Average Compound-Nucleus Cross Sections*

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Energy averages of resonant or fluctuating compound-nucleus cross sections were first discussed by Wolfenstein[1] and by Hauser and Feshbach.[2] They postulated that, as in the case of an isolated compound-state resonance, the contribution of each total angular momentum and parity of the system to the average partial reaction cross section could be factored into an average compound-nucleus formation cross section and an independent decay branching ratio. Together with the reciprocity theorem,[3] this led to the well-known Hauser–Feshbach formula which in the channel spin representation may be written as follows:

$$\left\langle\frac{d\sigma_{\alpha\alpha'}}{d\Omega}\right\rangle=\frac{\lambda\!\!\!^{-}{}_{\alpha}^{2}}{(2I_{\alpha}+1)(2i_{\alpha}+1)}\sum\frac{(-1)^{s-s'}}{4}\frac{T_{\alpha ls}{}^{J}T_{\alpha' l' s'}{}^{J}}{\sum T_{\alpha'' l'' s''}{}^{J}}\times\bar{Z}(lJlJ,sL)\bar{Z}(l'Jl'J,s'L)P_{L}(\cos\theta). \quad (1)$$

Here $\langle\ \ \rangle$ denotes an energy average over resonance fluctuations; $d\sigma_{\alpha\alpha'}$ is the differential cross section for the reaction proceeding from entrance channel α with fragment spins I_α and i_α to exit channel α'; l, s and l', s' are relative orbital angular momenta and channel spins in the entrance and exit channels, and J is the total angular momentum. The first sum is over L, J, and all combinations of partial waves l, s, l', s' consistent with conservation laws, and the sum in the denominator is over all partial waves and channels which compete in the decay of the compound state. The corresponding integrated cross section is

$$\langle\sigma_{\alpha\alpha'}\rangle=\pi\lambda\!\!\!^{-}{}_{\alpha}^{2}\sum g_{\alpha}{}^{J}\frac{T_{\alpha ls}{}^{J}T_{\alpha' l' s'}{}^{J}}{\sum T_{\alpha'' l'' s''}{}^{J}}, \quad (2)$$

where $g_\alpha{}^J=(2J+1)/(2I_\alpha+1)(2i_\alpha+1)$. The total compound-nucleus formation cross section in channel α is obtained by summing Eq. (2) over all α'

$$\langle\sigma_{\alpha}{}^{CN}\rangle=\pi\lambda\!\!\!^{-}{}_{\alpha}^{2}\sum g_{\alpha}{}^{J}T_{\alpha ls}{}^{J}. \quad (3)$$

The corresponding compound-nucleus formation cross section $\pi\lambda\!\!\!^{-}{}_\alpha^2 T_{\alpha ls}{}^J$ for the partial wave (αlsJ) will serve for the present to define the transmission coefficients $T_{\alpha ls}{}^J$. If the compound-nucleus formation cross section may be identified with the optical-model absorption cross section,[4] then

$$T_{\alpha ls}{}^{J}=1-\exp\left(-4\,\mathrm{Im}\,\delta_{\alpha ls}{}^{J}\right), \quad (4)$$

where Im δ is the imaginary part of the optical-model complex phase shift.

Two points require further examination. One is the assumption of the independence of formation and decay on the average, the other is the relationship between average compound-nucleus formation and optical-model absorption. That the first assumption does not have unlimited validity was pointed out by Lane and Lynn[5] and by Dresner,[6] who showed that in the limiting case of isolated resonances, where the compound-nucleus contribution to the cross section may be approximated by a sum over single-level Breit–Wigner resonance terms, the average cross sections are given by Eqs. (1) and (2), provided one makes the substitution

$$\frac{T_{\alpha ls}{}^{J}T_{\alpha' l' s'}{}^{J}}{\sum T_{\alpha'' l'' s''}{}^{J}}\rightarrow\frac{2\pi}{D_{J\Pi}}\left\langle\frac{\Gamma_{\mu,\alpha ls}\Gamma_{\mu,\alpha' l' s'}}{\Gamma_{\mu}}\right\rangle_{\mu(J,\Pi)}, \quad (5)$$

where $D_{J\Pi}$ is the mean spacing of resonances with angular momentum J and parity Π; Γ_μ is the total width of the resonance μ, $\Gamma_{\mu,\alpha ls}$ is a partial width and $\langle\ \ \rangle_{\mu(J\Pi)}$ denotes an average over resonances with angular momentum J and parity Π. Summing this new average cross section over exit channels and comparing with Eq. (3) yields in this single-level limit

$$T_{\alpha ls}{}^{J}=(2\pi/D_{J\Pi})\langle\Gamma_{\mu,\alpha ls}\rangle_{\mu(J\Pi)}. \quad (6)$$

Following Porter and Thomas[7] we assume that the values of the partial widths are distributed in μ and are also not completely correlated with respect to channel indices α, l, s. Therefore, the average compound-nucleus cross section can no longer be factored into independent formation and decay terms and, in fact, the substitution (5) leads to a width fluctuation correction[5,6] to the Hauser–Feshbach formula as specified by Eqs. (1), (2), and (6). If the partial widths have the Porter–Thomas distribution, this correction varies from a factor of unity to $\frac{1}{2}$ for $\alpha\neq\alpha'$, and from a factor of unity to three in the case of elastic scattering. The relative magnitude of the width fluctuation correction decreases with increasing positive correlation of partial widths and with increasing numbers of competing decay channels.[8] It vanishes in the limits of completely correlated partial widths and of very many competing channels. However, as the number of channels in-

* Work performed under the auspices of the U.S. Atomic Energy Commission.

