

# Tables of Clebsch-Gordan Coefficients of $SU_3$

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## INTRODUCTION

The group  $SU_3$  has lately been of considerable interest in regard to the description of elementary particle symmetries.<sup>1,2</sup> Especially a crucial experiment, the finding of the  $\Omega^-$ , seems indicative of the relevance of  $SU_3$ .<sup>3</sup>

Our original motivation in compiling these tables of the Clebsch-Gordan coefficients of  $SU_3$  resulted from the observation that many of our calculations on elementary particle processes which required some labor, if attacked directly using tensorial or projection operator methods, turned out to be almost trivial if one had already a complete set of tables of the Clebsch-Gordan coefficients. For this reason, and since there are no other publicly circulated tables, we publish our tables here, hopefully in a form which permits efficient use. Our purpose in the discussion below is just to present the tables with a summary of the important properties of the coefficients necessary to using the tables. For a more detailed discussion of the properties of the Clebsch-Gordan coefficients of  $SU_3$  see the review of de Swart.<sup>4</sup>

## TABLES

These tables were constructed primarily from the isoscalar factors given by de Swart.<sup>4,5</sup> The tables were extensively checked using the fact that each submatrix is unitary. The  $8 \otimes 8$  tables were also checked against a table constructed by Oakes<sup>6</sup> using tensorial methods. Spot checks were also made using  $U$ -spin and  $V$ -spin operators.

Our states are labeled in conformance with the usage of  $SU_3$  as an elementary particle symmetry,

$$| \mathbf{N}, Y, I, I_3 \rangle,$$

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<sup>1</sup> For a discussion of the relevance of  $SU_3$  to elementary particle symmetries see M. Gell-Mann, California Institute of Technology Report, CTSL-20 (unpublished) and Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

<sup>2</sup> For a review of the properties of Lie groups aimed at their use in elementary particle symmetries see R. E. Behrends, J. Dreitlein, C. Fronsdal, and B. W. Lee, Rev. Mod. Phys. **34**, 1 (1962).

<sup>3</sup> V. E. Barnes, P. I. Connolly, D. J. Crennell, B. B. Culwick, W. C. Delaney, *et al.*, Phys. Rev. Letters **12**, 204 (1964).

<sup>4</sup> J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963). In preparing our tables, three misprints in de Swart's tables were found and corrected.

<sup>5</sup> There are other tables of isoscalar factors available, to mention: A. R. Edwards, Proc. Roy. Soc. (London) **A268**, 567 (1962); M. A. Rashid, Nuovo Cimento **26**, 118 (1962); Y. Dothan and H. Harari, Israel Atomic Energy Commission Report IA-777 (unpublished). The reader should be warned of the fact that conventions for  $SU_3$  are not yet standardized and none of the sets of conventions in Refs. 4 and 5 entirely agree.

<sup>6</sup> R. J. Oakes (private communication).

where  $\mathbf{N}$  = dimension of the representation,  $Y$  = hypercharge,  $I$  = total isospin,  $I_3 = Z$  component of isospin. The tables are in the form

$n \otimes n'$	$\mathbf{N} \dots$
	$Y \dots$
	$I \dots$
	$I_3 \dots$
$y \ i \ i_3 \quad y' \ i' \ i'_3$	

and each entry is the (Clebsch-Gordan) coefficient for the decomposition of  $| \mathbf{N}, Y, I, I_3 \rangle$  into  $| \mathbf{n} \ y \ i \ i_3 \rangle \otimes$

TABLE I. Appropriate signs for reversal of order of the states.

