Variations in the Earth's Upper Atmosphere as Revealed by Satellite Drag

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1. INTRODUCTION

NEARLY all we know today about the structure and the variations of the atmosphere above the height of 200 km has been obtained by analyzing the secular decrease in period of a number of artificial satellites. There are, of course, more direct methods for determining atmospheric densities, such as measuring the aerodynamic pressure—or a parameter related to it—on the satellite by means of a gauge that transmits signals to ground stations. Gauges of this kind flown on Sputnik III (Mikhnevich *et al.* 1961) and on the Samos II satellite (Sharp *et al.* 1962) yielded satisfactory, though limited, results, which have provided a valuable check on the data derived from satellite motion.

Most satellites, however, do not have such a gauge aboard. Even if they did, it is doubtful whether the telemetered material would be accurate and continuous enough. Furthermore, it certainly would at first be a very difficult task to make sense out of an array of data collected at widely different heights and times all over the globe. Finally, in view of the limited sensitivity of such gauges, the results would be reliable only at relatively low heights. The gauges on the two aforementioned satellites recorded pressures only up to 550 km, even though we know that orbital changes caused by atmospheric drag can be measured when the perigee of a satellite is 1000 km and more above the surface of the earth.

Atmospheric drag is one of several perturbing forces acting on a satellite in motion. By "perturbing force" I mean any force that has the effect of deflecting the satellite from the Keplerian orbit that a body would describe in the idealized two-body problem in which the bodies are point masses. Perturbing forces arise from the irregular shape of the earth, from the pressure of solar radiation, and from the attraction of the nearest celestial bodies. All these forces can cause both short-periodic and long-periodic, as well as secular, perturbations. (Perturbations are defined as short-periodic when they have characteristic periods equal to the period of revolution or a submultiple of it; as long-periodic when they have characteristic periods that are long compared with the period of revolution; and as secular when they monotonically increase with time.)

In view of the different ways in which these perturbing forces act on a satellite, their effects on a specific orbital element bear very little relation to their relative intensities. This, as we shall see, is a very fortunate circumstance for the determination of air drag. The perturbing forces arising from the departure of the earth's figure from spherical symmetry are of the order of 1 dyne, while the atmospheric drag of the Vanguard satellite is of the order of 10^{-5} dyn. Yet, the atmospheric drag on Vanguard I produces a very noticeable secular decrease in the semimajor axis of the orbit, while the gravitational anomalies leave the major axis quite undisturbed. As a matter of fact, Kozai (1959a, 1959b) and Brouwer (1959) have shown that gravitational perturbations, whether arising from the earth's figure or from the attraction of the sun and the moon, never produce any appreciable long-periodic perturbations upon the orbital semimajor axis (and, therefore, upon the anomalistic period) of a close earth satellite. Thus, provided that in the analysis of the period variations we do not go to time intervals smaller than one revolution of the satellite, we are left with only two contending effects: atmospheric drag and solar radiation pressure. The separation of these two effects is an easy matter when we have the right program to compute radiation pressure perturbations (Musen 1960; Parkinson, Jones, and Shapiro 1960; Kozai 1961a; Bryant 1961). We assume here that we have to deal with satellite observations accurate to not more than 1′ in position and $0^s.01$ in time, which do not reveal the small 12-h oscillations in satellite motion caused by the ellipticity of the equator, and other "beat" periods of gravity anomalies (Izsak 1961; Kaula 1961; Kozai 1961b). In high-precision work with accurate photographic observations, these perturbations must also be taken into account.

2. GRAVITATIONAL PERTURBATIONS

The earth's gravitational potential varies from point to point in space and can be represented by a series of tesseral harmonics. The rotation of the earth under the satellite orbit has the result of smoothing the effect of the irregularities in longitude on the orbit, so that zonal harmonics can be used to a good degree of approximation. The gravitational potential U can then be expressed in the form

$$U = \frac{GM_\oplus}{r}\left[1 - \sum_{n=2}^{\infty} \frac{J_n}{r^n} P_n(\sin\delta)\right], \qquad (1)$$

where G is the gravitational constant; $M_\oplus$, the mass of the earth; r, the geocentric distance; P_n, the zonal harmonic of the nth order; δ, the declination or geocentric latitude; and the J_n's empirical coefficients.

The second-order term in the series, the "second harmonic," essentially represents the oblateness of the geoid, and is by far the largest "perturbing" term. Its presence causes large precessional motions in the orbit—both a rotation of the orbital plane and a rotation of the line of apsides. Thus, for example, in the orbit of the Vanguard I satellite the intersection of the orbit with the equatorial plane moves westward at the rate of 3°.0 a day, while the perigee moves eastward in the orbital plane at the rate of 4°.4 a day. These are the "secular" perturbations in satellite orbits; all the higher even harmonics contribute to them, although in a much smaller measure than does the second harmonic.

The presence of a nonnegligible third harmonic causes a slow change in the orbital elements, primarily in the eccentricity, with a period equal to that of the rotation of the line of apsides. Thus, for example, the perigee distance of Vanguard I oscillates by about 7 km with a period of 82 days (360°/4°.4). All the higher odd harmonics contribute, in a much smaller measure, to this oscillation.

Other perturbations, both long- and short-periodic, are caused by even as well as by odd harmonics. The short-periodic perturbations are relatively small; in Vanguard I they amount to not more than a couple of miles in the satellite's position.

An orbit-computing program must take into account short-periodic perturbations. Long-periodic perturbations, however, are quite conveniently incorporated in the unknowns that represent the variations of the elements, provided the observations used for computing the orbit fall within an interval that is short compared to that of the perturbations. By this procedure it is possible to treat in a purely empirical fashion also other causes of slow perturbations, such as radiation pressure and luni-solar attraction (which both produce negligible short-periodic perturbations), not to mention forces as yet unknown.

3. SOLAR RADIATION PRESSURE

Solar radiation pressure affects the orbital period when the satellite spends part of the time in the earth's shadow and the orbit is not exactly circular, which is commonly the case. For a relatively close satellite with a moderately eccentric orbit ($0.1 < e < 0.2$), the variations in period dP/dt caused by solar radiation pressure are of the order of $\pm 1 \times 10^{-7}\, A/m$, when the area/mass ratio A/m is expressed in cm^2/g. For comparison, the atmospheric drag at intermediate heights gives rise to a value of dP/dt of the order of $-1 \times 10^9\, \rho\, A/M$, where ρ is the atmospheric density in g/cm^3. Thus, when ρ is of the order of $10^{-16}\, g/cm^3$, the effect of solar radiation pressure may equal that of atmospheric drag. At times of sunspot maximum, this will occur at a height of 900 km; at times of low solar activity, however, when the atmosphere is appreciably contracted, it will occur as low as 500 km above the earth. If we want to determine atmospheric drag with a 10% accuracy or better, we must take into account solar radiation pressure whenever the perigee height of the satellite is greater than 400 km.

The effect of solar radiation pressure can be computed with reasonable accuracy when the satellite is spherical and the type of reflection (specular or diffuse) is known. For elongated, tumbling objects it is possible to envisage situations in which the effect computed under assumption of random orientation bears little resemblance to the real effect.

4. ATMOSPHERIC DRAG

Atmospheric drag produces both short-periodic and slow perturbations. For orbit computing it is again convenient to account empirically for the slow effects; the short-periodic effects can be computed by theory with good approximation once the size of the slow perturbations has been established. In a stationary atmosphere the drag force is exactly in the direction of the satellite's motion, so that the position of the orbital plane is not affected by it. In a rotating atmosphere all the elements are perturbed by drag; since, however, the velocity of rotation of the atmosphere is small compared to the orbital velocity of the satellite, all these perturbations are small compared to those in the orbital position, i.e., in the *mean anomaly*.

In a Keplerian orbit, if P is the anomalistic period, i.e., the time interval between two successive perigee passages, the anomalistic *mean motion* n (in revolution) is defined as $n = 1/P$, and the *mean anomaly* M as $M = M_0 + n(t - t_0)$, where M_0 is the value of M at the time $t = t_0$. The slow, or secular, effect of atmospheric drag is one in which P continually decreases or n continually increases.

The work performed by the aerodynamic force on an element ds of satellite orbit is

$$dW = \tfrac{1}{2} C_D A \rho v^2 ds \,, \tag{2}$$

where C_D is the drag coefficient, A the effective cross section of the satellite, ρ the atmospheric density, and v the satellite velocity with respect to the atmosphere. The work during a complete orbital revolution is

$$\Delta W = \oint_{\text{orbit}} dW \,,$$

and the change of orbital period ΔP is directly proportional to ΔW.

For a satellite of mass m moving in a rotating atmosphere, the explicit expression for ΔP assumes the form (Sterne 1958a)

$$\frac{\Delta P}{P} = \frac{dP}{dt} = -\tfrac{3}{2} C_D \frac{A}{m} \rho_p a \int_0^{2\pi} \frac{\rho}{\rho_p} \frac{(1 + e\cos E)^{\frac{3}{2}}}{(1 - e\cos E)^{\frac{1}{2}}} \times \left(1 - d\,\frac{1 - e\cos E}{1 + e\cos E}\right)^2 dE \,,$$

$$d = P\omega_s(1 - e^2)^{\frac{1}{2}} \cos i \,. \tag{3}$$

Here E is the eccentric anomaly of the satellite in its orbit; a, e, and i are the orbit's semimajor axis, eccentricity, and inclination, respectively; ρ_p is the atmospheric density at perigee height; and ω_s the angular velocity of atmospheric rotation.

