Unified Theory of Nuclear Reactions*

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Archimedes is reputed to have said "Give me a place to stand and I will move the earth." This paper may be expressed in the paraphrase: "Given the appropriate projection operator one can solve the problems of nuclear structure and reactions." The development of this theme is contained in two papers[1] bearing the same title as this paper. The use of the word unified in the title is justified in two senses. It is shown that the method contains the formalism of Wigner and Eisenbud[2] and a generalized version of that of Kapur and Peierls.[3] Secondly, it presents a treatment from which direct interactions and compound nuclear processes are easily and naturally abstracted. Related discussions have been given by Newton and Fonda[4] and by Agodi and Eberle.[5] The applicability of the method to gamma-ray processes[6] has been shown. We shall only briefly review the principal ideas. Our main interest for the first part of this paper will be to discuss how the resultant framework may be exploited toward the calculation of various nuclear reactions. So far, these procedures have been applied to the calculation of neutron scattering and strength functions for the pure elastic case.[7-9] Calculations involving inelastic scattering are in progress.[10]

We are concerned with a reaction in which we proceed from an initial state consisting of the incident particle (which can be composite) and the target nuclei in its ground state to an emergent particle (or particles) plus a residual nucleus in a ground or excited state. The corresponding wavefunctions are

$$|i\rangle=\mathcal{A}[\chi_0(r_0)\phi_0(\mathbf{r}_1\cdots\mathbf{r}_A)], \tag{1}$$

$$|f\rangle=\mathcal{A}[\chi(\mathbf{r}_0\cdots\mathbf{r}_k)\phi(\mathbf{r}_{k+1}\cdots\mathbf{r}_A)], \tag{2}$$

where $\mathcal{A}$ is the antisymmetrization operator. The projection operator P is defined in terms of the possible residual nuclear states: Namely, taking an arbitrary antisymmetric function Φ,

$$P\Phi(r_0\cdots r_A)\rightarrow\mathcal{A}\{v_0(r_0)\phi_0(\mathbf{r}_1\cdots\mathbf{r}_A)+v_1(\mathbf{r}_0)\phi_1(\mathbf{r}_1\cdots\mathbf{r}_A)+\cdots+w_0(\mathbf{r}_0\mathbf{r}_1)\phi_0(\mathbf{r}_2\cdots\mathbf{r}_{A-1})+w_1(\mathbf{r}_0\mathbf{r}_1)\phi_1(\mathbf{r}_2\cdots\mathbf{r}_{A-1})\cdots\}. \tag{3}$$

The arrow refers to that part of configuration space in which the components are no longer interacting. Most of our ideas can be developed using a P which projects out all the open channels in the sum on the right-hand side of (3), all closed channels being excluded. Open channels are defined as those reactions which are energetically allowed, i.e., the amplitudes of ϕ_0 etc. are finite, or at worse decrease as $(1/r)$ in the asymptotic region. There are some occasions for which one might want to restrict P to only the elastic channel, or particularly near threshold to include some closed channels in P. These, however, are minor tactical variations and are not of interest to us at present. So let P be the projection on the open channel subspace defined by the possible residual nucleus states. We emphasize that P is not uniquely defined by Eq. (3). There are many P. For example, for the Wigner–Eisenbud theory

$$\begin{aligned} P&=1 \quad \text{outside } R,\\ &=0 \quad \text{inside } R, \end{aligned} \tag{4}$$

where R marks a region in configuration space outside of which there are no interactions between the component systems. But there are other possibilities for P and the selection of the best one is a point to which we shall return later.

For the moment, however, we shall ignore this issue and see where the introduction of the P operator leads. If Ψ is the wavefunction for the entire system, the function $P\Psi$ contains all the amplitudes which are observable at infinity so that all we need for comparison with experiment is to determine $P\Psi$.

The equation for $P\Psi$ is obtained directly from the Schrödinger equation. We find

$$[E-H_{PP}-H_{PQ}(E-H_{QQ})^{-1}H_{QP}](P\Psi)=0,$$

$$PHP\equiv H_{PP}, \qquad PHQ\equiv H_{PQ}.$$

We see that there is an effective potential—or, if you wish, an effective index of refraction—which is energy dependent. This energy dependence is associated with a time delay, as is indicated by the propagator $(E-H_{QQ})^{-1}$. The very long time delays are associated with the poles of this propagator. In the situation where one pole dominates the energy dependence, the case

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[5] A. Agodi and E. Eberle, Nuovo Cimento **18**, 718 (1960).
[6] L. Estrada and H. Feshbach, Ann. Phys. (N. Y.) **23**, 123 (1963).
[7] B. Block and H. Feshbach, Ann. Phys. (N. Y.) **23**, 47 (1963).
[8] C. M. Shakin, Ann. Phys. **22**, No. 3 (1963).
[9] C. M. Shakin and R. Lemmer (to be published).
[10] C. M. Shakin and R. Lemmer (private communication).