[1] L. Wolfenstein, Phys. Rev. **82**, 690 (1951).
[2] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).
[3] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952).
[4] H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. **96**, 448 (1954).
[5] A. M. Lane and J. E. Lynn, Proc. Phys. Soc. (London) **A70**, 557 (1957).
[6] L. Dresner, Proceedings of the International Conference on Neutron Interactions with the Nucleus, Columbia University Report CU-175 (1957), p. 71.
[7] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).
[8] See, however, G. R. Satchler, Phys. Letters **7**, 55 (1963), where it is shown that correlation effects in elastic scattering may persist for large numbers of competing channels.

creases, the average total width to spacing ratio Γ/D of the resonances will increase, invalidating the assumptions underlying the above discussion.

The investigation of these questions in more general circumstances is handicapped by the great complexity of the theories of compound nucleus reactions. Two methods based on R-matrix theory[9] exist which extend the scope of the above results. One, derived by Thomas,[10] is based on an expansion about the limiting case of one or two open channels and becomes unreliable in the presence of additional channels which compete strongly in the decay. The other method[11] is effectively an expansion in Γ/D and becomes useless when that ratio is too large. These difficulties are not removed in a simple way by resorting either to the Kapur–Peierls[12] or Feshbach[13] formalisms, since the implicit energy dependences of the resonance parameters of these theories must be investigated before reliable energy averages can be obtained. We shall instead employ the Humblet–Rosenfeld resonance expansion[14] as a formal representation of the reaction process within a restricted energy averaging interval. We will then be hampered somewhat by our lack of knowledge regarding the detailed statistical properties of the resonance parameters in that formalism. However, we will be able to express the effects of these uncertainties largely in terms of a single parameter which can be evaluated by comparison with experimental data.[15]

The cross sections are bilinear functions of the elements of the transition matrix $\mathfrak{I}_{cc'}$, where the index c denotes a particular partial wave (α, l, s, J). We assume that the reaction mechanism can be separated into direct and compound-nucleus processes, so that we may write

$$\mathfrak{I}_{cc'}=\mathfrak{I}_{cc'}{}^{\text{direct}}+\mathfrak{I}_{cc'}{}^{\text{compound}}, \tag{7}$$

where $\mathfrak{I}_{cc'}{}^{\text{direct}}$ is sufficiently smoothly varying with energy that we may regard it as a constant within our averaging interval, and $\mathfrak{I}_{cc'}{}^{\text{compound}}$ is the compound-nucleus reaction amplitude which is assumed to vary rapidly with energy. The average integrated partial reaction cross section

$$\langle\sigma_{cc'}\rangle=\pi\lambda\!\!\!^{-}{}_c{}^2\langle|\,\mathfrak{I}_{cc'}\,|^2\rangle \tag{8}$$

may then be separated into the following four terms[16]:

$$\langle\sigma_{cc'}\rangle=\sigma_{cc'}{}^{\text{direct}}+\sigma_{cc'}{}^{\text{correlation}}+\sigma_{cc'}{}^{\text{fluctuation}}+\langle\sigma_{cc'}{}^{\text{interference}}\rangle, \tag{9}$$

where

$$\sigma_{cc'}{}^{\text{direct}}\equiv\pi\lambda\!\!\!^{-}{}_c{}^2\,|\,\mathfrak{I}_{cc'}{}^{\text{direct}}\,|^2, \tag{10a}$$

$$\sigma_{cc'}{}^{\text{correlation}}\equiv\pi\lambda\!\!\!^{-}{}_c{}^2\,|\,\langle\mathfrak{I}_{cc'}{}^{\text{compound}}\rangle\,|^2, \tag{10b}$$

$$\sigma_{cc'}{}^{\text{fluctuation}}\equiv\pi\lambda\!\!\!^{-}{}_c{}^2\langle|\,\mathfrak{I}_{cc'}{}^{\text{compound}}-\langle\mathfrak{I}_{cc'}{}^{\text{compound}}\rangle\,|^2\rangle, \tag{10c}$$

$$\langle\sigma_{cc'}{}^{\text{interference}}\rangle\equiv\pi\lambda\!\!\!^{-}{}_c{}^2 2\,\text{Re}\,(\mathfrak{I}_{cc'}{}^{\text{direct}})^*\langle\mathfrak{I}_{cc'}{}^{\text{compound}}\rangle. \tag{10d}$$