5. PRESENTATION AREA A

For a moving sphere the presentation area is obviously constant and equal to one-fourth the total area. It can be shown (see, for example, van de Hulst 1957) that the *average* presentation area of a randomly oriented body is also equal to one-fourth the total area, provided the surface has no concavities. Unfortunately, cylindrical satellites rotating around a transverse axis with a period that is short compared with the period of revolution (as is generally the case) cannot be considered to be randomly oriented with respect to the direction of motion. If the motion of the cylinder with respect to the air is like that of an airplane propeller, the presentation area is the greatest possible and is equal to ld, where l is the length of the cylinder and d its diameter. The minimum presentation area occurs when the cylinder tumbles end over end; in that case the presentation area is $ld\,(2/\pi + d/2l)$. We see, then, that for narrow cylinders we can be in error by as much as 20% if we assume an average presentation area. Oscillations of this order of magnitude with a period of 3.5 days were occasionally found in the drag of Explorer I (Satellite 1958 α) and attributed to a precessional motion of the axis of rotation of the satellite (Jacchia and Slowey 1961). This precessional motion has been found to be caused mainly by the interaction of the ferromagnetic components of the satellite and the magnetic field of the earth (Bandeen and Manger 1960; Colombo 1961, 1962). For satellites heavier than Explorer I (as most satellites are), the precessional period is likely to be longer, of the order of weeks; under these conditions the drag variations caused by variable presentation area cannot be separated from those caused by atmospheric-density variations unless there is a complete record of the satellite orientation derived from data transmitted to ground stations. For Explorer XI (Satellite 1961 ν), for example, the precessional period is about 55 days (Naumann *et al.* 1962).

For a charged satellite moving in an ionized medium the *effective* cross section may differ from the geometric presentation area. In this case it is generally preferred to keep for A its meaning of geometric presentation area and to account for the effect by changing the drag coefficient C_D.

6. DRAG COEFFICIENT C_D

The drag coefficient C_D plays a vital role in evaluating the drag force acting on a satellite. In view of the complexity of air drag theory, the drag coefficient is determined by experiments whenever such a procedure is feasible—as, for example, in the case of ordnance projectiles. Unfortunately, laboratory conditions are so different from those encountered by a satellite in motion that the drag coefficient for satellites must be evaluated by theory alone.

An important parameter to consider for the evaluation of the drag is the ratio of the mean free path of atmospheric molecules to the characteristic linear dimension of the moving body. When the ratio is much larger than unity, free-molecule flow prevails; when the ratio is smaller than unity, we approach the conditions of continuum aerodynamic regime, in which the drag coefficient is less than half that in free-molecule flow. Above 200 km all satellites launched so far should have been in free-molecule flow; on the other hand, some of the larger rocket bodies in orbit lower than 200 km should have been in continuum regime, at least during the last days of their lifetime. The conditions for the transition from one regime to the other and the manner in which the drag coefficient varies in the transition are known only with a poor degree of approximation.

Satellite drag in free-molecule flow has been investigated theoretically by Cook (1959) and Stirton (1960); the transitional regime is the subject of papers by Baker (1959) and by Probstein and Kemp (1960). In free-molecule flow the drag depends on the mechanism of molecular reflection, on the accommodation coefficient, on the ratio between the mean molecular speed and the satellite velocity, on the skin temperature of the satellite, and on molecular dissociation by impact. It appears, however, that even large variations of these conditions within common-sense limits have relatively little effect on C_D. According to Cook (1959), C_D should lie between 2.1 and 2.3 for satellites of conventional shapes rotating about a transverse axis. If we assume that we have to deal only with neutral drag, no great uncertainty should arise, then, through the drag coefficient in the determination of atmospheric densities for satellites with perigee heights above 200 km. In the region below 200 km, however, the drag coefficient may contribute to the many other difficulties that beset these determinations at those critical heights.

7. CHARGE DRAG

The role of charge drag was investigated by Jastrow and Pearse (1957) in pre-satellite days. As long as high electric charges on orbiting satellites seemed a distinct possibility, several investigators feared that it could seriously affect satellite motion, and thus impair atmospheric-density results. Perhaps the most pessimistic appraisal of the situation was that of Chopra (1961); after a detailed study of all possible electrohydrodynamic and magnetohydrodynamic phenomena that could affect satellite drag, he concluded that "it appears that the attempts made thus far to construct models of the terrestrial atmosphere have been mere exercises in curve-fitting." To most experimental workers the specter of charge drag never appeared quite so sinister, in view of the inner consistency of the data derived from satellites of different shapes and in different orbits.

In a recent theoretical paper, Hohl and Wood (1962) have evaluated the drag forces acting, as a result of its electric charges, on a spherical satellite 4 m in diameter at a 1500-km altitude. They found that the combined effect of increased ion impacts, ion scattering, and induction due to the earth's magnetic field amounts to an increase of only 3.5% with respect to the corresponding uncharged-sphere drag for average conditions of sunspot activity; the increase could be doubled at sunspot maximum and halved around sunspot minimum. From experimental studies on a large dipole orbiting at a height of 3100 km, Shapiro *et al.* (1962) could find no evidence of charge drag and concluded that the potential on the object must be less than 0.6 V.

From all the available evidence it would appear reasonable to conclude that charge drag is not a major source of error in the determination of atmospheric densities, at least up to orbital heights comparable with those of the Echo satellite (1500 km).

8. ATMOSPHERIC-DENSITY DETERMINATIONS

The expression for ΔP in Eq. (3) involves the integral of the product ρv^2 along the orbit. The velocity v varies somewhat in the course of a revolution, but its variation is trifling compared to that of ρ, which may easily reach a factor of 10^6 or more even for moderate orbital eccentricities. As an illustration, Fig. 1 shows the variation of ρv^2 during one revolution

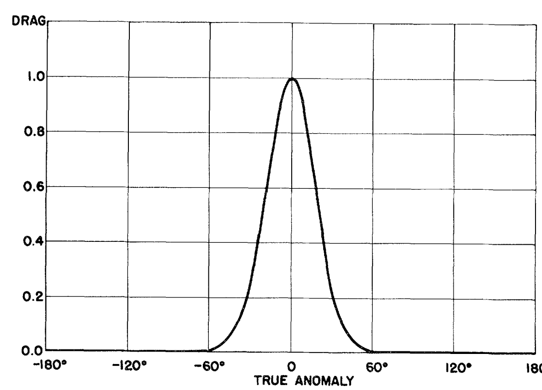

FIG. 1. Atmospheric drag on a satellite in a moderately eccentric orbit. Relative values of the drag are plotted against the true anomaly—i.e., the geocentric angular distance from perigee. The assumed perigee height is 370 km, the orbital eccentricity 0.1.

of the Vanguard I satellite. As can be seen, most of the drag occurs around perigee.

Equation (3) can be used to determine the density at perigee height ρ_p, provided we know the law that governs the variation of ρ along the satellite orbit. If we do not know the law, we have first to assume one to obtain provisional values of ρ_p and try to improve on it by successive approximations. If we make the simplifying assumptions that the atmosphere is nonrotating and spherically symmetric, and that its density decreases exponentially with height, solutions of Eq. (3) can be found in terms of Bessel functions of imaginary argument, for which, in turn, asymptotic expansion can be used if the orbital eccentricity is not too small. Formulas of this type have been developed by Sterne (1958b), by Groves (1958) and by Cook, King-Hele, and Walker (1960). Since the error introduced by these simplifying assumptions is not very large, most investigators have made use of them in deriving atmospheric densities from satellite accelerations. Neglecting terms in e^2, one has

$$\rho_P = -\frac{\dot{P}}{3\pi a C_D}\,\frac{m}{A}\,\frac{\exp\,(ae/H)}{I_0(ae/H) + 2eI_1(ae/H)}\,, \quad (4)$$

where H is the density scale height of the atmosphere. Using asymptotic expansions, one obtains

$$\rho_P = -\frac{\dot{P}}{3C_D}\,\frac{m}{A}\left(\frac{2e}{\pi a H}\right)^{\frac{1}{2}} \times\left[1 - 2e - \frac{H}{8ae} + O\left(e^2, \frac{H^2}{a^2e^2}\right)\right]. \quad (5)$$

Formulas that take into account terms in e^2 have been derived by Cook, King-Hele, and Walker (1960).

The term $H/8ae$ in the square brackets is generally quite small, of the order of 0.01. We can see, then, that for a specific satellite we have, very nearly, $\dot{P} \propto \rho_P H^{\frac{1}{2}}$; i.e., we must know H before we can determine ρ_P. It is true that the relative error in ρ_P caused by an error in the assumed value of H is only half the relative error in H, but still this is the greatest source of error in atmospheric-density determinations.

A way out of this difficulty was found by Whitney (1959) and by King-Hele (1959), although the procedure had been empirically foreshadowed by Siry (1958). It consists in evaluating the density not at perigee height Z_P, but at a point of height $Z_P + \frac{1}{2}H$, where a change of 50% in H will cause a change of less than 3% in ρ. The reason for the smaller error is that this point is close to the weighted mean of the heights over which the drag is effective.

Following King-Hele, let H^* be the best estimate for the value of H. Then, from Eq. (5), the air density ρ_λ at a height $Z_P + \lambda H^*$ is given by

$$\rho_\lambda = \rho_P \exp\,(-\lambda H^*/H)\,,$$

and, when ρ_P is replaced with the expression of Eq. (5),

$$\rho_P = -\frac{\dot{P}}{3C_D}\,\frac{m}{A}\left(\frac{2e}{\pi a H^*}\right)^{\frac{1}{2}}\left[\frac{H^*}{H}\exp\left(-\lambda\frac{H^*}{H}\right)\right] \times \{1 - \cdots\}\,. \quad (6)$$

If $\lambda = \frac{1}{2}$, the expression in the square brackets shows very little change when H^*/H varies between 0.6 and 1.5. Its mean value over this range is 0.593, with an error less than 2.5%; the replacement of H by H^* in the curly brackets adds only imperceptibly to this error. We can thus write

$$\rho_\lambda = -\frac{0.158\,\dot{P}}{C_D}\,\frac{m}{A}\left(\frac{e}{aH^*}\right)^{\frac{1}{2}} \times\left[1 - 2e - \frac{H^*}{8ae} + O\left(e^2, \frac{H^{*2}}{a^2e^2}\right)\right]. \quad (7)$$

Corrections to Eq. (5) to account for the oblateness and the rotation of the atmosphere have been derived by Sterne (1959) and by Cook, King-Hele, and Walker (1961); corrections for a linear increase of H with height have been evaluated by Jacchia (1960a) and Groves (1961a). Wyatt (1961) and Davies (1962) have derived corrections for the presence of a diurnal bulge in the atmosphere. Taking simultaneously into account all the distortions of the atmosphere from spherical symmetry is an almost hopeless task. If a good atmospheric-density model is available, numerical integration of Eq. (2) appears to be the safest way to derive densities from observed drag data.