$\mathbf{N}$ $\mathbf{n} \otimes \mathbf{n}'$	1	8	8'	10	$\bar{10}$	27	27'	35	$\bar{35}$	64
$8 \otimes 8$	1	1	-1	-1	-1	1				
$8 \otimes 10$		1		-1		-1		1		
$8 \otimes 27$		1		1	1	1	-1	-1	-1	1
$10 \otimes \bar{10}$	-1	1				-1				1

$| \mathbf{n}' \ y' \ i' \ i'_3 \rangle$ . Tables are presented here for the following Clebsch-Gordan series

Table II:  $8 \otimes 8 = 27 \oplus 10 \oplus \bar{10} \oplus 8 \oplus 8' \oplus 1$

Table III:  $8 \otimes 10 = 35 \oplus 27 \oplus 10 \oplus 8$

Table IV:  $8 \otimes 27 = 64 \oplus 35 \oplus \bar{35} \oplus 27 \oplus 27' \oplus 10 \oplus \bar{10} \oplus 8$

Table V:  $10 \otimes \bar{10} = 64 \oplus 27 \oplus 8 \oplus 1$ .

Where there are two representations, e.g.,  $8$  and  $8'$ , in an outer product we have distinguished them, as is conventional, by  $R$  symmetry.<sup>1</sup>

## SYMMETRY PROPERTIES

The complex conjugate of a state can be determined from

$$| \mathbf{N} \ Y \ I \ I_3 \rangle = (-1)^{I_3+Y/2} | \bar{\mathbf{N}} \ -Y \ I \ -I_3 \rangle.$$

Often, e.g., in calculating crossing-matrices, one is interested in the Clebsch-Gordan coefficient with the states in reversed order. This reversal of order introduces a sign of  $\pm 1$ . Table I lists the relevant signs.









































### PHYSICAL STATES

The states we use enjoy the standard (Condon and Shortley<sup>7</sup>)  $SU_2$  phase conventions. This means that not all of the mathematical states will have the phases which would be most convenient for  $SU_3$ . For example, it would be convenient for the assignment of the pseudoscalar mesons to an octet to be such that the octet was self-adjoint. But because of our convention for the complex conjugate state there will be some minus signs which enter. These minus signs would be compensated if we made the following assignment of physical states to our formal states:

$$\begin{aligned} |K^0\rangle &= |\mathbf{8} \ 1 \ \frac{1}{2} \ -\frac{1}{2}\rangle, & |K^+\rangle &= |\mathbf{8} \ 1 \ \frac{1}{2} \ \frac{1}{2}\rangle \\ |\pi^-\rangle &= |\mathbf{8} \ 0 \ 1 \ -1\rangle, & |\pi^0\rangle &= |\mathbf{8} \ 0 \ 1 \ 0\rangle, \\ |\pi^+\rangle &= -|\mathbf{8} \ 0 \ 1 \ 1\rangle, & |\eta\rangle &= |\mathbf{8} \ 0 \ 0 \ 0\rangle, \\ |K^-\rangle &= -|\mathbf{8} \ -1 \ \frac{1}{2} \ -\frac{1}{2}\rangle, & |K^0\rangle &= |\mathbf{8} \ -1 \ \frac{1}{2} \ \frac{1}{2}\rangle. \end{aligned}$$

This difference of assignment is more clear if we examine the unitary singlet resulting from the product  $\mathbf{8} \otimes \mathbf{8}$ . As the appropriate column of Table II shows,

<sup>7</sup> E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, Cambridge, England, 1935).

there are a number of minus signs which occur when  $|1 \ 0 \ 0 \ 0\rangle$  is written as a sum of products  $|\mathbf{8} \ y \ i \ i_3\rangle \otimes |\mathbf{8} \ -y \ i \ -i_3\rangle$ . These minus signs would also be compensated by the assignment above, making the unitary singlet a completely symmetric combination of the physical states. Since under the usual conventions for  $SU_3$  tensors the unitary singlet is also completely symmetric, we see our assignment above is just the relation between our  $SU_2$ -convention states and the usual  $SU_3$  tensors.

For many applications it is more convenient to simply make the assignment of physical states directly to the mathematical states with all plus signs and to remember that the octet, indeed even the  $SU_2$  triplet ( $\pi$ ), is not manifestly self-adjoint.

*Note added in proof.* Dr. P. Tarjanne has kindly informed us of his tables of isoscalar factors and Clebsch-Gordan coefficients available as Carnegie Institute of Technology Reports NYO-9290 and NYO-9290A (unpublished).

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