Clearly, the average compound-nucleus cross section must be considered as given by

$$\langle\sigma_{cc'}{}^{\text{compound}}\rangle=\sigma_{cc'}{}^{\text{correlation}}+\sigma_{cc'}{}^{\text{fluctuation}}, \tag{11a}$$

so that

$$\langle\sigma_{cc'}\rangle=\sigma_{cc'}{}^{\text{direct}}+\langle\sigma_{cc'}{}^{\text{compound}}\rangle+\langle\sigma_{cc'}{}^{\text{interference}}\rangle. \tag{11b}$$

The conventional optical model is a theory of scattering described by the average diagonal elements $\langle\mathfrak{I}_{cc}\rangle$. A generalized optical model involving coupled channels, is obtained by considering the scattering and reaction processes described by the complete average transition matrix.[17] The corresponding generalized optical-model cross section is

$$\sigma_{cc'}{}^{\text{optical}}=\sigma_{cc'}{}^{\text{direct}}+\sigma_{cc'}{}^{\text{correlation}}+\langle\sigma_{cc'}{}^{\text{interference}}\rangle, \tag{12a}$$

leading to a second decomposition of the average cross section:

$$\langle\sigma_{cc'}\rangle=\sigma_{cc'}{}^{\text{optical}}+\sigma_{cc'}{}^{\text{fluctuation}}. \tag{12b}$$

In the case where

$$\langle\mathfrak{I}_{cc'}{}^{\text{compound}}\rangle\to 0, \tag{13a}$$

we find that

$$\sigma_{cc'}{}^{\text{correlation}}\to 0, \tag{13b}$$

$$\langle\sigma_{cc'}{}^{\text{interference}}\rangle\to 0, \tag{13c}$$

$$\sigma_{cc'}{}^{\text{optical}}\to\sigma_{cc'}{}^{\text{direct}}, \tag{13d}$$

$$\langle\sigma_{cc'}{}^{\text{compound}}\rangle\to\sigma_{cc'}{}^{\text{fluctuation}}. \tag{13e}$$

Entirely analogous decompositions of the angular distributions can be made. The average total cross section is given by

$$\langle\sigma_\alpha{}^{\text{total}}\rangle=2\pi\lambda\!\!\!^{-}{}_\alpha{}^2\sum_c \text{Re}\,(\mathfrak{I}_{cc}{}^{\text{direct}}+\langle\mathfrak{I}_{cc}{}^{\text{compound}}\rangle), \tag{14}$$

which shows that the limit (13) for the diagonal transition elements would imply no net compound-nucleus contribution to the average total cross section.

The direct amplitudes $\mathfrak{I}^{\text{direct}}$ include the effects of nuclear potential scattering and point-charge Coulomb scattering, as well as processes which may be discussed in terms of direct-reaction models such as the

[9] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947); A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).
[10] R. G. Thomas, Phys. Rev. **97**, 224 (1955).
[11] P. A. Moldauer, Phys. Rev. **123**, 968 (1961).
[12] P. L. Kapur and R. Peierls, Proc. Roy. Soc. (London) **A166**, 277 (1938).
[13] H. Feshbach, Ann. Phys. (N. Y.) **5**, 357 (1958); **19**, 287 (1962).
[14] J. Humblet and L. Rosenfeld, Nucl. Phys. **26**, 529 (1961).
[15] See also Ref. 18.
[16] The classification of cross sections given here enlarges somewhat on that used in the oral presentation of this paper at Gatlinburg. At that time the "direct cross section" was identified with what is here called σ^{optical}.
[17] S. Yoshida, Proc. Phys. Soc. (London) **A69**, 668 (1956); D. M. Chase, L. Wilets, and A. R. Edmonds, Phys. Rev. **110**, 1080 (1958).

distorted-wave Born approximation. These will not concern us further here. In order to be able to discuss $\mathfrak{J}^{\text{compound}}$ further, we make the assumption[18] that within a suitable averaging interval we can write[19]

$$\mathfrak{J}_{cc'}{}^{\text{compound}}=i\sum_{\mu}[g_{\mu c}g_{\mu c'}/(E-E_{\mu}+\tfrac{1}{2}i\Gamma_{\mu})] \quad (15)$$

with constant parameters $g_{\mu c}$, E_{μ}, and Γ_{μ}. The equation (15) has very general validity as a representation of the fluctuating part of a causal transition amplitude on the real energy axis within a finite interval which is very far from the thresholds of certain channels.[18] The physical interpretation of this representation has been discussed by Humblet and Rosenfeld.[14] We shall make the additional assumption that the real resonance energies E_{μ} may be considered to be distributed with uniform density and correlations from $E=-\infty$ to $+\infty$ and that the real Γ_{μ} and the complex $g_{\mu c}$ have uniform statistical distributions over the range of all resonance indices μ.[18] These distributions will, of course, depend on the energy interval being considered.