It should be pointed out that because of the aforementioned distortions in the atmosphere the variation of density with height along the satellite orbit can, on occasion, be quite different from that along a vertical passing through the perigee. This fact may cause an appreciable error when densities are determined from low-eccentricity satellites by use of formulas that assume spherical symmetry of the atmosphere. Similarly, any atmospheric distortion that is not taken into account in the formulas will be somewhat smoothed out in the results from satellite drag regardless of orbital eccentricity. For example, when the perigee of a satellite in its precessional motion slowly crosses the diurnal bulge (see Sec. 12), we should obtain a density profile of the bulge along the path of the perigee. If, however, the bulge is ignored in the formulas, the peak of the profile will appear to be less sharp and lower than it actually is. Only a process of successive approximations can extract all the information contained in the observational drag data.

9. DETERMINATIONS OF SATELLITE ACCELERATIONS

The determination of the period change dP/dt is not a simple operation, and we must start by examining some of the prerequisites for the determination of the orbital period itself. If we write

$$M = M_0 + \dot{M}_0(t - t_0) + \tfrac{1}{2}\ddot{M}_0(t - t_0)^2 + \cdots, \quad (8)$$

where M is the mean anomaly, and the subscript 0 refers to the time $t = t_0$, we clearly have

$$\dot{M}_0 = n_0 = 1/P_0,$$

and

$$\dot{P}_0 = -\ddot{M}_0/\dot{M}_0^2. \quad (9)$$

$\left(\begin{array}{l} n \text{ is the mean motion, in revolutions} \\ P \text{ is the anomalistic period} \end{array}\right)$

The secular acceleration $\dot{P} = dP/dt$, a nondimensional quantity, is always relatively small, of the order of 10^{-5} for low satellites and 10^{-7} to 10^{-8} for higher ones; therefore, $\dot{M}$ changes very little compared to $\ddot{M}$, and over reasonable time intervals, $\dot{P}$ is proportional to $-\ddot{M}$.

Even for orbits of modest eccentricity, M does not vary smoothly in the course of one revolution because the atmospheric-drag perturbation is concentrated in the vicinity of the perigee. We can, however, treat this short-period variation of the drag in the same fashion as we do the gravitational short-periodic perturbations; a complete and practical theory of the effect has been given by Izsak (1960).

If M is expressed in revolutions, the maximum error ΔM that arises from neglecting the periodic drag perturbations is smaller than $\dot{P}/8$; this means that it can exceed 1 sec of arc only for values of $\dot{P}$ larger than 10^{-5}. Once we have removed the short-periodic drag perturbations, or satisfied ourselves that they are negligible, we can treat P as a slowly varying, continuous function of time and identify $\Delta P/\Delta t$ (the change of period during one revolution) with dP/dt.

The least laborious, but also the least accurate, way to determine $\dot{P}$ is to derive it directly, as a by-product of orbital computations, by treating as unknowns all the coefficients of the time expansion (8) of the mean anomaly M. Although for very low satellites, which are strongly affected by atmospheric drag, this method may yield satisfactory accelerations, it generally has the following disadvantages:

(1) Since at least 7 unknowns, and often several more (the other orbital elements and their time variations) are involved in the solution, the error in $\ddot{M}_0$ is, in part, a compound of the errors in the other unknowns.

(2) If the observations are not well distributed along the orbit, the orbital elements can become uncertain; this uncertainty is reflected in the determination of $\ddot{M}_0$.

(3) To obtain reliable orbital elements, we may need observations taken within a relatively long interval, often as long as several days. During this time the drag may undergo lively fluctuations, in which case the computed acceleration is a smoothed version of the true acceleration. We should not increase the number of unknowns in the M expression to account for such a possibility, because if we do, we may have, instead of a smoothed acceleration, merely a larger error caused by the excessive number of unknowns.

By this method, even quite satisfactory observations are wasted if they are not sufficiently numerous or sufficiently well distributed to allow a good orbit determination. Since the orbital elements of a satellite vary rather slowly, it appears more logical to accept only the most reliable orbits and use interpolated elements for an analysis of the individual observations. This is the basis of the method followed by the writer since the early days of atmospheric-drag determinations; for descriptions of the procedure, see Jacchia (1961a), Jacchia and Slowey (1962a), and Jacchia (1962b).

Computing the orbital acceleration of artificial satellites is a delicate operation that requires a considerable degree of skill, and it is essential to ascertain *how* an acceleration was derived before it is used for the determination of atmospheric densities. A cause of *spurious* acceleration is to be found in the acceleration of the perigee. Since the perigee is the origin of the mean anomaly, any nonlinear term in the equation that represents the motion of the perigee is reflected in the anomalistic acceleration. All orbital accelerations must be corrected for this effect, which is not negligible, especially when a large third-harmonic sine term appears in the argument of perigee of a low-drag satellite.

10. ATMOSPHERIC VARIATIONS REVEALED BY SATELLITES: CHRONOLOGICAL OUTLINE

Erratic fluctuations in the orbital acceleration of an artificial satellite were first noticed in Sputnik II (Jacchia 1958a,b; King-Hele 1958). Since this satellite was rocket-shaped, there seemed to be a possibility of explaining these fluctuations with a systematic change in the effective presentation area rather than by invoking atmospheric-density fluctuations. That this was not so became quite obvious when similar fluctuations were observed in the acceleration of the spherical Vanguard I (Satellite 1958 $\beta 2$, perigee height 656 km); a periodicity of about 27 days pointed to variable solar radiation as a cause of the density fluctuations (Jacchia and Briggs 1958). A comparison of the accelerations of Satellites 1958 $\beta 2$ and 1958 $\delta 1$ (Sputnik III rocket, perigee height 200 km) showed that the fluctuations were in phase for both satellites, proving their global character (Jacchia 1959b); the amplitude of the fluctuations was larger for the higher satellite.

A correlation was found by Priester (1959) between the acceleration determined by Jacchia for Satellite 1957 $\beta 1$ and the solar flux at the wavelength of 20 cm. The correlation was confirmed by Jacchia (1959b,c) for the 10.7-cm flux and the drag of satellite 1958 $\beta 2$; over an interval of 10 months all the individual maxima and minima of the solar-flux curve had their counterpart, in phase, in the drag curve.

Jacchia (1959c) found that two transient increases in the drag of satellite 1958 $\delta 1$ (on 9 July and 4 September, 1958) coincided in time and duration with two violent magnetic storms. Jacchia and Slowey (1962a) found later that all geomagnetic perturbations, even the smallest, affect the density of the upper atmosphere.

A slow variation of the drag of satellite 1958 $\beta 2$ by nearly one order of magnitude, with a maximum in December 1958 was recognized (Jacchia 1959d; Wyatt 1959; Priester and Martin 1960) to be owing to the precessional motion of the satellite perigee through a permanent thermal bulge in the atmosphere, located in the bright hemisphere. At the lower heights of Sputnik II and Sputnik III (200 km) the effect of the diurnal bulge was barely detectable (Jacchia 1959a, 1960b).

A semiannual variation in the atmospheric densities derived from satellite drag was detected by Paetzold and Zschörner (1960) and named by them the "plasma effect," since it appears to be caused by the solar wind; the minima occur in January and July, the maxima in April and October, in phase with the well-known statistical variation of the planetary geomagnetic index K_p. The effect was confirmed by Jacchia (1962a). According to Paetzold and Zschörner (1961) the July minimum is more pronounced than the December minimum, suggesting that an annual variation is superimposed on the semiannual effect.

Small latitudinal and seasonal variations have been announced at various times by several authors (Priester, Martin, and Kramp 1960; Groves 1961b; Paetzold and Zschörner 1961), but the results are contradictory; King-Hele and Walker (1961) find no evidence for them from a study of Discoverer VI and Sputnik III. A spurious latitudinal and seasonal effect arises from the latitude variation of the diurnal bulge in the course of the solar year. Similarly unconfirmed is a correlation found by Rasool (1961) between atmospheric density variations and the occurrence of large meteor showers.

Because of the magnitude of the various types of atmospheric fluctuations, empirical density models of the upper atmosphere must be referred to standard parameters of solar activity (Priester and Martin 1960; Jacchia 1960b; Paetzold and Zschörner 1961); otherwise, if they are obtained by simply averaging values over a given interval of time (CIRA 1961; King-Hele and Walker 1962), they must be considered as referring to that given time only. All atmospheric models agree in showing a density scale height that increases with height. Nicolet (1960) showed that this increase can be satisfactorily accounted for by a decrease of molecular weight with height, if the temperature is kept constant at heights above 200 km. Consequently, he assumed that the solar radiation responsible for the heating of the upper atmosphere is absorbed mainly in the region between 100 and 200 km; above this level the atmosphere, which is in diffusion equilibrium, is heated by conduction and is isothermal for any given geographic location. The diurnal bulge then is a direct consequence of conduction heating. A change in solar radiation will cause a change ΔT of upper-atmospheric temperature, and this will result in a density change $\Delta\rho$ which will be different at different heights, according to the value of $d\rho/dT$ that corresponds to diffusion-equilibrium conditions. The different amplitudes of the atmospheric-density fluctuations revealed by satellites orbiting at different heights are satisfactorily represented by this mechanism (Jacchia 1961b).

