions: compound-nucleus theories postulate a time delay; direct-interaction theories presume no time delay. The amount (if any) of time delay and also the simplicity or complexity of compound-nucleus reactions are of importance since many of the extant compound-nucleus theories have assumed either very long times of interaction, or have assumed that the interaction is delayed by incomprehensibly complex and statistical processes. Thus, both the time delay and the complexity of reactions are worthy of study.

On the other hand, the cluster model[3,4] (which embraces many aspects of the intermediate coupling shell model and the resonating-group model,[5,6]) treats nuclear processes as being compound-nucleus, time-delayed but adds physical assumptions about the nature of the time delay. Further, this theory supposes that the time delay may be of any amount lying between instantaneous (direct interaction) to very long (narrow compound-nucleus resonances).

This paper will discuss the cluster model and the time delay aspects of nuclear reactions and will seek to show that the compound-nucleus–cluster-model view of the processes accounts for many observed phenomena, predicts as yet unobserved phenomena, and provides a guide to interpretation of nuclear data.

II. THE MEASUREMENT OF TIME DELAY IN TWO-BODY NUCLEAR REACTIONS

Most nuclear processes available to study are very fast; that is, they are much faster than can be measured directly by available, direct techniques of time measurement. Direct-reaction processes occur in times comparable to the transit time of nuclear projectiles across nuclear dimensions. These correspond to times of about 10^{-22} sec. On the other hand, slow-neutron resonances of 1 mV width correspond to about the slowest nuclear times available (for dynamical study): these resonances are of about 10^{-12}-sec. lifetime. Actually, most resonances are much wider and of much shorter life than slow neutron resonances. Many resonances in the light and medium weight nuclei are known whose width is about 1 MeV in energy and whose lifetime corresponds to about 10^{-21} sec. These times are about one order of magnitude slower than direct-interaction times, and yet they correspond to important physical processes. As is known, in many cases these broad resonances comprise most of the cross section of nuclear reactions. These resonant cross sections frequently approach the geometrical cross section. To put it simply, most nuclear reactions would not be of much importance if it were not for these broad, short time-delayed resonances. These resonances are fundamental to an understanding of nuclear processes.

* Work supported by the U.S. Atomic Energy Commission.

[1] N. Bohr, Nature **137**, 344 (1936).

[2] G. Breit and E. Wigner, Phys. Rev. **49**, 519 (1936).

[3] K. Wildermuth and T. Kanellopoulos, Nucl. Phys. **7**, 150 (1958).

[4] G. C. Phillips and T. A. Tombrello, Nucl. Phys. **19**, 555 (1960); G. C. Phillips, *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach Publishers, Inc., New York, 1963), p. 869.

[5] Cf. B. H. Flowers, Progr. Nucl. Phys. **2**, 235 (1952).

[6] J. A. Wheeler, Phys. Rev. **52**, 1107 (1937).

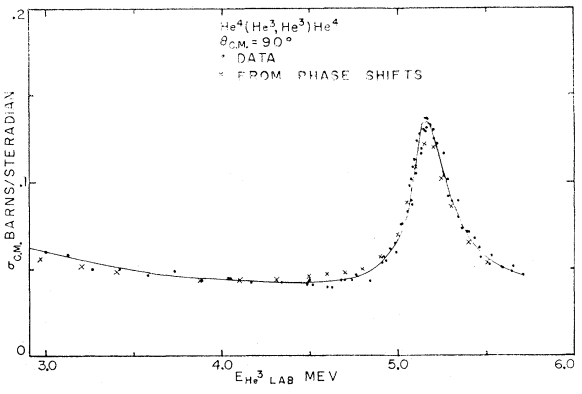

FIG. 1. Excitation curve for elastic scattering of He^3 from He^4 at a c.m. angle of 90°. The strong resonance at about 5.25 MeV corresponds to the $\frac{7}{2}^-$, second excited state of Be^7. The width of the state is about 40% of the Wigner limit which shows the state to be a cluster state of the form He^3+He^4 (Refs. 7 and 8).

For these resonant processes a simple physical picture is provided by the cluster model: the projectile interacts with the target nucleus; a simple state of resonant motion is formed whose time delay is representative of the compound-nucleus features of the reaction, and the great width is representative of the simplicity of the wave function of the interacting nucleons. In many situations that have been studied for the light elements the state consists principally of a few, simple vibrations of two clusters of nucleons about each other. These facts form the experimental basis of the cluster model.[4,7]

Unfortunately, it is presently impossible to measure times of the order of 10^{-18}–10^{-21} sec directly. These short times must be measured to distinguish between direct and compound-nucleus processes and to identify the physical nature (simple or complex) of the delayed, compound (perhaps statistical) nucleus processes. For these reasons, one must seek other methods. Since time is the conjugate quantity to energy, then short time intervals may be measured for quantum systems by energy distributions. Thus, the study of the time delay of two-particle nuclear processes must presently reside in three types of measurements: (a) the energy widths of resonant states, (b) the absolute cross sections, and (c) the comparison of partial-energy widths to the Wigner limit. For three-particle processes other dynamical variables are available to aid in these studies and additional methods of lifetime measurement become possible: this point will be discussed below in some detail.

It is to be emphasized that in nuclear measurements one should not be concerned with only the measure of the time of the interaction; one must also determine the degree of simplicity or complexity of the process. This corresponds in making a *complete* set of measurements and, for example, in determining the value of *all* partial widths of a resonance, not just the total width. Very few such measurements have been performed, but a few will be listed below.

A. Elastic Scattering

The simplest of two-body final states is that of elastic scattering of two spinless particles or of a spinless and a spin-$\frac{1}{2}$ particle; both can be described exactly in terms of phase shifts derived from the scattering cross sections. Recently a dramatic example of cluster structure has been examined by such a study. At Rice[7,8] and at Cal Tech[9,10] the study of the reactions

$$\begin{array}{ccc} He^3 + He^4 & \longrightarrow & He^3 + He^4 \\ & \times & \\ Li^6 + P & \longrightarrow & Li^6 + P \end{array} \qquad (1)$$

has been carried out. First it was learned that the compound nucleus Be^7 has excited states that are rather purely of a form given by the relative motion of He^3 about He^4 or of P about Li^6. The latter is not so surprising, but the former perhaps is. One of these states, the second excited state in Be^7, is shown in Fig. 1.[7,8] This state has about 40% of the Wigner limit for the F-wave motion of an He^3 particle about He^4. It is clear that this state is a good cluster-model state of He^3+He^4 motion.