In the case of an energy interval containing isolated resonances for which the average total width Γ is small compared to the average spacing D of the resonance levels, Eq. (15) follows directly from R-matrix theory. The E_{μ} are then distributed like the poles of the R-matrix, exhibiting the well-known repulsion of neighboring levels and other correlation effects.[20] The $g_{\mu c}$ can be written $\gamma_{\mu c}\exp(i\varphi_c)$, where φ_c is constant in μ and the real $\gamma_{\mu c}$ are normally distributed with zero mean. The partial widths $\Gamma_{\mu c}=\gamma_{\mu c}{}^2=|g_{\mu c}|^2$ follow the Porter–Thomas distribution[7] (χ^2 distribution with one degree of freedom) and $\Gamma_{\mu}=\sum_c\Gamma_{\mu c}$, where the sum is over all open channels.

For intervals in which Γ/D is not small, the distribution laws of the resonance parameters in (15) may differ from those discussed above. It is no longer certain that the resonance levels E_{μ} will exhibit repulsion. The distribution of the complex $g_{\mu c}$ is also not known, but it appears likely on statistical grounds that both their real and imaginary parts are normally distributed, so that the $|g_{\mu c}|^2$ follow a distribution with characteristics somewhere between a χ^2 distribution with one and with two degrees of freedom, depending on the relative magnitudes and correlations of the real and imaginary parts. A two degrees of freedom distribution of $|g_{\mu c}|^2$ results from an isotropic normal distribution of the $g_{\mu c}$. The partial widths must be defined in terms of a real normalization constant[21] $N_{\mu}\geq 1$,

$$\Gamma_{\mu c}=|g_{\mu c}|^2/N_{\mu}, \quad (16)$$

which makes

$$\Gamma_{\mu}=\sum_c\Gamma_{\mu c}. \quad (17)$$

The distribution of the N_{μ} and their correlations with the $|g_{\mu c}|^2$ are important because of their effects on the distributions and correlations of partial and total widths. Since N_{μ} is essentially the normalization of the eigenfunction of a very complex boundary value problem of high dimension and may be considered as a sum of a large number of random contributions, we may suppose that the N_{μ} are fairly constant in μ. We also suppose that the distribution laws of widths and spacings deviate from their known isolated resonance limits in proportion to the magnitude of Γ/D. Fortunately, we also find that as Γ/D becomes large, the values of average cross sections do not depend strongly on the precise details of the statistics of the resonance parameters in (15). To some extent this is also true of cross-section fluctuations, so that it may be difficult to obtain detailed information on resonance parameter statistics in the region of overlapping resonances.

Averaging (15) gives

$$\langle\mathfrak{J}_{cc'}{}^{\text{compound}}\rangle=(\pi/D)\langle g_{\mu c}g_{\mu c'}\rangle_{\mu}, \quad (18)$$

where it will be taken for granted that $\mu\equiv\mu(J\Pi)$, and hence

$$\sigma_{cc'}{}^{\text{correlation}}=\pi\lambda\!\!\!^{-}{}_c{}^2(\pi^2/D^2)\;|\langle g_{\mu c}g_{\mu c'}\rangle_{\mu}|^2. \quad (19)$$

The fluctuation cross section is given by

$$\sigma_{cc'}{}^{\text{fluctuation}}=\pi\lambda\!\!\!^{-}{}_c{}^2\left\{\frac{2\pi}{D}\left\langle\frac{|g_{\mu c}|^2\,|g_{\mu c'}|^2}{\Gamma_{\mu}}\right\rangle_{\mu}\right.$$

$$\left.-\frac{2\pi^2}{D^2}\left\langle g_{\mu c}g_{\mu c'}g_{\nu c}{}^*g_{\nu c'}{}^*\left[1-\Phi_0\left(\frac{\Gamma_{\mu}+\Gamma_{\nu}}{2D}\right)\right]\right\rangle_{\mu\neq\nu}\right\}, \quad (20)$$

where Φ_0 is defined in terms of the resonance energy pair correlation function[22] $R_2(E_{\mu}-E_{\nu})$ to be

$$\Phi_0\left(\frac{\Gamma}{D}\right)=-i\frac{D}{\pi}\int_{-\infty}^{+\infty}\frac{d\epsilon R_2(\epsilon)}{\epsilon-i\Gamma}, \quad (21)$$

and has the properties of tending to zero for small Γ/D and maximum level repulsion[23] and approaching unity for large Γ/D or when the resonance level energies E_{μ} are uncorrelated. Therefore, as Γ/D becomes large, $1-\Phi_0$ tends to zero regardless of the level spacing distribution law. Using the fact that in any case Φ_0 is a slowly varying function of its argument and assuming

[18] P. A. Moldauer, Phys. Rev. (to be published). Questions of validity of these assumptions and their relation to the R-matrix theory (Ref. 9) and Kapur–Peierls Theory (Ref. 12) are discussed there, as well as methods for treating the statistical properties of the parameters of Eq. (15).