11. ERRATIC OR "27-DAY" ATMOSPHERIC FLUCTUATIONS AND THE 11-YEAR CYCLE: DECIMETRIC SOLAR FLUX

The atmospheric oscillations that are in phase with the decimetric solar flux are often referred to as the "erratic" or the "27-day" fluctuations. The latter designation comes from the fact that the oscillations sometimes exhibit a semiregular character for intervals of several months, during which a period of 27 days is easily recognizable—as is often the case with the sunspot numbers, with which the decimetric solar flux is strongly correlated. Actually, the parameter with which the 10.7-cm flux seems to be correlated best is the Greenwich sunspot area; Ward and Shapiro (1962) found that the coefficient of correlation between the two quantities is almost unity. According to Waldmeier and Müller (1950) the decimetric flux, apart from infrequent and short-lived bursts, is due to thermal emission from coronal condensations, which cluster above sunspots; this would account for the correlation. On the other hand, Elwert (1956) showed that the coronal condensations must also be emitters of soft x rays and extreme ultraviolet radiation, which should contribute to the heating of the upper atmosphere; hence a second-hand relation between the decimetric solar flux and atmospheric temperatures. X rays are mainly absorbed in the E layer, and their total energy is not sufficient to explain heating at greater heights. Most of the heating must be ascribed to radiation of wavelength greater than 200 Å, which is absorbed in or above the F1 layer. Photon-flux measurements from rockets by Hinteregger *et al.* (1960) and Hinteregger (1961) have shown that there is enough energy in solar lines such as λ304 (He II) λ584 (He I) and others in the range 170Å–600Å to allow them to play an important role in the heating process.

The correlation between the 10.7-cm solar flux and atmospheric densities is extraordinarily good; even minor details in the flux curve can be recognized in the density curves, when these are derived from accurate determinations of satellite drag (see Figs. 2, 3, and 4). Also, the atmospheric curve shows no appreciable lag with respect to the flux curve. Just as for the sunspot numbers, the "27-day" fluctuations in the solar-flux curve are superimposed on a larger 11-yr oscillation (Fig. 5); the same correlation occurs in the atmospheric oscillations. When the first satellites were launched, in 1957, solar activity was near maximum and the upper atmosphere was hot and in an expanded state. It has been cooling and contracting ever since; at the perigee height of the Vanguard I satellite, densities decreased by a factor of 10 from 1958 to 1961.

In their atmospheric model Priester and Martin (1960) assumed a linear correlation between the 20-cm solar flux F_{20} and the atmospheric density ρ at any given height. Jacchia (1960b) put $\rho \backsim F_{20}^m$ and found that for Satellite 1957 β1 (perigee height 200 km) the best agreement with observations was obtained with $m = 0.7$, while for satellites 1958 δ1 and 1958 δ2 (perigee height also 200 km) $m = 1$ was satisfactory; as a consequence he also used $m = 1$. Subsequently, Priester (1961) found that for heights between 350 km (Satellite 1958 α) and 1300 km (Satellite 1960 ι1) m shows a general increase with height and moreover varies with the hour of the day, being larger at night than in daytime. This behavior, as Priester pointed out, must be expected in an atmosphere in diffusion equilibrium possessing a thermal bulge.

Jacchia (1961b) showed that Priester's values of m could be reproduced almost perfectly on the basis of Nicolet's (1961c) atmospheric model, assuming a linear correlation between solar flux and atmospheric temperatures. Any empirical correlations between the solar flux and atmospheric densities are obviously

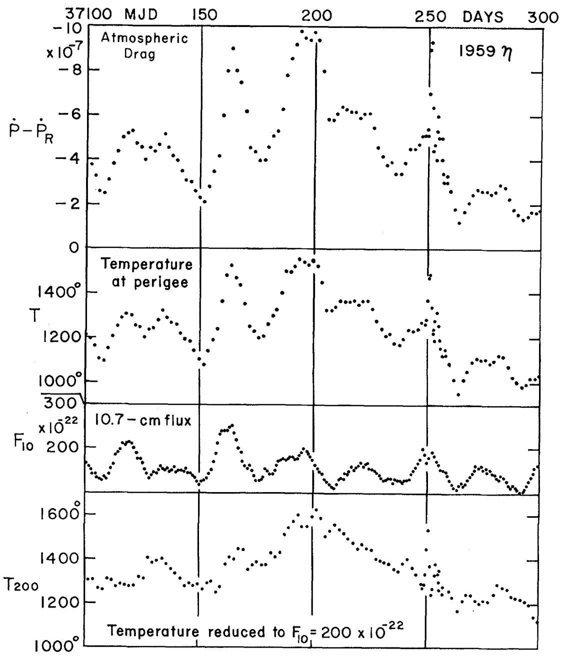

FIG. 2. Atmospheric drag of satellite 1959 η (Vanguard III) during a period of lively "27-day" fluctuations, compared with the 10.7-cm solar flux. In the top section the orbital acceleration $\dot{P}$ is corrected for the effect of radiation pressure ($\dot{P}_R$). The atmospheric temperatures (°K) in the second section are computed from the densities derived from the drag, using Nicolet's (1961) multitemperature model. In the bottom section the temperatures are reduced to a standard 10.7-cm flux value of 200×10^{-22}, using $dT/dF_{10.7} = 2\overset{\circ}{.}5$. The slow variations after correction are mainly due to the passage of the satellite perigee through the diurnal bulge. The sudden jump at MJD 37251 corresponds to a violent magnetic storm (from Jacchia and Slowey 1962b).

poor substitutes for the more basic correlation that must exist between the solar flux and atmospheric temperatures. As atmospheric models improve, it becomes increasingly feasible to use the temperature, rather than the density, as the fundamental parameter in the study of upper-atmosphere variations. A second step forward will be possible with the monitoring of the extreme ultraviolet solar radiation by means of artificial satellites.

According to recent determinations the "27-day"

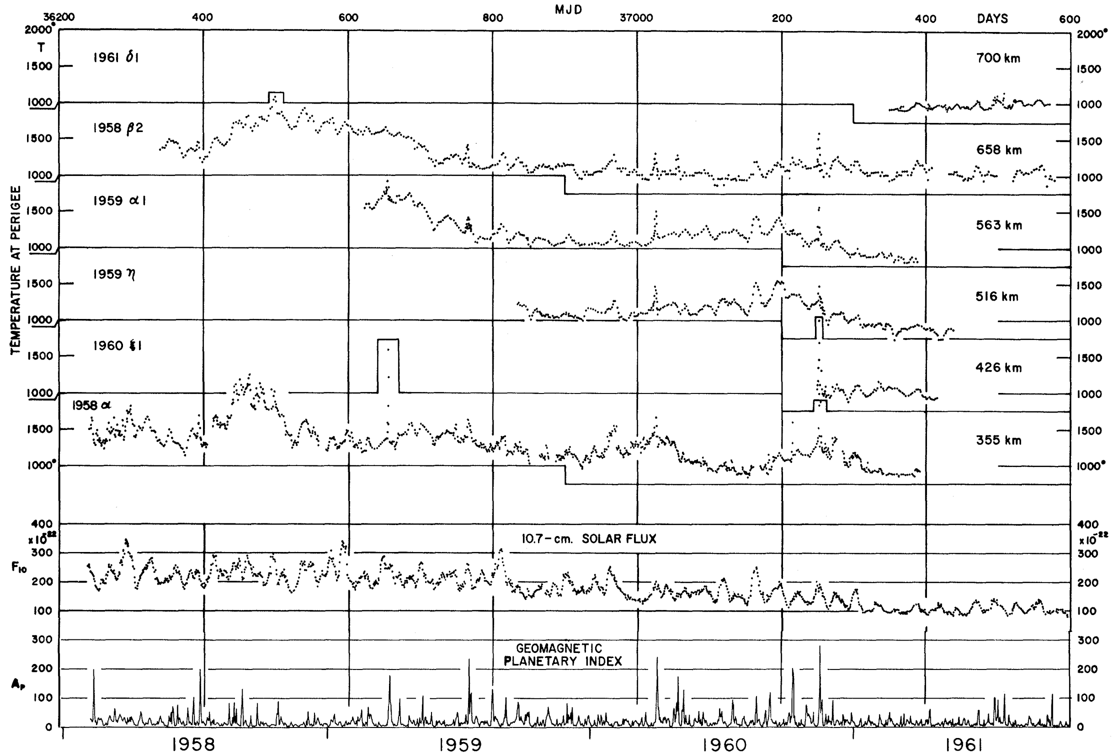

FIG. 3. Atmospheric temperatures (°K) derived from the drag of six satellites using Nicolet's 1961 model atmosphere. Average perigee heights are shown at right. The 10.7-cm solar flux and the daily geomagnetic index A_p are shown for comparison. The slow temperature oscillations—as those that produced the four pronounced maxima about 275 days apart in 1958 α—are due to the motion of the satellite perigee with respect to the diurnal bulge (from Jacchia and Slowey 1962b).