of nonoverlapping resonances (the weak coupling case of Lemmer and Shakin), we can separate out this pole term, i.e., $H_{\text{eff}}=H'+$pole term in which H' varies only slowly with energy. We then find that the transition amplitude consists of two terms

$$T(f\mid i)=T_{NR}(f\mid i)+\frac{\langle\psi_f^{(-)}H_{PQ}\Phi_S\rangle\langle\Phi_S H_{QP}\psi_i^{(+)}\rangle}{E-E_S+i\Gamma_{S/2}}, \tag{5}$$

with

$$\Gamma_S=2\pi\int\sum_{\mu}\mid\langle\psi_f^{(-)}H_{PQ}\Phi_S\rangle\mid^2 d\Omega, \tag{6}$$

$$H_{QQ}\Phi_S=\mathcal{E}_S\Phi_S,\qquad E_S=\mathcal{E}_S+\Delta_S, \tag{7}$$

$$(E-H')\psi_{i,f}^{(\pm)}=0.$$

This is an *exact* expression. The sum here is over all possible exit channels, and defines the width for emission into a given solid angle. Note that these expressions are independent of the angular momentum coupling scheme. One can now insert channel spin scheme of Wigner and Eisenbud or the chirality scheme of Jacob and Wick[11] and so on. T_{NR} gives the nonresonant amplitude associated with H'. It therefore contains the direct inelastic and potential scattering processes. The extension of this expression to the case of overlapping levels (Lemmer and Shakin's strong coupling case) can and has been made.[1,2] We turn now to the matter of choosing the correct P. We emphasize that up to this point our results are independent of P except for one condition. It is necessary for the validity of (5) that the effective Hamiltonian be short ranged, i.e., decrease more rapidly than $(1/r)$ except for the known Coulomb term. But we also have some empirical requirements. For example, for low-energy elastic neutron scattering, the potential scattering is substantially given by the shell model potential extrapolated to positive energies. This must be modified, of course, when dealing with aspherical nuclei or situations in which vibrational or other collective modes of excitation are important, where a more complete model Hamiltonian must be used. These are cases in which it pays to place some of the closed channels into the domain spanned by P. In the simpler case then, empirically $H'=T+V_{\text{shell}}$.

What about H_{PQ}? The operator H_{PP} did not involve any excitation of closed channels. The operator H_{PQ} does involve such excitations and it is thus to be identified with the deviation of H_{eff} from the average field. The latter does not produce any excitation. This suggests that H_{PQ} and the residual interaction are directly related:

$$H_{PQ}\simeq PV_RQ.$$

Finally, we turn to H_{QQ}. Here we are dealing with the closed channels—all transitions to the open channels are forbidden—and because the channels are closed, the wavefunctions for physical energies of interest decay exponentially at infinity. We are interested in the bound eigenstates of H_{QQ}. We may, for example, employ the full shell model potential here for H_{QQ}, just making certain that transitions to the states included in P are omitted. Shakin and Lemmer use the harmonic oscillator well with a pairing force.

With these assumptions, we have then a defined problem whose solutions will give the various desired reaction amplitudes and one which is as solvable as the shell model. These assumptions can and should, of course, be modified for different nuclei. We have already alluded to one such possibility. But the general illustration which we have given indicates how one employs the empirical knowledge generally available to add to the kinematics of the formalism, the dynamics required to make predictions of reaction amplitudes. It is probably unnecessary to add that these remarks do not pretend to be a solution of the problem of what P is. We are hard at work on this problem. Our point, rather, is that our formalism is so constituted that one can make use of empirical data to make further explorations into nuclear structure.

The first application of these ideas were made by Block,[7] later by Shakin[8] in their calculation of the s-wave strength function for thermal neutrons. We will not review these papers here but will restrict our discussion to the concept of *doorway states* and to an important experimental possibility. Briefly, we note that if we make the critical assumption that V_R is a sum of two-body potentials, and if we describe the target nucleus in terms of a shell model, that $V_R\psi_i^{(+)}$ will contain only two-particle–one-hole states or three quasiparticle states. These are the *doorway states*, for only when there is at least one such state of appropriate energy will the process of forming the compound state proceed. It is through this same transition that the compound state decays. In the expression for the width $\mid\langle\Phi_S V_R\psi_i^{(+)}\rangle\mid^2$ we see that only the three-quasiparticle part of Φ_S counts. We first consider the simplest (and most unusual) case for which there is only one three-quasiparticle state χ_n with energy E_n close to E. Then $\Gamma_S=2\pi\mid a_{Sn}\mid^2\mid\langle\chi_n V_R\psi_i^{(+)}\rangle\mid^2$. $\mid a_{Sn}\mid^2$ has a Lorentzian shape decreasing as the difference between $\mathcal{E}_S$ and E_n increases. On the wings its decrease is more rapid than Lorentzian. The width Δ_{nS}, associated with this Lorentzian form, is just the width of the three-quasiparticle state against decay into more complex configurations.