[19] The notation used here is related to that of Humblet and Rosenfeld as follows: $\mathfrak{J}_{cc'}{}^{\text{direct}}$ and $g_{\mu c}$ are equivalent to $-(k_ck_{c'})^{\frac{1}{2}}\bar{Q}_{cc'}$ and $(k_c)^{\frac{1}{2}}G_{c\mu}\exp i\xi_{c\mu}$ of Ref. 14, respectively. E_{μ} and Γ_{μ} mean the same as in Ref. 14.

[20] E. P. Wigner, *Fourth Canadian Mathematical Congress Proceedings* (University of Toronto Press, Toronto, 1957), p. 174; M. L. Mehta, Nucl. Phys. **18**, 395 (1960); M. L. Mehta and M. Gaudin, *ibid.* **18**, 420 (1960); M. Gaudin, *ibid.* **25**, 447 (1961).

[21] The parameter N_{μ} is equivalent to the $q_{\mu}{}^2$ of Ref. 14 and to the $|N_{\mu}|^{-1}$ of Ref. 12.

[22] Freeman J. Dyson, J. Math. Phys. **3**, 166 (1962).

[23] P. A. Moldauer, Phys. Letters **8**, 70 (1964).

that a sum of two total widths does not fluctuate very much, we may take Φ_0 to be a function of the average Γ/D. Defining also

$$\Theta_{\mu c}=(2\pi/D)N_\mu^2\Gamma_{\mu c}, \qquad \Theta_\mu=\sum_c \Theta_{\mu c}, \tag{22}$$

we may write with the help of (16) and (17)

$$\sigma_{cc'}{}^{\text{fluctuation}}=\pi\lambda\!\!\!^{-}{}_c^2\left\langle\frac{\Theta_{\mu c}\Theta_{\mu c'}}{\Theta_\mu}\right\rangle_\mu-2(1-\Phi_0)\sigma_{cc'}{}^{\text{correlation}}. \tag{23}$$

Invoking flux conservation we find that

$$\langle\Theta_{\mu c}\rangle_\mu=T_c{}^{\text{compound}}+2(1-\Phi_0)\sum_{c''}\frac{\sigma_{cc''}{}^{\text{correlation}}}{\pi\lambda\!\!\!^{-}{}_c^2}, \tag{24}$$

where $T_c{}^{\text{compound}}$ is the transmission coefficient of the generalized optical model which is related to the transmission coefficient T_c of the ordinary single-channel optical model, defined in Eq. (4) by

$$T_c{}^{\text{compound}}=T_c-\sum_{c''\neq c}(\sigma_{cc''}{}^{\text{optical}}/\pi\lambda\!\!\!^{-}{}_c^2). \tag{25}$$

By Eq. (22) the distribution of the fluctuations of $\Theta_{\mu c}$ about $\langle\Theta_{\mu c}\rangle_\mu$ is governed chiefly by the distribution of the $\Gamma_{\mu c}$ which we expect to follow a χ^2 distribution with one degree of freedom at low Γ/D and with between one and two degrees of freedom at larger Γ/D. In general, when $\Theta_{\mu c}$ follows a χ^2 distribution with ν_c degrees of freedom, the first term in Eq. (23) can be evaluated in terms of the following integral[11]

$$\frac{\langle\Theta_{\mu c}\Theta_{\mu c'}/\Theta_\mu\rangle_\mu}{\langle\Theta_{\mu c}\rangle_\mu\langle\Theta_{\mu c'}\rangle_\mu/\langle\Theta_\mu\rangle_\mu}=\int_0^\infty\frac{dt}{f_cf_{c'}\prod_{\text{all }c''}f_{c''}{}^{\nu_{c''}/2}}, \tag{26}$$

where

$$f_c=1+\frac{2}{\nu_c}\frac{\langle\Theta_{\mu c}\rangle_\mu}{\langle\Theta_\mu\rangle_\mu}t.$$

As the number of competing channels becomes large, the above integral is expected to go to unity, and at the same time Γ/D is expected to become large so that Φ_0 approaches unity. In this continuum limit we then have

$$\sigma_{cc'}{}^{\text{fluctuation}}\to\pi\lambda\!\!\!^{-}{}_c^2\frac{T_c{}^{\text{compound}}T_{c'}{}^{\text{compound}}}{\sum_{c''}T_{c''}{}^{\text{compound}}}, \tag{27}$$

which defines the meaning and conditions of validity of the Hauser–Feshbach formula (2).