Now the higher energy levels of Be^7, shown in Fig. 2, have been recently examined by Tombrello and Parker[10] with $He^3+\alpha$ scattering up to about 8 MeV of excitation. It had been known[9] for some time that Be^7 had a state at 7.18 MeV of spin-parity $\frac{5}{2}^-$. The recent data[10] revealed another $\frac{5}{2}^-$ state at about 6.55 MeV. The interesting thing about these two states is that they both have the same angular momentum and parity and yet do not interfere detectably. This is shown in Fig. 3 where no evidence is seen, above the He^3+He^4

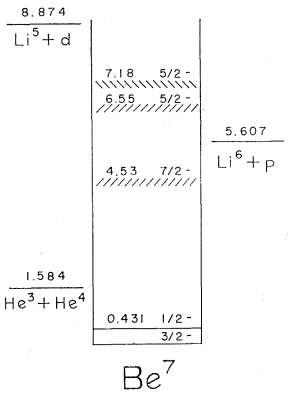

FIG. 2. Energy levels of the Be^7 nucleus up to about 8 MeV. The energies, angular momenta, and parities of the states discussed in the text are shown (Refs. 7–10).

[7] P. D. Miller and G. C. Phillips, Phys. Rev. **112**, 2048 (1958).

[8] A. C. L. Barnard, C. M. Jones, and G. C. Phillips, Nucl. Phys. **50**, 629 (1964).

[9] J. B. Marion, G. Weber, and F. S. Mozer, Phys. Rev. **104**, 1402 (1956).

[10] T. A. Tombrello and P. D. Parker, Phys. Rev. **130**, 1112 (1963).

cluster resonance, of $\frac{5}{2}^-$ nature at 8.7-MeV He^3 energy, for the $P+Li^6$ $\frac{5}{2}^-$ cluster resonance that might be expected to be observed at 9.8-MeV He^3 energy.

This set of experiments provides a remarkable example of the experimental determination of cluster states. These states are compound-nucleus states; they are time-delayed. Since their delay is as small as is possible for a resonant state, their nature is *simple*; they are cluster states. The cross section at resonance assumes the classical value; it is as large as possible. The states are very purely of one form or another: they are either He^3+He^4 cluster-model–compound-nuclear states or they are $P+Li^6$ cluster-model–compound-nuclear states. The lifetime of these states has been determined to be only slightly longer than direct-interaction times and yet shorter than times sometimes, improperly, associated with compound nuclear processes. Most important, the *physical nature* of the states has been determined and this nature was found to be simple.

B. Time Delayed Nuclear Reactions

Frequently, for high-energy processes, one uses either the direct or the statistical compound-nucleus approach

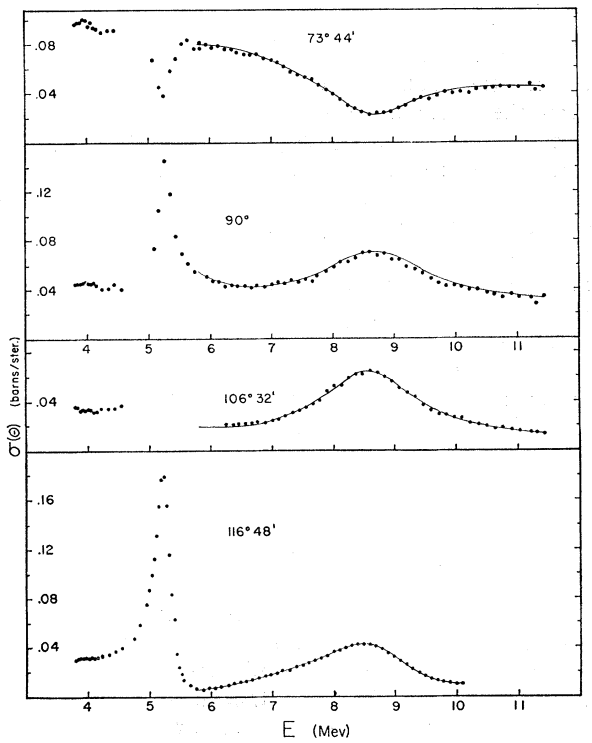

FIG. 3. Excitation curves at four c.m. angles for the elastic scattering of He^3 from He^4 (Ref. 10). The resonance shown in Fig. 1 is observed and an additional resonance, of the same l value (f waves) is detected at about 8.7-MeV He^3 energy of $\frac{5}{2}^-$ character. This broad resonance has about 50% of the Wigner limit for the He^3+He^4 channel and is a cluster state in that channel. Note that no evidence for a resonance at about 9.8 MeV is detected where another broad $\frac{5}{2}^-$ level (Ref. 9) is known to have the cluster structure $p+Li^6$.

TABLE I. Reduced widths θ^2 for emission of protons, neutrons, and deuterons from excited states in N^{14} (Ref. 11).

Excitation energy (MeV) in N^{14}	θ_p^2	θ_n^2	θ_d^2
11.29	0.02	0.13	0.24
11.39	0.001	0.003	0.20
11.80	0.003	0	0.08
11.96	0.04	0.11	0.06
12.22	0.03	0.09	1.0

to treat the reaction. Actually, there are many experiments where neither work well. An example is N^{14} in the range 10–14-MeV excitation; however, other examples could be cited. The N^{14} nucleus has many reaction channels open in this energy region; the angular distributions sometimes appear to be direct interaction ones: they have strong forward peaks in the angular distributions, and sometimes they do not have forward peaks, but rather they may be backward peaked. The excitation curves appear to show very broad, overlapping resonances. Optical-model calculations, ubiquitously, will fit many of the strange things measured.

Careful measurements have been made on the N^{14} nucleus[11]; all available reaction channels have been studied with carefully measured excitation curves and angular distributions. The results demonstrate the importance of cluster structure for the N^{14} nucleus. The widths of some of these states are shown in Table I.[11] Here it is seen that states exist that have their widths in certain channels near the Wigner limit. Since the total cross sections, at resonance, approach the geometrical limit and since this cross section is frequently in one, or a few channels, one concludes that (a) there is a significant time delay—a compound nuclear process, and (b) the important processes are simple, corresponding to the relative motion of one cluster about another: d about C^{12}, P about C^{13}, etc.