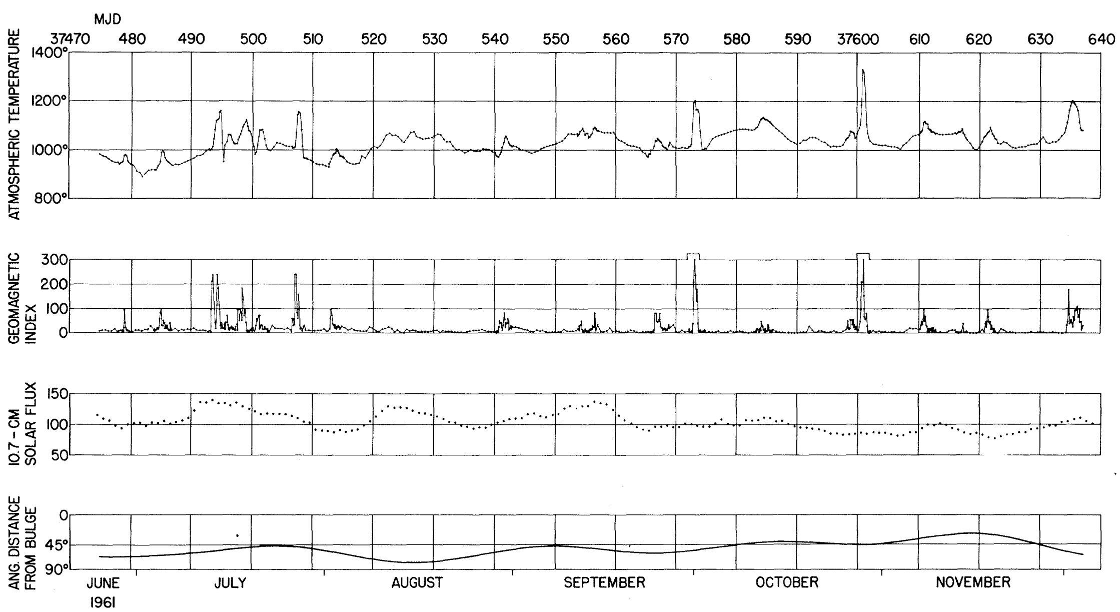

FIG. 4. Atmospheric temperature (°K) derived from precise drag determinations on 1961 δ1 (Explorer IX, the 12-ft balloon satellite) using Nicolet's 1961 model atmosphere. Notice the fluctuations in phase with the 10.7-cm solar flux and the perturbations corresponding to magnetic storms. The slow general increase in temperature is due to the approach of the perigee to the diurnal bulge; the angular distance between perigee and bulge is plotted in the bottom section. The observations used to obtain the drag are all precision-reduced photographs taken with the Baker–Nunn camera (from Jacchia and Slowey 1962c).

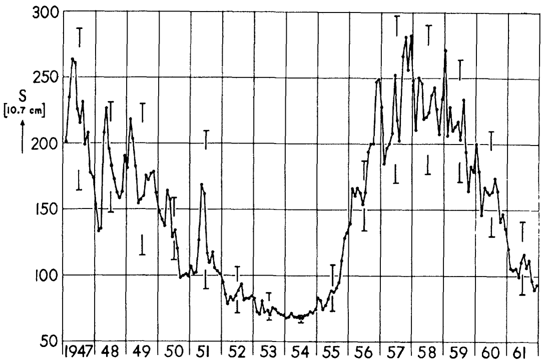

FIG. 5. Variation of the monthly averages of the 10.7-cm solar flux according to the measurements of the National Research Council of Canada. The bars indicate the extent of the monthly fluctuations in June and July of each year (from Harris and Priester 1962b).

oscillations in density are explained if the atmospheric temperature increases on the average by 2°.5 (Jacchia and Slowey 1962b; Paetzold 1962a) when the 10.7-cm solar flux increases by 1×10^{-22} W m^{-2} cps^{-1} bandwidth. While this relation holds within one cycle of the "27-day" fluctuation, it cannot be interpreted as a general relation between solar flux and atmospheric temperatures over long time intervals, because of another effect connected with the 11-yr solar cycle and probably originating with the solar wind (see Sec. 14). It may be added that 2°.5 is probably only a mean between the values that the coefficient $dT/dF_{10.7}$ reaches during the day and during the night. More about this point is given in the following section.

12. DIURNAL BULGE

The difference in density between the dark side of the earth and the sunlit hemisphere is frequently called the "diurnal" or "day-and-night" effect. Local solar time is often used as a parameter against which atmospheric densities and temperatures are plotted to analyze the effect. Early investigations (Jacchia 1959d; Priester and Martin 1960; Wyatt 1959) showed that the density at any given height above the earth peaks rather sharply around 2 p.m. solar time; the nighttime minimum is flatter than the

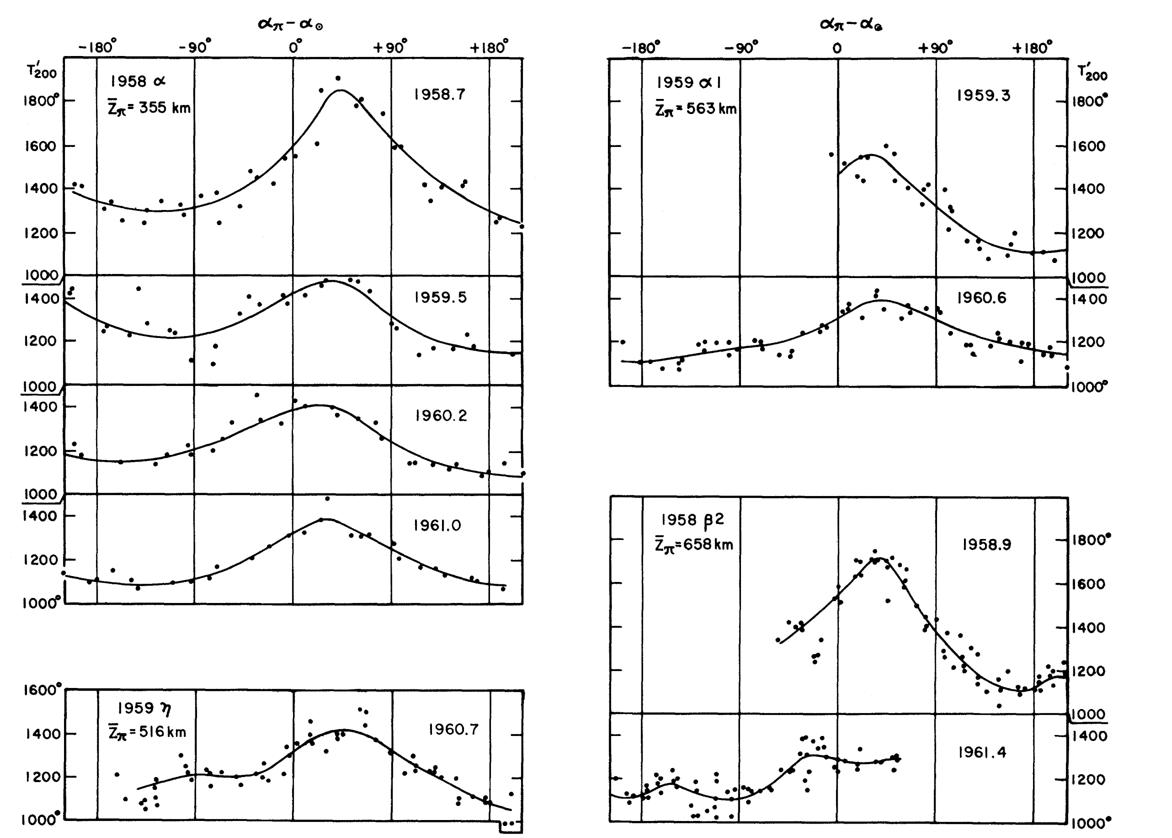

FIG. 6. The diurnal effect for four satellites. Temperatures (°K) derived from the drag in the same manner as in Figs. 2, 3, and 4, are here reduced to a standard 10.7-cm flux of 200×10^{-22} (using $dT/dF_{10.7} = 2°.5$), corrected for the semiannual effect and plotted against the difference in right ascension between the satellite's perigee and the sun. The approximate date of the maximum of the curve is shown on the right.

maximum and occurs in the second half of the night, between 2 a.m. (Jacchia 1960b; Jacchia and Slowey 1962b) and 5 a.m. (Priester *et al.* 1960); see Fig. 6.

A plot of atmospheric densities against solar time would give a correct picture of the day-and-night effect only if the latter consisted of a wave with a longitudinal front. The correct picture is, instead, that of an atmospheric swelling with a peak at the latitude of the subsolar point. For any point at a given height above a given geographic location on earth the amplitude of the wave depends on the difference between the latitude of this location and the latitude of the subsolar point, which varies in the course of the seasons. If, instead of having a fixed point above the earth, we have to deal with a satellite perigee, which has latitude variations of its own, the result will be a badly distorted curve when the densities derived over a limited time interval are plotted against solar time (which is simply a *longitude* difference between the given location and the subsolar point). In particular, since the satellite perigee may never cross the center of the bulge, the amplitude of the wave derived in this fashion is bound to be smaller than in reality.

An obvious solution would be to draw lines of equal density, or temperature, on a map or a globe on which the origin of the coordinates moves with the subsolar point. Unfortunately, this procedure generally fails because of the peculiar distribution of points coming from individual satellites and because of the large scatter introduced by all the various solar effects during the relatively long time interval it takes a satellite perigee to cross the bulge region. This method for mapping the shape of the diurnal bulge may, however, give satisfactory results during the coming period of the quiet sun.

Although there is some indication (Priester *et al.* 1960) that in the diurnal wave the rise to maximum density is a little faster than the decline after the maximum, the departure from symmetry around the maximum should be relatively small. Assuming, for simplicity, that the diurnal bulge possesses rotational symmetry, Jacchia (1960b) used the geocentric angular distance ψ' from the center of the bulge as the independent parameter for analyzing its shape, and found that in the height region from 200 to 700 km atmospheric densities at a given height z could be satisfactorily represented by the equation

$$\rho = \rho_0[1 + f(z) \cos^6 \tfrac{1}{2} \psi'] , \qquad (10)$$

where ρ_0 is the minimum nighttime density, and $f(z)$ increases as z increases.

The amplitude $f(z)$, for which Jacchia also gave an empirical expression, is actually a complicated function of height that varies with time as the structure of the atmosphere changes with solar radiation. With Nicolet's (1961c) model atmosphere, in which the atmospheric-top temperature is constant above any given geographic location, the density profile of the bulge is automatically determined when we specify the distribution of temperatures around the center of the bulge. Although no detailed analysis has been made so far, it looks as though a distribution following the equation:

$$T = T_0(1 + 0.4 \cos^4 \tfrac{1}{2} \psi') \qquad (11)$$

is a good approximation. Here T_0 is the temperature corresponding to the nighttime minimum. The temperature at the center of the bulge should then be 40% higher than that at the opposite point of the globe (Jacchia and Slowey 1962b).