It is often assumed, and likely to be true at low energies, that the width amplitudes for different channels are uncorrelated, in which case we may write

$$\langle g_{\mu c}g_{\mu c'}\rangle_\mu=\delta_{cc'}\langle g_{\mu c}^2\rangle_\mu=\delta_{cc'}b_c\langle N_\mu\Gamma_{\mu c}\rangle, \tag{28}$$

where b_c is a generally complex number whose magnitude varies between unity for small Γ/D and zero if the $g_{\mu c}$ are distributed isotropically in the complex plane. The assumption (28) means that the relations (13) hold for $c\neq c'$. If we also assume that

$$N_\mu\approx\langle N_\mu\rangle_\mu\equiv N, \tag{29}$$

we can write

$$\sigma_{cc'}{}^{\text{correlation}}=\pi\lambda\!\!\!^{-}{}_c^2\delta_{cc'}(|b_c|^2/4N^2)\langle\Theta_{\mu c}\rangle_\mu^2, \tag{30}$$

$$\sigma_{cc'}{}^{\text{fluctuation}}=\pi\lambda\!\!\!^{-}{}_c^2\{\langle\Theta_{\mu c}\Theta_{\mu c'}/\Theta_\mu\rangle_\mu-\delta_{cc'}\tfrac{1}{4}Q_c\langle\Theta_{\mu c}\rangle_\mu^2\}, \tag{31}$$

$$\langle\Theta_{\mu c}\rangle_\mu=(2/Q_c)[1-(1-Q_cT_c{}^{\text{compound}})^{\frac{1}{2}}], \tag{32}$$

where

$$Q_c=2\,|\,b_c\,|^2(1-\Phi_0)/N^2. \tag{33}$$

Clearly $\langle\Theta_{\mu c}\rangle_\mu\to T_c{}^{\text{compound}}$ when either $T_c{}^{\text{compound}}$ or Q_c is small. The former condition is expected to hold near the threshold of channel c giving rise to the relation (6), the latter for large Γ/D, leading again to Eq. (27).

It should be noted that even in the limiting case of Eq. (27), the assumption of statistical independence of the average decay probabilities governs the magnitude of the fluctuation cross section and that the average compound-nucleus cross section (11a) exceeds the fluctuation cross section by the amount of the correlation cross section for those decay channels whose amplitudes are correlated with the entrance channel amplitudes. This preference for decay into correlated channels is due to the interference effects which were discussed by Professor Breit.[24] Among these preferred decay channels will ordinarily be the entrance channel itself. However, when the cross-section decomposition (12) is used, then the enhancement $\sigma_{cc}{}^{\text{correlation}}$ of the compound elastic scattering cross section is included in the optical-model scattering cross section $\sigma_{cc}{}^{\text{optical}}$, even in the case of the conventional single channel optical model. In fact, $\sigma_{cc}{}^{\text{correlation}}$ is just the difference between what is meant by the optical-model shape elastic scattering cross section and the potential scatter-cross section of resonance theory.

Observable integrated cross sections are obtained by summing the above partial cross sections over appropriate exit channels and, averaging over entrance channels using $g_\alpha{}^J$ as a weighting factor as in Eqs. (2) and (3). The differential fluctuation cross section is given by

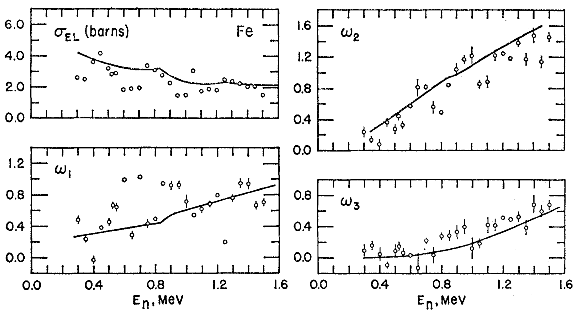
$$\frac{d\sigma_{\alpha\alpha'}{}^{\text{fluctuation}}}{d\Omega}=\frac{\lambda\!\!\!^{-}{}_\alpha^2}{4(2I_\alpha+1)(2i_\alpha+1)}\sum_{cc'L}P_L(\cos\theta)$$
$$\times\Bigg\{(-1)^{s-s'}\bar{Z}(lJlJ,sL)\bar{Z}(l'Jl'J,s'L)\left\langle\frac{\Theta_{\mu c}\Theta_{\mu c'}}{\Theta_\mu}\right\rangle_\mu$$
$$+\delta_{\alpha\alpha'}\delta_{ss'}\bar{Z}^2(lJl'J',sL)\Bigg[(1-\delta_{cc'})\Bigg(\left\langle\frac{\Theta_{\mu c}\Theta_{\mu c'}}{\Theta_\mu}\right\rangle_\mu$$
$$+\frac{2\pi}{D}\operatorname{Re}\left\langle\frac{g_{\mu c}^2g_{\mu c'}{}^{*2}}{\Gamma_\mu}\right\rangle_\mu\Bigg)$$
$$-\operatorname{Re}b_cb_{c'}{}^*\frac{(1-\Phi_0)}{2N^2}\langle\Theta_{\mu c}\rangle_\mu\langle\Theta_{\mu c'}\rangle_\mu\Bigg]\Bigg\}, \tag{34}$$

[24] G. Breit, *Definitions of Compound States*, Topical Conference on Compound Nuclear States, Gatlinburg, Tennessee, 10 October 1963; and *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1959), Vol. 41/1.