Thus, the study of elastic scattering resonances and of nuclear reaction resonances provides a measure of time delay of nuclear processes. Further, a study of the absolute cross sections and of the reduced widths provides a measure of the simplicity of the delayed reactions.

C. Reactions of Unknown Time Delay and High-Energy Reactions

The category of reactions of unknown time delay is very large. This group includes most higher energy reactions. Most of them can be fitted, in some sense, with direct-interaction calculations. Frequently these

[11] E. Kashy, R. R. Perry, R. L. Steele, and J. R. Risser, Phys. Rev. 122, 884 (1961); E. Kashy, Ph.D. thesis, Rice University, 1960 (unpublished); T. A. Belote, Ph.D. thesis, Rice University, 1962 (unpublished).

TABLE II. Time-delay mechanisms for a reaction producing three particles in the final state by Eq. (2) or (3) (see text).

Reaction mechanism	First emission mechanism	Second emission mechanism	τ	τ_1	τ_2	Spectrum of b
Simultaneous	direct	direct	0	0	0	bell shaped
	direct	delayed	>0	0	>0	peaked
Sequential	delayed	delayed	>0	>0	>0	peaked
	delayed	direct	>0	>0	0	broad continuum

reactions show weak resonant structure that perhaps indicates some time delay. In cases that have been examined thoroughly these reactions often show dominant resonant structure at certain energies, and in certain channels. This fact, sometimes ignored, makes one doubt the direct-interaction calculations. Alternatively one has statistical[12] or semistatistical[13] theories of delayed processes. These theories are also subject to criticism. They suppose a growth of density of energy levels in nuclei that has not been verified experimentally, and they ignore clustering phenomena that could seriously destroy their validity. An illustration of this point is the density and widths of states in Mg^{24}. The Chalk River experiments[14] show C^{12}–C^{12} resonances whose simpliest interpretation violates the accepted level densities for Mg^{24}. E. Vogt has discussed this point.[15] These experiments show that the Mg^{24} nucleus at high excitations, far from behaving as a Fermi gas, behaves as two C^{12} clusters. Clearly, additional measurements of the partial widths of these states are needed.

However, for most reactions at high bombarding energies, high-energy release, and very short lifetimes, the experimenter cannot use the methods discussed above and is faced with the problem of devising new techniques to measure the times of the reaction.

III. TIME DELAY FOR THREE-BODY FINAL STATES

The decay of a nuclear system into three (or more) particles is complicated both experimentally and theoretically. Consider the *sequential* reaction

$$a+A\rightarrow {}^*D \xrightarrow{\tau_1} b+{}^*B$$
$$ {}^*B \xrightarrow{\tau_2} c+C_1, \qquad (2)$$

and the *simultaneous* reaction

$$a+A \xrightarrow{\tau=0} b+c+C_1, \qquad (3)$$

where the τ represents the *over-all* time delay; the reaction time minus the transit times, for the reactions $\tau=\tau_1+\tau_2$. The *simultaneous* reaction has been discussed,[16] and it has been shown that the spectral distribution of any particle should be a broad, bell-shaped curve extending between zero and the maximum energy available. On the other hand, the *sequential* reaction has been shown[17,18] to have structure in the energy spectrum of the first emitted particle, b, due to the final state interaction of the nucleus $^*B$. This structure of the spectrum of b is closely related to the lifetime τ_2 of $^*B$.[4,18]

Now it is clear that if $\tau\approx 0$ (a simultaneous reaction occurs) then both τ_1 and τ_2 are zero. Alternatively, if $\tau\neq 0$, then a simultaneous reaction does not occur and certainly either τ_1 or τ_2 are nonzero; or both are nonzero. These classifications are summarized in Table II. Thus, if measurements can be made that, firstly, show structure in the energy spectra of particle b corresponding to metastable states of $^*B$ and if, secondly, the measurements demonstrate the order of emission of the particles (i.e., can prove that b is emitted first), then one can assert that a sequential mechanism has been observed and that $\tau_2\neq 0$. The order of emission of the particles is, of course, important since rather than the *sequential* reaction (2) the process may be

$$a+A\rightarrow {}^*D\rightarrow c+{}^*E; \qquad {}^*E\rightarrow b+C, \qquad (4)$$

or

$$a+A\rightarrow {}^*D\rightarrow C+{}^*F; \qquad {}^*F\rightarrow b+c. \qquad (5)$$

Finally, if these criteria are demonstrated, then it may be possible to interpret the data to obtain a quantitative measure of the lifetime τ_2 of $^*B$.

The above argument shows that three-body measurements, while complicated, provide an interesting additional method of studying time delay. These points are discussed below.

[12] J. B. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952).
[13] T. Ericson, *Proceedings of the International Conference on Nuclear Structure, 1960* (Kingston), edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960), p. 697; See also DA-4, 5, this conference; T. Ericson, Advan. Phys. **9**, 415 (1960).
[14] E. Almqvist, D. A. Bromley, J. A. Kuehner, and B. Whalen, Phys. Rev. **130**, 1140 (1963); *Proceedings of the International Conference on Nuclear Structure, 1960* (Kingston), edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960), B-6 and CB-2, 4, 5, 6.
[15] E. Vogt, this conference.
[16] G. E. Uhlenbeck and S. Goudsmit, *Verhandelingen van Dr. P. Zeeman* (Martimus Nijhoff, The Hague, 1935).
[17] K. M. Watson, Phys. Rev. **88**, 1163 (1952).
[18] G. C. Phillips, T. A. Griffy, and L. C. Biedenharn, Nucl. Phys. **21**, 327 (1960).

A. Determination of Time Delay by Nuclear Bremsstrahlung

Suppose that the particle labeled b in Eqs. (2) and (3) is a γ ray. Now since particle emission is highly favored over electromagnetic transitions, it follows that for first emission of the γ ray the process must be sequential and be delayed-delayed (see Table II). This process has been discussed by several authors.[19] The method is a very attractive one since it is rather model-independent, relying principally on known electromagnetic processes; however, the method suffers the inconvenience of abnormally small cross sections because of suppression of the γ-ray emission due to competition with particle emission. Eisberg discusses this method in a paper in this volume.[20]

B. Emission of Three Nuclear Particles and Time Delay

According to the two-body cluster model,[4] the nature of the wave function of a nuclear state is a linear combination of two-body cluster wave functions. Thus in the cases cited above the nature of Be^7 is a mixture of He^3+He^4 and of $P+Li^6$ cluster wave functions. The amplitude in a particular two-body channel is expected to be largest when the absolute value of the channel kinetic energy is small. Thus highly excited states, where several two-body and three-body channels are open, are expected to decay into states where a particle is first-emitted and a metastable nucleus is produced which later decays by particle emission. This situation corresponds to either the direct-delayed or delayed-delayed categories of the sequential reactions of Table II. To ascertain that such a process has occurred, one must observe an energy structure and demonstrate the order of emission of the particles in the reaction. Clearly the kinematics must be completely determined if the state is to be defined. Since three final momenta are involved, there are nine scalar variables to be determined; the conservation of energy and momenta reduce this to five. This can be accomplished, for example, by measuring the momentum of one particle and the direction of a second. In the Rice measurements, described below, two momenta have been measured and six (an overdetermination by one) variables fixed. The overdetermination of variables is very helpful in subtracting background effects.