According to Nicolet's (1961c) tables, $d \log \rho / d \log T$ is just about zero at $z = 200$ km for temperatures higher than 1000° K, which would make the amplitude of the diurnal effect zero at such heights, in agreement with data from Sputnik II and III, which were launched near the sunspot maxima. For temperatures between 800° and 1000° Nicolet's tables give, for the same height, $d \log \rho / d \log T = 2$; assuming a maximum daytime temperature 40% higher than the nighttime temperature, this would produce a detectable diurnal bulge (density increased by a factor of 2) when solar activity is low. At greater heights $d \log \rho / d \log T$ becomes quite large, reaching values as high as 7, but never exceeding this limit. This would limit the amplitude of the diurnal bulge to a maximum factor of 9 in the densities, in agreement with satellite observations so far (see Figs. 7, 8, and 9).

If the diurnal bulge were to be considered as caused entirely by the conduction of heat generated by the absorption of extreme ultraviolet radiation and if the "27-day" fluctuation were due entirely to variations in the same radiation, we should expect that the response of the temperature T to a variation of the 10.7-cm flux $F_{10.7}$ must be different in daytime and at night. Specifically, if the temperature at the center of the bulge is 40% higher than the nighttime temperature, then also the coefficient $dT/dF_{10.7}$ at that point must be 40% larger than its nighttime value. Jacchia and Slowey (1962b) find that such a hypothesis is not contradicted by the observations; in that case the value 2°5 they obtain for $dT/dF_{10.7}$ should be the average between the extremes 2°1 and 2°9. Paetzold (1962b) actually believes he can determine these extremes, for which he gives 2°0

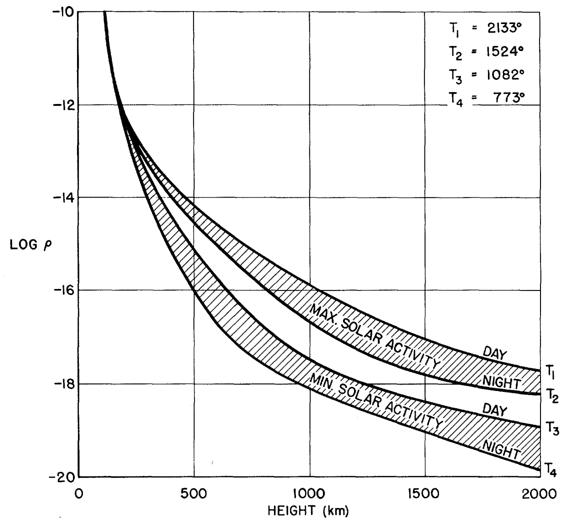

FIG. 7. Day and night density profiles in the upper atmosphere at sunspot maximum and at sunspot minimum. The profiles were computed from Nicolet's 1961 model atmosphere, assuming a maximum day temperature 40% higher than the corresponding night minimum.

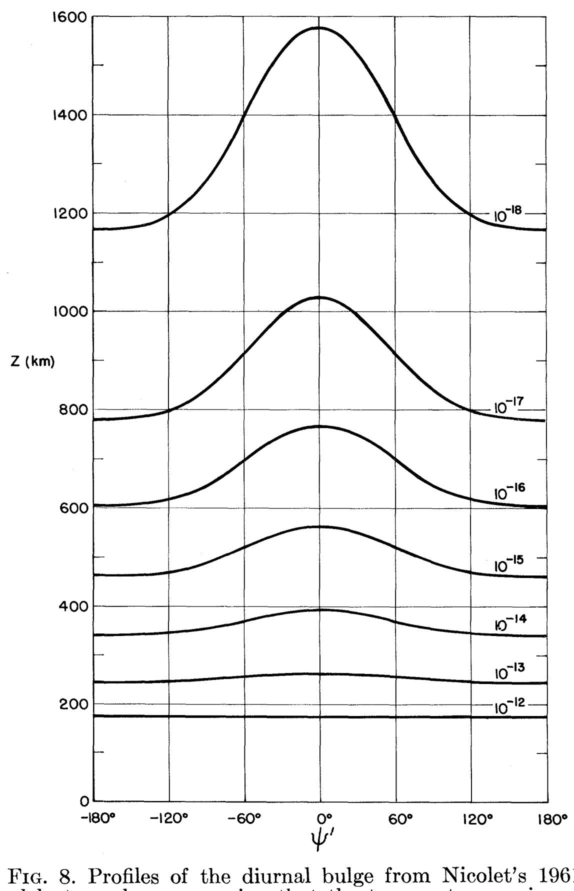

FIG. 8. Profiles of the diurnal bulge from Nicolet's 1961 model atmosphere, assuming that the temperature varies as $T = T_0(1 + 0.4 \cos^4 \frac{1}{2}\psi')$, where ψ' is the geocentric angular distance from the center of the bulge. Plotted are the heights of isopycnic surfaces above a great circle across the center of the diurnal bulge. T_0 is taken to be 1000° K.

and 3°.4; this would entail a somewhat larger temperature range in the diurnal effect.

13. ATMOSPHERIC VARIATIONS CONNECTED WITH MAGNETIC STORMS

The atmospheric perturbations connected with magnetic storms are, on occasion, of spectacular character. During the most violent storms the density may first increase and then decrease by an order of magnitude in a matter of hours. What makes these perturbations difficult to detect is their short duration. Suppose the acceleration of a satellite is temporarily increased by a constant amount during a time interval τ. The effect of this increase on the position of the satellite, which is the observed

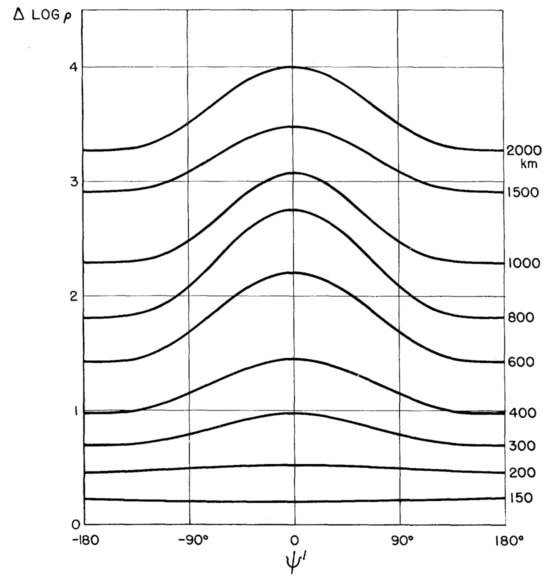

FIG. 9. Density variations along a great circle across the center of the diurnal bulge at different heights, using the same assumption as for Fig. 10.

quantity, will be proportional to τ^2. The magnetic-storm perturbation in the atmosphere usually lasts only 1 or 2 days; therefore the effect of such a perturbation on the mean anomaly of a satellite is several hundred times smaller than that of a 27-day oscillation with the same amplitude in the atmospheric density. This should explain why the literature on the subject of this type of perturbation is so meager.

Jacchia (1961a) analyzed the drag perturbation of seven satellites during the November 1960 solar events (see also Groves 1961c) and found that the maximum of the atmospheric perturbation occurs at nearly the same time all over the globe and approximately coincides with the time of maximum of the magnetic storm. A plot of the satellite accelerations during the storm looks like a replica of the plot of the 3-h planetary indices a_p (see Figs. 4 and 10). The 12-f balloon satellite, Explorer IX, is ideal for studying this type of perturbation on account of its large area-to-mass ratio, which makes it sensitive even to minor atmospheric perturbations. Jacchia and Slowey (1962b) found peaks corresponding to 12 magnetic storms in the acceleration curve for this satellite in an interval of about 7 months in 1961, and this was done using only rough, field-reduced photographic observations. More recently, using precision-reduced observations, it became possible to increase the resolution and make more detailed studies of individual magnetic storms (Jacchia and Slowey 1962c); as a result of this investigation it appears that the atmospheric perturbation lags about 5 h with respect to the a_p indices.

The amplitude of the perturbation generally increases with height in the same fashion as the diurnal bulge and it is possible to give a fair representation of the observed amplitudes by assuming that there is a global temperature increase in the atmosphere above 200 km by an amount proportional to a_p (Jacchia 1961b). Using Nicolet's (1961c) atmosphere, the increase turns out to be approximately 1°2 for every a_p unit, at least in the a_p range between 0 and 300; above this limit a_p should lose its quantitative significance, because 400 is the upper limit, corresponding to any exceptional magnetic storm ($K_p = 9$). The atmospheric perturbation of 13 November 1960 can be accounted for by a temperature increase of about 500° or 600°. Since this was the strongest magnetic storm that occurred during the present solar cycle, such a temperature range should be considered as an upper limit for the effect (larger temperature ranges have been derived from the accelerations of individual satellites, but no great significance should be attached to them in view of the uncertainty in the determinations).

The orbital inclination of the Explorer IX satellite is 38°8, and all the preceding statements are valid only for low geomagnetic latitudes. Recent drag data from the Injun III satellite (Jacchia and Slowey 1963) have shown that in the auroral zones the heating that accompanies geomagnetic perturbations is four or five times greater than at low latitudes.

The mechanism by which the atmosphere is heated during magnetic storms is not well understood. Since the observed amplitudes in the drag perturbations can be explained by one and the same increase of temperature at all satellite heights, the dissipation of energy must occur mainly at heights below 200 km. Joule heating, as suggested by Cole (1962), would satisfy this condition; hydromagnetic waves as a source of heat have been suggested by Dessler (1959).

14. VARIATIONS CONNECTED WITH THE "SOLAR WIND"

It has been known for about 30 years (Bartels 1928, 1932) that geomagnetic activity undergoes a semiannual oscillation, with maxima around the equinoxes and minima around the solstices. This oscillation is small compared to the erratic, day-to-day fluctuations; in order to reveal it, one must take observations extending over long series of years and work on averaged values. Thus, analyzing the daily A_p indices—which can jump from near zero to a peak of 200 and more during magnetic storms—Bell and Glazer (1958) found a systematic semiannual variation with an amplitude of 8 units (from 13 at minima to 21 at maxima). Analogous variations are found for the C_i index (Shapiro and Ward 1960) and the u index (Priester and Cattani 1962).