In a more realistic case, there are several three-quasiparticle states, i.e., several doorway states which are within each other's width. This gives rise to a structure in the strength function associated with the consequent overlap which can be observed by examining the energy dependence of the strength function. To begin with, there is of course the simplest structure,

[11] G. Jacobs and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

the giant resonance. But, in addition, the giant resonance itself will have structure which will depend upon the number of three-quasiparticle states which at that energy can serve as doorway states. It is this structure which was described in Block's paper and which has recently been discussed by Kerman, Rodberg, and Young.[12] The widths associated with the structure can be estimated from Shakin's calculation, and for tin in the thermal region are of the order of 100 keV. It may be that this structure has already been observed. For example, Kerman *et al.* point to an old experiment of Schieffer, Moore, and Class[13] in 1956 on ^{58}Ni+p. We would also expect structure for states which are observed by stripping processes. There is no need for us to comment on the need for further experiments to examine this conjecture.

We turn now to the last section of this paper. This matter was discussed in some part in our paper in Padua,[14] but we have made a little progress since that time. Consider now the direct interactions. Since we are, in general, dealing with several, say, N open channels, the effective Schrödinger equation for $\psi^{(+)}$ will be a set of N coupled equations. Rather than considering the nonresonant terms only, we shall add on the proper average of the resonance terms and then the N equations become modified by the addition of complex diagonal and coupling potentials. These complex terms will have structure effects as discussed just above, i.e., they will be energy-dependent. It is also clear that even in the absence of this compound nuclear effect, there will be structure arising from the coupling between the channels whose nature depends very much upon the strength of the coupling terms, and on the number of such coupled equations. The maxima in the cross sections correspond to a long time delay, though it will generally not be so long as to approach that of a narrow compound nuclear state. This time delay may be thought of as occurring when it is possible for the system to move from one open channel to another without actually decaying. Or, stated in another way, there are combinations of the open channel wavefunctions which in the nuclear interior are long lived. At the appropriate energy this combination is the principal one excited and therefore the long time delay and the maximum in the cross section. These are the generalization of the doorway states to the continuum.

Furthermore, suppose the incident channel is coupled strongly to ν other channels either directly, or, if not directly, via only a few intermediate strong links. Then we shall find that the density of maxima in the elastic cross section will go up by at least a factor of ν. When ν is small the various cross sections will show structure which is characteristic of a few degrees of freedom. The maxima will be sparse, and between them we would have regions in which the cross section would vary slowly with the energy. The statistical hypothesis will fail and there will be asymmetric angular distributions, polarization, and so on. Even though we may be dealing with a heavy target nucleus, this part of the cross section will behave very much like those of light nuclei at relatively low energies of excitation for which the compound nuclear density is low. Because this is a type of direct interaction, and since some channels are weakly coupled to others, the independence hypothesis may be expected to fail, i.e., the relative probability for the formation of two final states will depend on the nature of the incident particle, and not only on some appropriately chosen excitation energy. As the density of open channels increases, these statements need to be modified. Particularly if there are a large number of channels with emergent particles of very low energy, we may expect very narrow maxima and a return to approximate validity of the statistical and independence hypothoses. It is in this region that Ericson's statistical theory of fluctuations should apply, although our fluctuations are not of the same origin as the ones Ericson is treating. Ericson's fluctuations are connected with the compound nucleus and are related, in the present formulation, to the energy dependence of the complex part of the potentials in the coupled equations. But, where the random phase hypothesis fails, his theory does not apply and how his predictions are to be modified is not as yet clear. We can of course count the number of open channels, and at least *a posteriori* see which are closely coupled and which are not. On a purely statistical basis we can also estimate that the width of the maxima will follow a distribution away from the average similiar to Wigner semicircle law, so that there will be an appreciable number of widths as narrow as the single particle width divided by $(\nu)^{\frac{1}{2}}$, i.e., $\Gamma_{s.p}/(\nu)^{\frac{1}{2}}$ and even a few as small as $\Gamma_{s.p}/\nu$. Of course, as anyone who has worked with coupled channels knows, occasionally very narrow maxima can occur. The observation of these phenomena experimentally would, of course, be of great interest.

[12] A. Kerman, L. Rodberg, and J. Young, Phys. Rev. Letters (to be published).
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[14] H. Feshbach, in *Proceeding of the Conference on Direct Interactions and Nuclear Reaction Mechanisms September* 1962 (Gordon and Breach Science Publishers, Inc., New York, 1962).