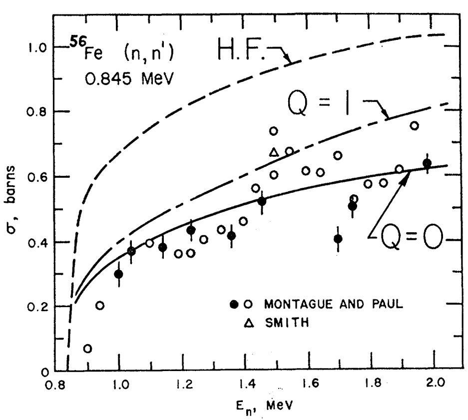
when the condition (28) is satisfied. The first term in the brackets is analogous to Eq. (1). The remainder of the expression affects only elastic processes and does not necessarily vanish even in the limit of large Γ/D.

The above methods may also be used to calculate cross-section fluctuations. These results are reported elsewhere.[18,23]

We finally turn to some calculations in which consequences of some of the above formulas are compared with measured low-energy neutron elastic and inelastic scattering cross sections in iron, copper, zirconium, and niobium. The procedure used was to obtain first good optical-model fits to the elastic scattering data of Smith and collaborators.[25–27] These fits were based on the low-energy neutron optical model of Moldauer.[28] Deviations from that model are indicated in the captions to Figs. 1(a), 2(a), 3(a), 4(a), where the calculations are compared with the measured total elastic scattering cross sections σ(ELASTIC) and the



(a)



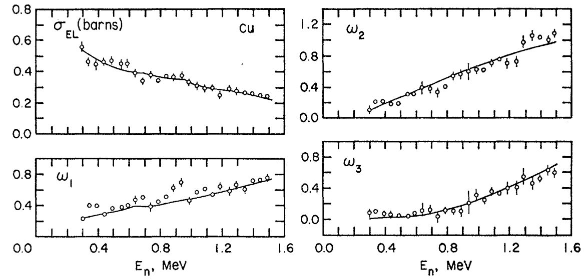
(b)

FIG. 1. (a) Elastic scattering of neutrons by iron. Data from Ref. 25, calculations by optical model of Ref. 28 with the substitution $R=4.8$ F. (b) Inelastic neutron scattering to the 0.845-MeV level in ^{56}Fe. Data from Refs. 25, 29. Calculations as described in the text based on the same optical model as in Fig. 1(a).

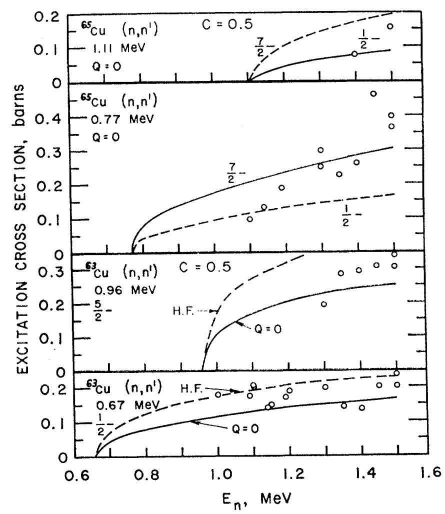
[25] A. B. Smith, "Scattering of Fast Neutrons from Iron" (to be published).

[26] A. B. Smith, C. A. Engelbrecht, and D. Reitmann, "Elastic and Inelastic Scattering of Fast Neutrons from Co, Cu, and Zn" (to be published).

[27] D. Reitmann, C. A. Engelbrecht, and A. B. Smith, Nucl. Phys. **48**, 593 (1963).



(a)



(b)

FIG. 2. (a) Elastic scattering of neutrons by copper. Data from Ref. 26, calculation by optical model of Ref. 28. (b) Inelastic neutron scattering to the first two excited states in each of the stable isotopes of copper. Data from Ref. 26. Calculations as described in text based on optical model of Ref. 28 with spin and parity assignments as shown.

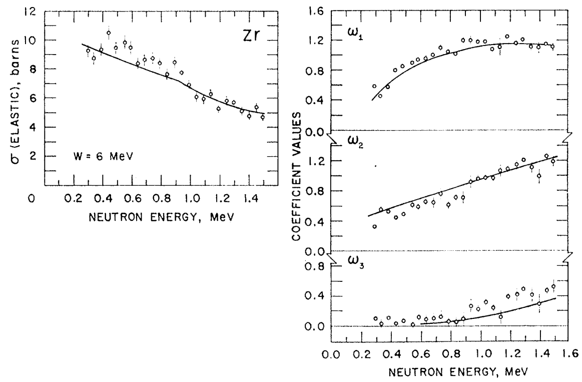
Legendre polynomial expansion coefficients ω_L of the angular distribution defined by

$$d\sigma(EL)/d\Omega=[\sigma(EL)/4\pi]\sum_L \omega_L P_L(\cos\theta). \quad (35)$$