The reactions

$$p+B^{11}\rightarrow{}^*C^{12}\rightarrow 3\alpha \qquad (6)$$

and

$$p+d\rightarrow{}^*He^3\rightarrow 2p+n \qquad (7)$$

[19] R. M. Eisberg, *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Elementel and C. Villi (Gordon and Breach Publishers, Inc., New York, 1963), p. 352.

[20] R. M. Eisberg, this conference, Rev. Mod. Phys. **36**, 1100 (1964).

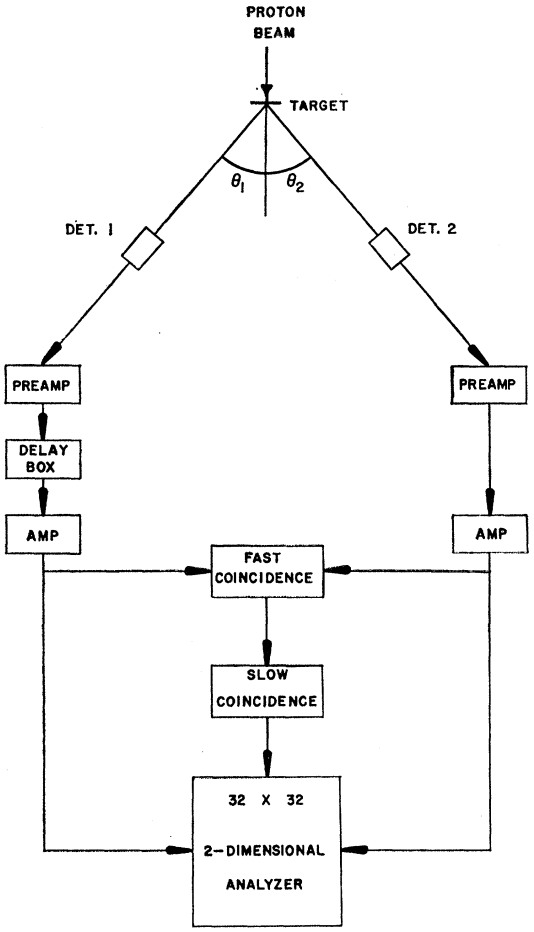

FIG. 4. Schematic diagram for the experimental arrangement used at Rice University to study three-body final states. Solid-state detectors were used. The beam was from the Rice Van de Graaff accelerators. The resolving time of coincidence was about 30 nsec.

have been studied at a variety of bombarding energies and angles for the two counters at Rice University. The experimental arrangement is shown in Fig. 4. Since these experiments give new evidence for cluster structure and since they allow new ways of measuring time delays, they will be discussed in detail below.

1. *The B^{11} $(p, 3\alpha)$ Reaction*

This reaction has been studied at Rice University by similar methods to those described by Dehnhard *et al.*[21] who concluded that the reaction was dominantly *simultaneous* when the bombarding energy was upon the 163-keV, $T=1$, resonance in C^{12}. On the contrary, they found that for other energies the decay was *via* a *sequential* mode proceeding through well-known states in Be^8. Thus, one concludes that this resonance in C^{12} is produced by a delayed-direct reaction. This

[21] D. Dehnhard, D. Kamke, and P. Kramer, Phys. Letters **3**, 52 (1962).

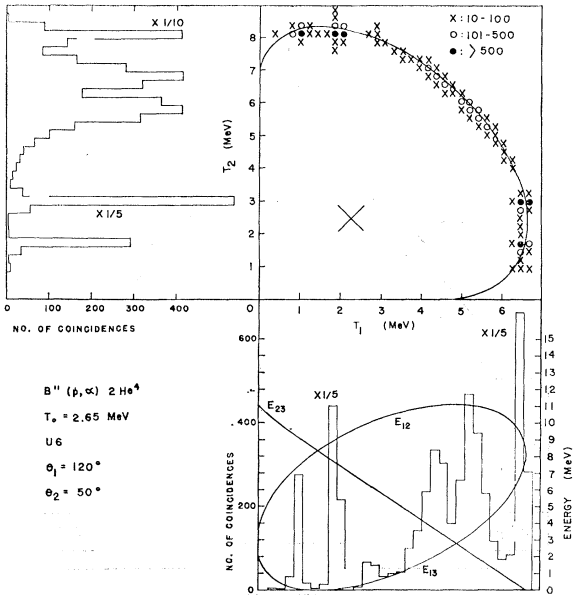

FIG. 5. Coincidence spectra in two dimensions for the reaction B^{11} $(p, 3\alpha)$. The data were obtained as shown in Fig. 4. The bombarding energy and angles are indicated. In the upper right-hand corner the axes T_1 and T_2 are the energies deposited in counters 1 and 2 by coincident α particles. The counts, indicated by symbols, are seen to fall along the unlabeled smooth curve that represents the calculated kinematic locus of coincident events. These observed counts are projected onto the T_1 or T_2 axes and presented as histograms below or to the left. The calculated curves labeled E_{12}, etc., are used for interpretation as follows: an event of energy T_1, projected up to the E_{12} curve may be read (on the T_2 axis) as the energy of excitation of the composite particle (1, 2) *provided* particle (3) was emitted first, etc. Accidental coincidences have been subtracted.

implies that $\tau_1 > 0$, $\tau_2 \approx 0$. For other nearby energies in C^{12} the reaction was found to be either direct-delayed or delayed-delayed.[21] This assertion will be proved below by the recent Rice experiments. The special nature $(T=1)$ of the C^{12} resonance accounts for its