In the two possible explanations advanced for the effect, the corpuscular stream from the sun (the "solar wind") is held to be the primary agent. Bartels (1928) and McIntosh (1959) believe that the interaction between the corpuscular stream and the magnetic field of the earth varies with the angle between the earth's dipole and the line joining earth and sun (this would imply also a 24-h variation, for which McIntosh finds some evidence). Priester and Cattani (1962) think that the variations of the earth's heliographic latitude in the course of the year may explain the phenomenon; the observed maxima and minima occur one month later than predicted by the theory, but this may be due to the retention of particles in the earth's magnetosphere.

The semiannual variation in atmospheric temperature and density detected by Paetzold and Zschörner (1960) is roughly in phase with the variation of geomagnetic indices. Shapiro and Ward's (1960) C_i curve shows maxima around 15 March and 1 October; atmospheric temperatures show maxima around 7 April and 7 October (Jacchia and Slowey 1962b). In 1928, when solar activity was near maximum, the amplitude of the semiannual temperature oscillation was about 250° K; since then it has been steadily decreasing, and was down to about 100° K or less in 1962 (Jacchia and Slowey 1962b; see Fig. 11).

These large temperature ranges show that the relation between geomagnetic indices and atmospheric temperatures is radically different in the magnetic-storm effect and in the semiannual effect.

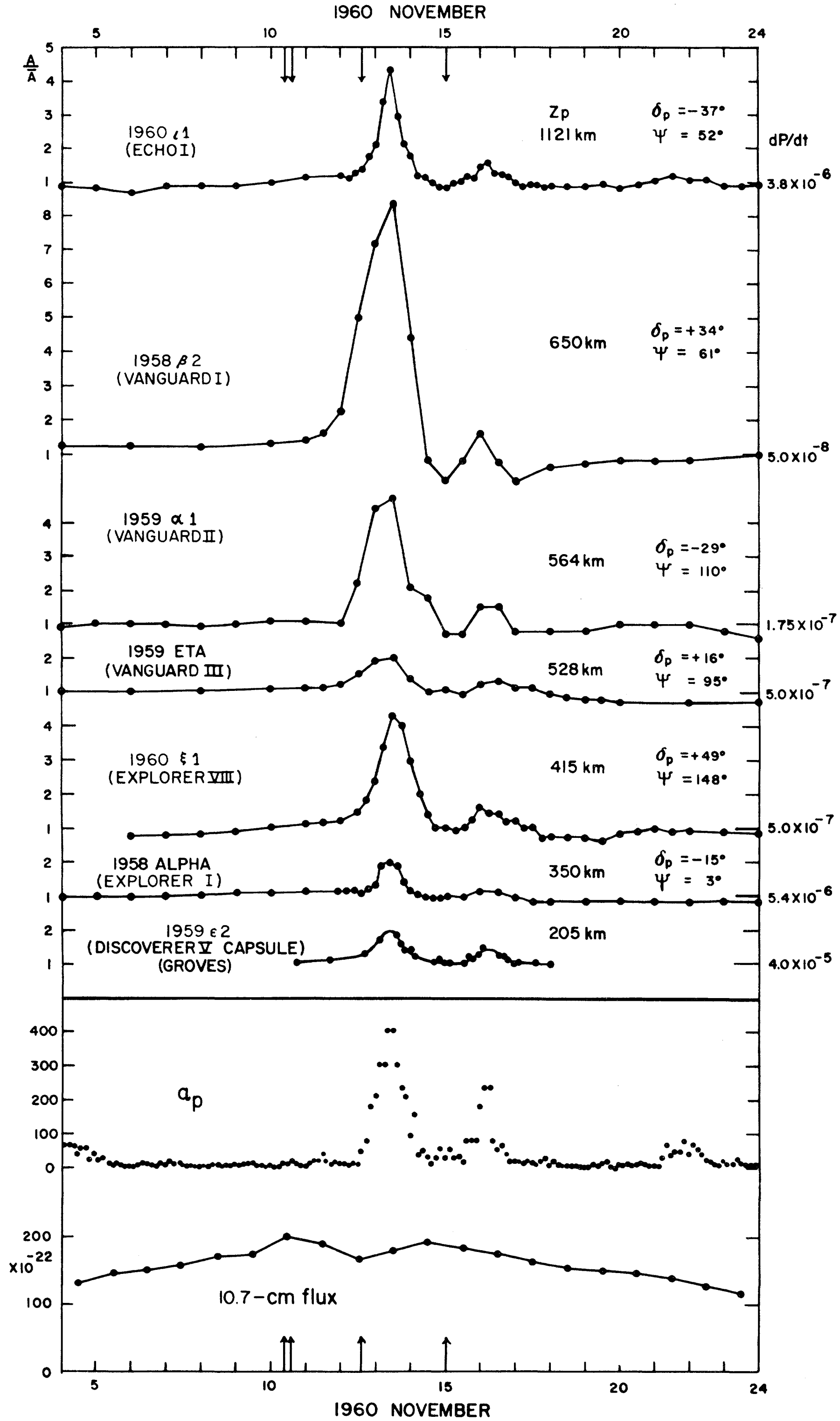

FIG. 10. Atmospheric drag of seven artificial satellites during the November 1960 events, compared with the 3-hourly geomagnetic index a_p and the solar flux at 10.7 cm. The figures outside the right margin give the mean value $\bar{A}$ of the acceleration for each satellite before and after the perturbations of 12 through 17 November. The plotted points represent the ratio of the instantaneous acceleration A to this mean value $\bar{A}$. Other pertinent data—the declination of the satellite perigee δ_p and the geocentric angle ψ between the perigee and the sun—are given inside the right margin of the diagram. The arrows at the top and at the bottom of the diagram mark the instants corresponding to the appearance of 3^+ flares on the sun (from Jacchia 1961a).

If the value of $dT/dA_{\text{p}} = 1\overset{\circ}{.}2$, found for magnetic storms, were to hold also for the semiannual effect, the amplitude of the latter would be of 10° only! Also, the amplitude of the semiannual oscillation of the A_{p} index is not greater at sunspot maximum than at sunspot minimum—actually, the opposite seems to be true (Bell and Glazer 1959). We must conclude that the geomagnetic indices, while strongly correlated with the source of energy that heats the atmosphere during magnetic storms, are very poorly correlated with the solar wind. Whether the mechanism by which the atmosphere is heated by the solar wind on quiet days is or is not the same as that which operates during magnetic storms remains an open question.

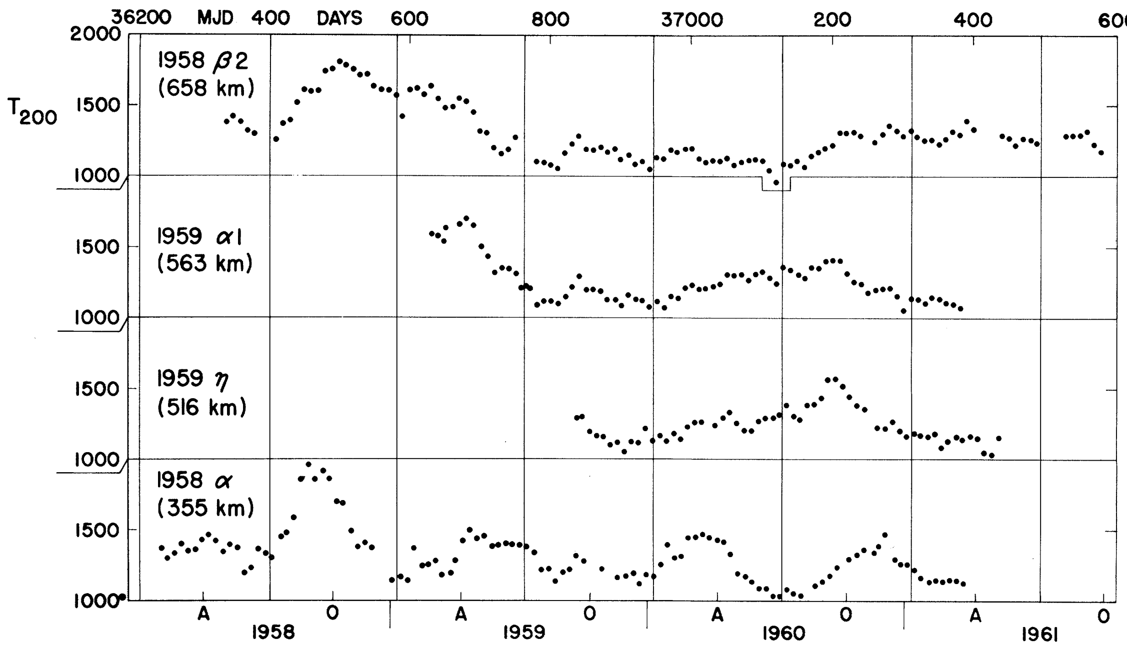

FIG. 11. The "solar-wind" effect. Atmospheric temperatures, derived from the drag of four satellites in the same manner as in Figs. 2, 3, and 4, are here reduced to a standard 10.7-cm flux of 200×10^{-22}, using $dT/dF_{10.7} = 2\overset{\circ}{.}5$. The vertical lines are drawn in correspondence with the minima (January and July) of the semiannual variation, which can be discerned superimposed on the slower oscillation caused by the diurnal-bulge effect. Notice the systematic decrease in temperature that accompanied the waning amplitude of the semiannual variation, as the solar wind decreased from 1958 to 1961 (from Jacchia and Slowey 1962b).