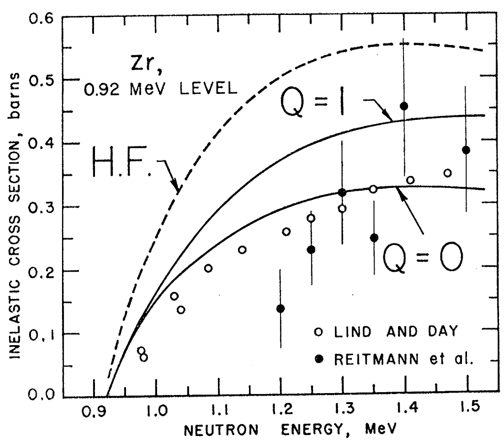
Particular attention was paid to energies below inelastic thresholds where all of the optical-model absorption cross section was assumed to produce elastic fluctuation scattering, the angular distribution of which was calculated by means of the formulas in the Appendix of Ref. 28.

The transmission coefficients of these optical models were then used in Eqs. (31) and (32) to calculate partial inelastic scattering cross sections in the respective nuclides. It was assumed that no direct or correlation cross sections contribute to the inelastic scattering and that all $\Theta_{\mu c}$ have a Porter–Thomas distribution. In general, calculations were carried out for the cases of all $Q_c=0$ and all $Q_c=1$. The latter value would be approximately applicable for most of the important

[28] P. A. Moldauer, Nucl. Phys. **47**, 65 (1963).



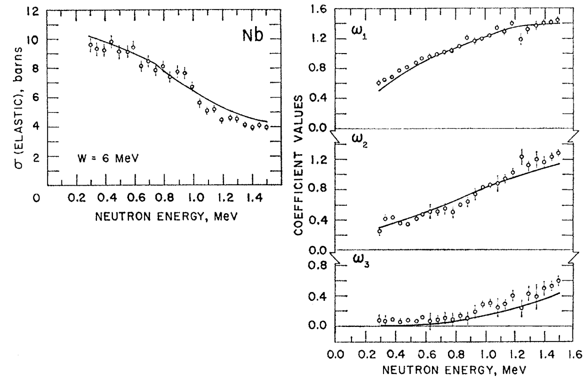
(a)



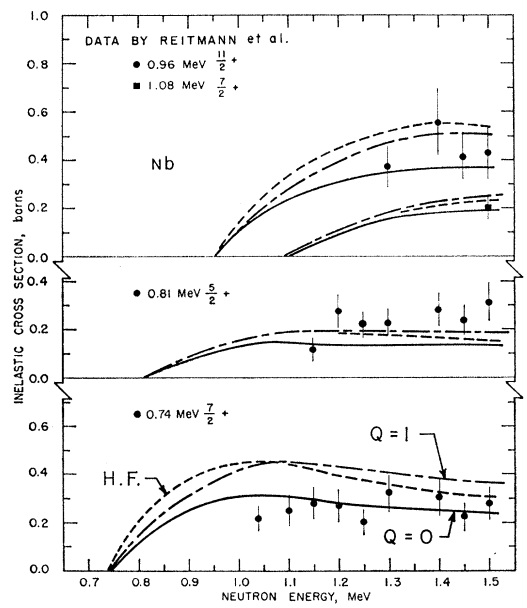
(b)

Fig. 3. (a) Elastic scattering of neutrons by zirconium. Data from Ref. 27, calculations by optical model of Ref. 28 with the substitution $W=6$ MeV. (b) Combined inelastic neutron scattering to the excited levels near 0.92 MeV in all isotopes of zirconium. Data from Refs. 27, 30. Calculations as described in text based on same optical model as in Fig. 3(a).

channels in these reactions if $|b_c|^2/N^2$ were unity and if the full Wigner resonance level repulsion applied.[20] The results of these calculations, together with the predictions of Eq. (2) (labeled H.F.) are shown in Figs. 1(b), 2(b), 3(b), 4(b), and are compared there with the measurements of Smith and collaborators,[25–27] of Montague and Paul,[29] and of Lind and Day.[30] The results suggest that the neutron partial-widths are uncorrelated in this region and that their distribution



(a)



(b)

Fig. 4. (a) Elastic scattering of neutrons by niobium. Data from Ref. 27, calculations by optical model of Ref. 28 with the substitution $W=6$ MeV. (b) Inelastic neutron scattering to the first four excited states in niobium. Data from Ref. 27, calculations as described in text based on same optical model as in Fig. 4(a).

is close to that suggested by Porter and Thomas. Any serious deviations from these conditions would cause an increase in the values of most of the inelastic cross sections. The comparison also suggests that for the important neutron channels in this region, Q_c is close to zero for zirconium and niobium and small for iron and copper.

[29] J. H. Montague and E. B. Paul, Nucl. Phys. **30**, 93 (1962).
[30] D. A. Lind and R. B. Day, Ann. Phys. (N.Y.) **12**, 485 (1961).