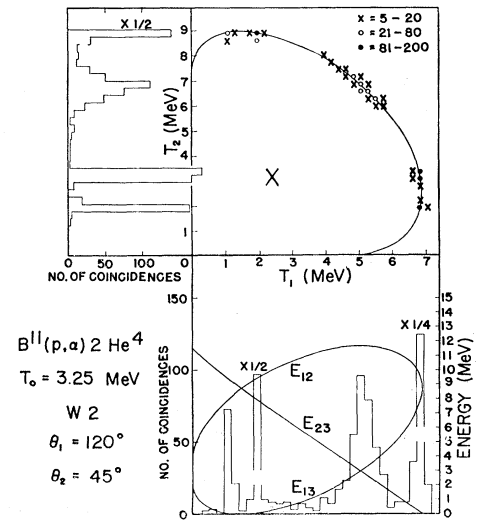

FIG. 6. Coincidence spectra in two dimensions for the reaction B^{11} $(p, 3\alpha)$. See caption of Fig. 5.

unusual decay properties: there are no low-lying $T=1$ states in Be^8; the lowest ones occur above 15 MeV of excitation. Only the "tails" of these $T=1$ Be^8 states are energetically available for α decay. This fact apparently accounts for the nonpeaked alpha spectra at the $T=1$ C^{12} resonance. Some of the Rice data[22] is shown in Figs. 5, 6, 7, and 8 for this reaction. The results of all the data may be summarized as follows:

(*a*) *Sequential decay.* The decay proceeds overwhelmingly *via* a *sequential* mechanism to the resonant ground state or first excited state of Be^8. No more than 5% of the $3-\alpha$ cross section can be attributed to a

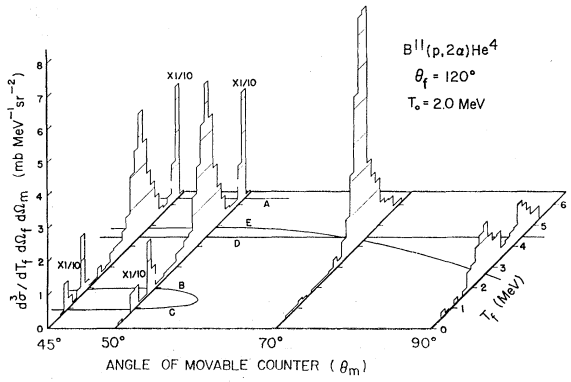

FIG. 7. A three-dimensional, isometric projection of coincidence counts for the B^{11} $(p, 3\alpha)$ reaction in the E_{fixed}, θ_{moving} plane. The values of bombarding energy and θ_{fixed} are indicated. Only events falling on the kinematic loci (see caption Fig. 5) have been considered. The solid curves on the E_f–θ_m plane are the calculated loci of the "bands" of energy where events proceeding through the 0^+ and 2^+ ground and first excited states of Be^8 should occur. When a "band" is parallel to the θ_m axis, the fixed counter has received a first-emitted event, when the "band" is curved the events correspond to the moving counter receiving the first-emitted particle. Note the low yield observed between the calculated "bands." Note that intensity peaks are only observed above the "bands" corresponding to either leaving Be^8 in the ground state (curves *A*, *B*, *C*) or the 2^+, 3-MeV, first excited state (curves *D*, *E*). The sharper peaks are always the ground-state group. This low yield between "bands" indicates that the reaction is principally sequential.

simultaneous mechanism. No evidence is found for any significant *simultaneous* decay. This does not contradict the data of Dehnhart *et al.* since their results are apparently due to the purity of the $T=1$ nature of the resonant C^{12} state they studied. All of the resonances studied at Rice have been $T=0$ states.

(*b*) *Order of particle emission-time interval measurement.* The experiments not only show peaks in the distributions but they also demonstrate that the *order* of emission of the particles can be determined uniquely when peaks are observed. Figure 7 at an angle of 90° shows two resolved peaks: the peak at $T_f=4.4$-MeV corresponds to a *first-emitted* α particle being detected in the fixed counter; the peak at $T_f=2.6$ MeV corresponds to a *second-emitted* α particle being detected in the fixed counter.

[22] J. D. Bronson, Jr., Ph.D. thesis, Rice University, 1963 (unpublished). J. B. Bronson, W. D. Simpson, W. R. Jackson, and G. C. Phillips, Bull. Am. Phys. Soc. **9**, 406 (1964).

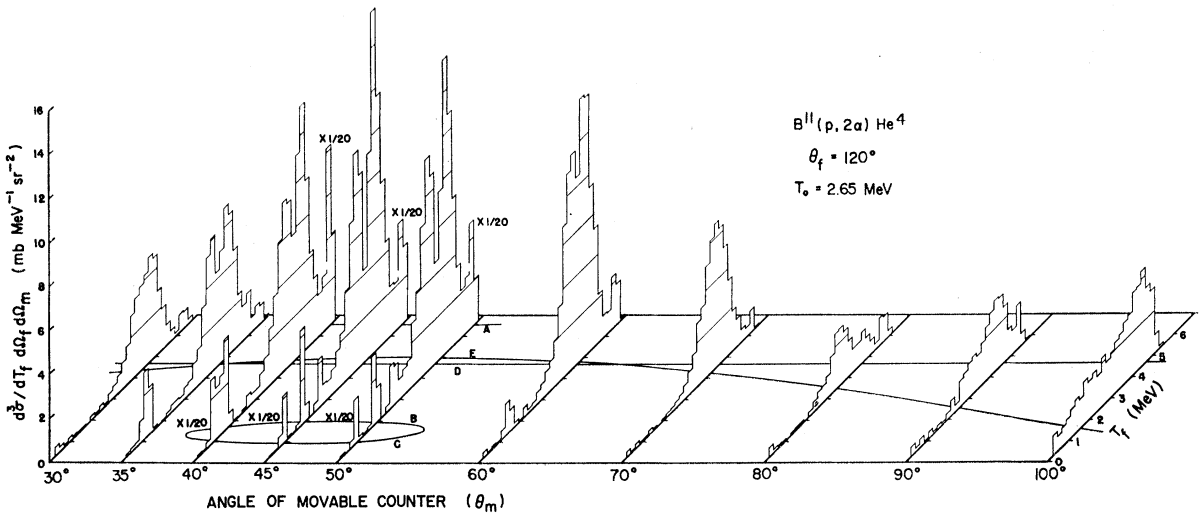

FIG. 8. A three-dimensional, isometric projection of coincidence counts for the B^{11} $(p, 3\alpha)$ reaction in the E_{fixed}, θ_{moving} plane. See caption of Fig. 7.

These experiments give a measure of a *time interval*: since the order of emission of the two detected α particles is in some cases determined, one can assert that the two particles were emitted with a time difference of $\tau_2 \simeq \hbar/\Gamma$, where Γ is the energy width of the Be^8 state. For 0^+ ground state, this time interval is about 2×10^{-16} sec ($\Gamma\simeq3$ eV) and for the 2^+ first excited state the time interval is about 0.6×10^{-21} sec ($\Gamma\simeq1$ MeV).

While this measure of time interval is necessarily a construction from energy–angle spectra, using the uncertainty principle, it is nevertheless a real time delay. The whole diagram, such as Fig. 7, accomplishes the measure of *time interval.*

When a two-body scattering or reaction resonance is observed, it is usual to construct a time delay[23] by the relation $\tau\simeq\hbar/\Gamma$. This construction, however, does not represent any real time interval that occurs during the course of the measurement; since the energy is an eigenvalue, the time is indeterminate. In the three-body case discussed above the time interval is real and is determined.

(*c*) *Cluster structure.* The cross section for this sequential process is large (about 200 mb for the excitation of the 2^+, 3-MeV Be^8 state), approaches the geometrical cross section, and thus confirms that the continuum C^{12} states studied have a large amplitude for the cluster structure $\alpha+{}^*Be^8$.

(*d*) *Interference effects.* Interference effects peculiar to the three particle sequential breakup mechanism have been observed for the B^{11} (p, α) $^*Be^8$ reaction. Figures 7 and 8 show one manifestation of these effects. Note that for the part of the energy–angle plane where the kinematics do not allow a clear determination of which α particle is first emitted [where the curved (E) and straight (D) loci are near to each other], the shape of the 2^+, 3-MeV, first-excited-state peak is strongly distorted. In Fig. 8, for angles of about 30–70° for the moving counter one would expect the two peaks, due to the two possible orders of α emission into the fixed counter, to overlap almost exactly; classically their intensities should add. Actually, it is seen that an intensity minimum occurs at the angle-energy region where a peak is expected. This clearly is evidence for destructive interference.

In Fig. 7, in the angular range 45–70°, constructive interference is seen to occur since the energy half-width of the observed peak is only about 0.5 MeV, while the half-width of the Be^8 state is known to be about 1 MeV.[24] There seems to be constructive interference in Fig. 8 in the angular range 80–100°, between the 2^+, 3-MeV, Be^8 peaks.

These effects came about from the fact that the cross section at any bombarding energy T_0, fixed and moving counter angles θ_f, θ_m, ϕ_m and fixed energy T_f is

$$\sigma(T_0, \theta_f, \theta_m, \phi_m, T_f) = \beta \mid M_{1,2}+M_{2,1} \mid^2$$
$$= \beta[\mid M_{1,2} \mid^2 + \mid M_{2,1} \mid^2 + 2 \operatorname{Re} M_{1,2}M_{2,1}^*],$$

where β is a number and M_{ij} represents the matrix element for the detection of particle i in the fixed counter and j in the moving counter. The symbols i, j refer to the order of emission: 1 referring to first emission, 2 to second emission. Thus, in Figs. 7 and 8, $\mid M_{1,2} \mid^2$ is calculated to have a maximum along the lines D, while $\mid M_{2,1} \mid^2$ should have a maximum along the lines E. The cross term, however, may be negative or positive and causes the interference effect. This effect will be called *the order of emission interference effect.* In general, other matrix elements also occur, but this point will be discussed elsewhere. An attempt at a quantitative description of these effects is underway.[25]

An important new feature of these interference effects

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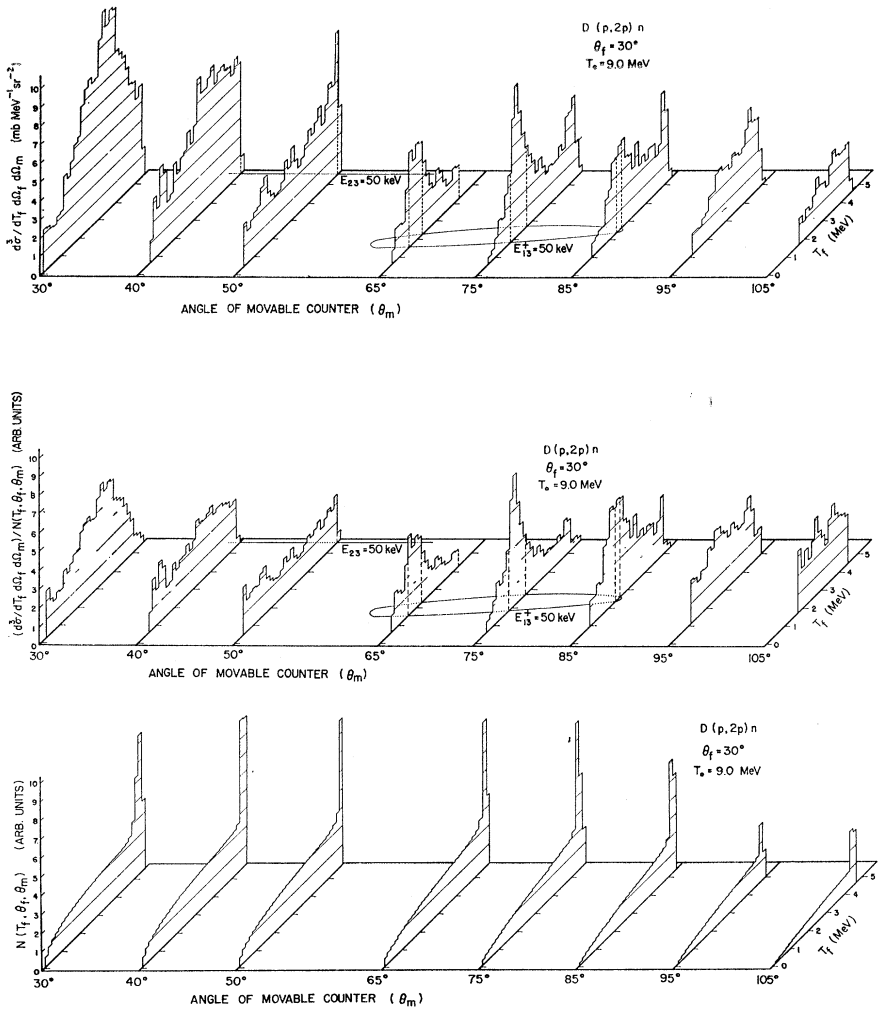

FIG. 9. A three-dimensional, isometric projection of coincidence counts for the $D(p, 2p)n$ reaction in the E_{fixed}, θ_{moving} plane. The bombarding energy is 9.0 MeV and θ_{fixed} is 30°. The curve E_{23} indicates the locus of events proceeding via a sequential breakup in which the proton detected in the fixed counter (1) is first emitted, leaving the system (2, 3) with 50-keV internal energy. The curve E_{13} indicates the locus of events proceeding via a sequential breakup in which the proton detected in the movable counter (2) is first emitted, leaving the system (1, 3) with 50-keV internal energy. The upper diagram is the data as collected, with cross section plotted vs the energy of the proton detected in the fixed counter and vs the angle of the movable counter. The center diagram is a similar display, but with the available phase space divided out. The lower diagram indicates this available phase space, i.e., the distribution expected if the reaction were to proceed via simultaneous emission and with intensity proportional to the phase space available.

resides in the possibility of employing them to obtain an independent measure of nuclear lifetimes or widths. This comes about from the following consideration: the order of emission interference effect is expected to be large only when the two matrix elements $|M_{1,2}|$ and $|M_{2,1}|$ (with all other quantum numbers j, π, T, etc. degenerate) are large at the same T_f, θ_m. This should occur when the $|M_{i,j}|$ peaks are within less than a level width of each other. Only in this case will the three-body results of reaction (9) differ from a proper superposition of *intensities* of the two-body reaction $c+C \rightarrow B^* \rightarrow c+C$. However, if such a difference is observed then it will be due to the interference effects.

Since the experimenter, within limits, may vary the separation energy of the two types of energy–angle loci of possible resonant states of B^* (see loci D, E, of Figs. 7 and 8), the variation of interference terms with this energy separation may be studied, and information about the half-width of states obtained. This half-width information is independent of that obtained from cross-section studies of the final-state system and in some cases may allow new information about the lifetimes of nuclear states. This will be discussed below.

2. *The $D(p; 2p, n)$ Reaction*

This reaction has also been studied at Rice using the coincidence methods described above. Typical data are shown in Figs. 9 and 10. In this case the data indicate a possible mixture of direct-direct and direct-delayed effects as is shown in the figures. It is interesting to note, however, the clear evidence of the singlet, virtual deuteron which is seen in Fig. 10. This implies that the continuum He^3 states studied have significant amplitudes for the two-body cluster configuration $p+d^{(1)}$, where $d^{(1)}$ represents the singlet deuteron and that a large part of the reaction mechanism is sequential. The sequential mode accounts for about 1 barn of the cross section at 9 MeV. It is interesting to compare this value with the total $n+d$ elastic cross section at 9 MeV of 1 barn. The fact that these cross sections are comparable suggests that the nucleon plus triplet deuteron $(N+d^{(3)})$ and nucleon plus singlet deuteron $(N+d^{(1)})$ have comparable amplitudes in these continuum excited states of the three-nucleon system. The large value of these cross sections (near geometrical) suggests that a two-body cluster-model description of

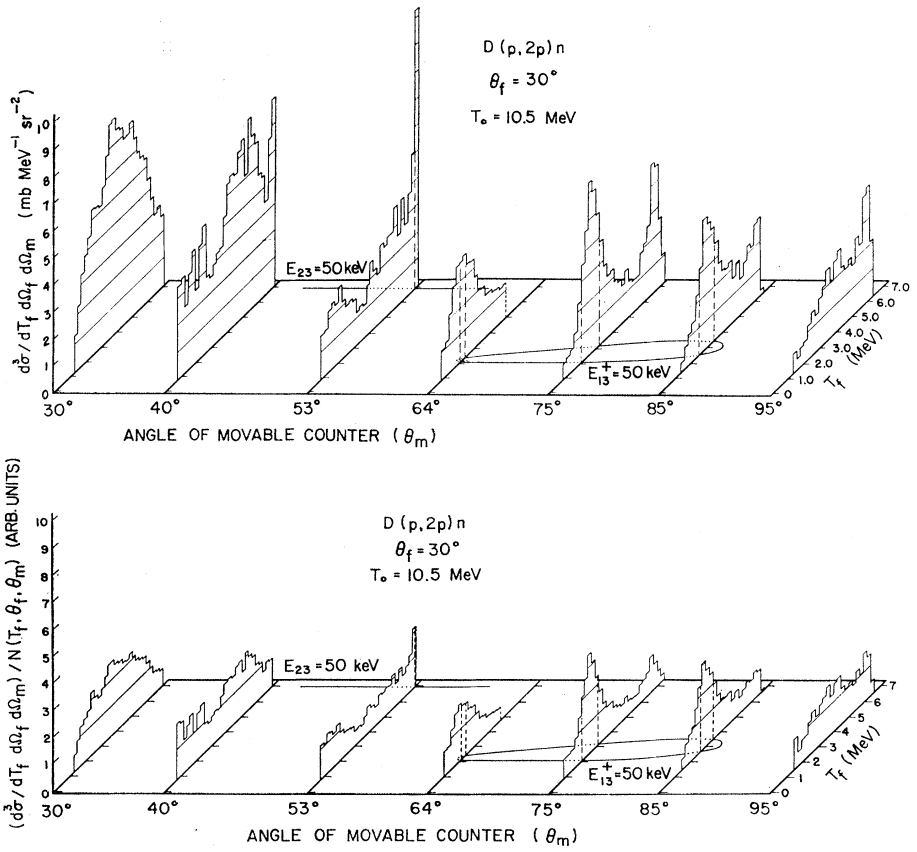

FIG. 10. A three-dimensional, isometric projection of coincidence counts for the $D(p, 2p)n$ reaction in the E_{fixed}, θ_{moving} plane (see Fig. 9). The bombarding energy is 10.5 MeV and θ_{fixed} is 30°. The upper diagram is the data as collected, while the lower diagram has the available phase space divided out.

these continuum states is reasonable. Thus one may expect the three-nucleon system in this energy range to be represented by a linear combination of $N+d^{(3)}$, $N+d^{(1)}$, and possibly $N+$diproton or $N+$dineutron amplitudes. There may be some true direct-direct, simultaneous breakup also which would need to be included in describing the mass-three states. However, it is possible that this will not be the case and a decision must await a quantitative three-body decay theory.

In summary one can conclude that the study of three-body processes allows an additional method of examining the compound-nucleus features and the cluster structure of nuclei, and provides a sensitive additional method of discerning nuclear time delays.

C. Possible New Methods of Measuring Time Delays

One of the most interesting of nuclear reaction problems has been mentioned above: are certain high-energy reactions direct, or are they time-delayed? Many (d, p), (d, n), (He^3, α), (α, α) processes are examples. These processes often show aspects of both types of mechanisms and thus constitute a dilemma.

In particular many high-energy processes show so-called *fluctuations* which are certainly evidence of resonant, compound-nucleus contributions to the cross section. There are always two variables that must be measured to describe these fluctuations: the energy separation D between interfering states and the partial widths Γ_λ for decay into channel λ. The observed fluctuations are a complicated function[13] of both of these quantities, and the measurement of the fluctuations does not provide a unique measure of either. Thus observation of a given fluctuating cross section in a single or in a few of all possible exit channels may be described as either the effect of many narrow, unresolved levels, or of a few broad, overlapping resonances (possibly with large widths in unobserved channels).

To provide new information for this problem the following method is proposed. Suppose one wishes to study the reaction

$$\begin{array}{l} g+H \xrightarrow{\tau_2=0} c+C \\ \quad \searrow B \nearrow \ \tau_2 \neq 0 \end{array} \tag{8}$$

to determine whether it is direct, $\tau_2=0$, or delayed, $\tau_2 \neq 0$. To do this, consider the three-body reactions $A(a, b)cC$:

$$a+A \xrightarrow[\tau_1=\tau_2=0]{} b+c+C, \tag{9a}$$

$$a+A \xrightarrow[\tau_1=0]{} b+B^*; \quad B^* \xrightarrow[\tau_2 \neq 0]{} c+C, \tag{9b}$$

$$a+A \to D \xrightarrow[\tau_1 \neq 0]{} b+B^*; \quad B^* \xrightarrow[\tau_2 \neq 0]{} c+C, \tag{9c}$$

$$a+A \to D \xrightarrow[\tau_1 \neq 0]{} b+c+C. \quad (\tau_2=0) \tag{9d}$$

Now there are four possibilities: if reactions (9) are a direct-direct, simultaneous process (9a), or are delayed-direct (9d), they will have a bell-shaped energy distribution of the coincidence spectra and imply that τ_2 is zero and thus reaction (8) is direct. If reactions (9) are a sequential reaction of either the direct-delayed (9b) or delayed-delayed (9c) type, their coincidence energy distributions will have peaks, implying that $\tau_2 \neq 0$ for reaction (8) as well as (9). Thus a measure of the relative amounts of broad and peaked coincidence energy distributions for the three-body process (9) should allow one to conclude the relative importance of the direct versus delayed nature of reaction (8). To make this determination effective, however, it will be necessary to ascertain which particle is emitted first if the reaction is sequential. As was seen in Figs. 5–8, this appears to be a possible measurement in some circumstances.

An example of such a possible measurement might be to study the time delay of (d, p) measurements. These reactions are generally assumed to be of the direct type; yet they frequently have fluctuating cross sections. If the three-body reaction (He^3, $2p$) were studied, and if it were to be found to be of the direct-direct, or delayed-direct type, when the emitted protons have the proper energy to simulate the (d, p) reaction, then the direct nature of the (d, p) reaction would be independently established. Alternatively, if the reaction showed peaks in the angle-energy spectra, then the reaction is either delayed-delayed or direct-delayed, and in either case the (d, p) process could be presumed to proceed with some time delay. Finally, as discussed above, a comparison of the (d, p) cross-section fluctuations with the energy–angle diagrams of the corresponding (He^3, p, p) reaction will provide new information about the widths of the states.

ACKNOWLEDGMENTS

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Statistical Properties of Compound States*

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I. INTRODUCTION

The concept of an energy-averaged cross section has been relevant to the understanding of experimental data for at least as long as the concept of scattering experiments. However, it has been mainly in the analysis of nuclear scattering experiments that the understanding of this concept has been sharpened to the point where the new ideas that have evolved in the rather specialized sphere of nuclear reactions may have much broader implications in other areas of physics and chemistry.

The cross section $\sigma_{cc'}(E)$ for the reaction leading from channel c to the channel c' is a function not only of the energy E (or the driving angular frequency $\omega=E/\hbar$), but also of the parameters E_λ (energy positions) and $\gamma_{\lambda c}$ (reduced-width amplitudes) for the states λ that are excited by driving channel c. Thus we write

$$\sigma_{cc'}=\sigma_{cc'}(E;\{E_\lambda\},\{\gamma_{\lambda c}\})$$
$$=\sigma_{cc'}(E;H), \tag{1}$$

where the sets of parameters $\{E_\lambda\}$ and $\{\gamma_{\lambda c}\}$ have been replaced symbolically by the Hamiltonian operator H which defines the eigenvalues E_λ and the eigenfunctions X_λ, the latter of which enter into the $\gamma_{\lambda c}$. Integrated cross sections of the form of (1) are additive with respect to symmetry (for example total angular momentum J and parity π):

$$\sigma_{cc'}=\sum_{J,\pi}\sigma_{cc'}{}^{J\pi}. \tag{2}$$

For one symmetry, the cross section can be written in terms of the scattering matrix element $U_{cc'}{}^{J\pi}$:

$$\sigma_{cc'}{}^{J\pi}=\pi\lambda_c{}^2 g_{J\pi}\,|\,\delta_{cc'}-U_{cc'}{}^{J\pi}\,|^2. \tag{3}$$

If we drop the $J\pi$ labels and set $g_{J\pi}=1$ (s waves), we can more conveniently examine the notion of averaged cross section. To average the cross section over energy we introduce an energy resolution function $R_I(E-E')$ which we take to be a function of the energy difference $(E-E')$ between the energy E' at which the unaveraged cross section is specified and the energy E at which the averaged cross section is determined. In addition, the resolution function R_I depends on the interval I over which the average is carried out. The interval I is important since it defines the time $\hbar/I$ which the measurement takes. If the time $\hbar/I$ is small compared to the Poincaré recurrence time $2\pi\hbar/D$, where D is the mean distance between the fine

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