If the semiannual temperature variation in the atmosphere is due to a modulation of the corpuscular heating from the sun, we must expect to find also an 11-yr temperature variation if the solar corpuscular stream varies in intensity with the solar cycle. This is precisely what has been found, independently, by Jacchia (1962a) and by Paetzold (1962a,b). When atmospheric temperatures are corrected for the "27-day" oscillation—i.e., for euv heating—using the value of $dT/dF_{10.7} = 2\overset{\circ}{.}5$, there remains a slow, systematic decrease which has amounted to 300° K from early 1958 to mid-1962. If we assume that the intensity of the solar wind is proportional to the monthly means $\bar{F}_{10.7}$ of the 10.7-cm flux, we obtain for this effect $dT/d\bar{F}_{10.7} = 2\overset{\circ}{.}0$ (Jacchia 1962a). From this we can infer that the contribution of the solar wind to the heating of the upper atmosphere must be quite considerable, possibly reaching 30 or 40%. If we also assume that the amplitude A_s of the semiannual temperature variation is linearly related to $\bar{F}_{10.7}$, we obtain roughly, $A_s = 0\overset{\circ}{.}5\ \bar{F}_{10.7}$, so the contribution of the solar wind to atmospheric heating can be written, approximately, as

$$\Delta T = \left(2\overset{\circ}{.}0 + 0\overset{\circ}{.}5 \cos 4\pi \frac{t - \text{Apr.7}}{365}\right) \bar{F}_{10.7}\,.$$

Paetzold and Zschörner (1961) find that in the semiannual oscillation the July minimum is systematically much deeper than the January minimum; the temperature rise between the July minimum and the October maximum is, according to them, three times greater than the temperature drop from October to January. This would imply an *annual* effect superimproved on the semiannual variation. For its explanation Paetzold (1962b) looks to the "interstellar wind" caused by the sun's motion relative to interstellar matter. In view of the difficulty of separating the semiannual effect from the variations caused by the diurnal bulge and from all the other variations, it may be premature to embark on speculations on this, as yet unconfirmed, annual effect.

In connection with the semiannual effect a few words should be said about a puzzle that involved the 20-cm solar flux in the year 1958. During that year, the 20-cm flux, which is measured at the Heinrich-Hertz-Institut in Berlin-Adlershof, showed a large, slow oscillation with maxima in April and October and a minimum in July. No oscillation of this sort could be detected in the 10.7-cm solar flux measured by the National Research Council in Ottawa, nor in the fluxes at wavelengths of 3.2, 8, 10, 15, 21, and 30 cm measured at various other stations. The singular behavior of the 20-cm flux did not extend beyond the end of 1958; after that time it agreed with the flux at other decimetric wavelengths. Nicolet (1960, 1961a), in pointing out the singularity, attributed it to a drift of instrumental origin that affected the 1958 measurements of the 20-cm flux. No other explanation appears possible today. At the time, however, owing to the fact that the spurious fluctuation mimicked the atmospheric semiannual effect both in phase and in amplitude, it appeared for a while as though the 20-cm flux were in better agreement with the observed atmospheric fluctuations than the 10.7-cm flux. This allowed Priester and Martin (1960) and Jacchia (1960b) to construct variable atmospheric-density models based on the 20-cm flux that were in good agreement with the observations, in spite of the fact that no allowance was made for a semiannual oscillation.

15. VARIABLE ATMOSPHERIC-DENSITY MODELS

In October 1958, at a time of high solar activity, the perigee of Satellite 1958 $\beta 2$ was rather close to the center of the diurnal bulge; at that time the atmospheric density derived from the drag for the perigee point of the satellite (658 km above the earth) was 1.5×10^{-15} g/cm^3. Less than 2 yr later, in June 1960, the perigee of the same satellite still at the same height above the earth was in the dark hemisphere and solar activity was rather low; the density derived from the drag was 2.5×10^{-17} g/cm^3 (Jacchia and Slowey 1962b). These are by no means the extremes of density to be expected at that height; it can be easily inferred that the maximum density range is close to a factor of 500. Such being the facts, it should be obvious that no static model of the atmosphere can even approximately represent atmospheric parameters in the general case.

In an atmosphere in hydrostatic equilibrium, the density ρ varies with the height z according to the equation

$$\frac{d \log \rho}{dz} = -\frac{g\mathfrak{M}}{kT} + \frac{d \log (\mathfrak{M}/T)}{dz}, \qquad (12)$$

where g is the acceleration of gravity, k the universal gas constant, $\mathfrak{M}$ the mean molecular mass, and T the kinetic temperature. If the composition of the atmosphere is governed by diffusion, $\mathfrak{M}$ itself is a function of T for each individual value of z. Thus, if the law governing the variation of T with height is known, we can compute a complete $\mathfrak{M}(z)$ profile starting from a single set of initial conditions and, by integration of Eq. (12) obtain $\rho(z)$.

The atmospheric parameter that is directly affected by solar heating is obviously T. We cannot expect satisfactory results from an atmospheric model unless T is taken as the independent variable, and its variations are related to some indicators of the variable solar-energy sources that are involved in the heating process. This, however, is not possible as long as we do not know how T varies with height. The solution became possible only after Nicolet's (1960) discovery that atmospheric densities from satellite drag observations could be satisfactorily represented on the basis of diffusion equilibrium assuming that T reaches a constant value for $z > 200$ km. Before this discovery, attempts were made by Priester and Martin (1960) and by Jacchia (1960) to construct variable atmospheric models in which the density was linearly related to the 20-cm solar flux. The results seemed rather satisfactory at the time because all the observations used were from the years 1958–59, a time of high solar activity, but as soon as the solar cycle had proceeded past the half-mark toward minimum, large systematic discrepancies started appearing. King-Hele and Walker (1961) tried to solve the problem in a purely empirical way, deriving average density profiles for each year from 1958 to 1960.

After it had been established that the upper atmosphere is locally isothermal, it became possible to construct atmospheric models for different temperatures and fit them to observed conditions. Thus Priester and collaborators (Martin *et al.* 1961) computed a daytime and a nighttime model valid for a 20-cm solar flux value of 170×10^{-22} W^{-2} cps^{-1} bandwidth, starting from pressure boundary conditions at 2100 km, derived by Römer (1961) from an analysis of Echo I observations. The top temperatures were 1892° and 1186° K, respectively; the hydrostatic equation was integrated downwards in height, following the observed density profile. The mean molecular mass $\mathfrak{M}$ was derived from the pressure scale height ($H_p = kT/\mathfrak{M}g$), assuming a constant value for T. A similar procedure was followed by Kallmann-Bijl and collaborators in the preparation of the CIRA (1961) tables, which give three atmospheric models, for top temperatures of 1186°, 1474°, and 1834° K, respectively. Paetzold and Zschörner (1961), in a somewhat similar fashion, computed two atmospheric profiles, for top temperatures of 1100° and 1350° K approximately (they did not attempt to keep the temperatures rigorously constant at the top).

In contrast with the previous models, Nicolet's 1961 model was constructed on an almost purely theoretical basis. Diffusion was assumed to start at a height of 105 km and a set of boundary condition was established at 120 km, temperatures were assumed to approach asymptotically a constant value for great heights. Mean-molecular-weight profiles were computed on the basis of diffusion for 12 asymptotic temperatures ranging from 773° to 2131° K; the inclusion of helium among the atmospheric constituents resulted in a primary role for this element at heights greater than 500 km (Nicolet 1961b).

Jacchia (1961b) showed that the asymptotic temperatures derived through Nicolet's model from satellite drag observations are linearly correlated with the 10.7-cm solar flux and are consistent at heights between 350 and 700 km. It appears, then, that in Nicolet's model we have a good tool for deriving the atmospheric temperatures that are needed for an analysis of atmospheric heating. Caution, however, should be used with this model for heights smaller than 300 km. As we mentioned in Sec. 12, at $z = 200$ km the densities in Nicolet's model are nearly independent of temperature, in agreement with the near-zero amplitude of the diurnal variation at that height. We must recognize, however, that lively "27-day" fluctuations were observed at the height of 200 km in the days of the early Sputniks (satellites 1957 β, 1958 δ1, and 1958 δ2), caused by temperature changes that were considerably smaller than the night-and-day effect. Also unaccounted for are the drag perturbations of 1958 δ1 during magnetic storms (Jacchia 1959). The reason for these discrepancies are probably to be found in conduction time and in systematic changes with temperature in the boundary conditions.

The most ambitious attempt to construct a variable atmospheric model was made by Harris and Priester (1962a,b), who performed a simultaneous integration of the heat-conduction equation and the hydrostatic equation, using an *ad hoc* variable heat source. The boundary conditions were taken from Nicolet (1961) at $z = 120$ km and diffusion was assumed to prevail above this height.

When the heat source was identified with the solar euv radiation alone, the authors found that with the energy necessary to give the observed average temperature, the computed variation would come out too large and wrongly phased, with a maximum at 5 p.m. solar time instead of 2 p.m. as observed. Consequently, they introduced a second heat source, of such intensity and direction as to give a diurnal temperature variation in agreement with the observations. This source turned out to have a maximum at 9 a.m., of intensity 10% greater than the maximum solar euv radiation and the authors believe it represents the corpuscular heating component. The authors found a linear correlation between the atmospheric temperature and the 10.7-cm solar flux, given by the following expression:

$$\text{Nighttime minimum: } T = 4\overset{\circ}{.}47\, F_{10.7} + 275^\circ,$$

$$\text{Daytime maximum : } T = 7\overset{\circ}{.}05\, F_{10.7} + 372^\circ.$$

As can be seen, the coefficient $dT/dF_{10.7}$ for the nighttime minimum is identical with the sum of the two coefficients, 2°.5 and 2°.0, found by Jacchia (see Sec. 14) for the contribution of the euv and the corpuscular heating; the diurnal temperature range, however, is somewhat greater than the one derived by Jacchia—a factor of 1.58 instead of 1.4. The atmospheric parameters from the numerical integrations by Harris and Priester are tabulated in great detail for every hour of solar time and for five different values of $F_{10.7}$.

This model represents fairly well the upper-atmosphere conditions averaged over several weeks; it is not intended to represent the "27-day" fluctuations, whose computed amplitude would turn out too large if daily values $F_{10.7}$ were placed in the equations instead of the monthly means. Also, we must remember that the fair over-all agreement with observations is not by itself a proof that the second heat source is, in phase and intensity, the one found by the authors. Phase and intensity were derived by subtracting from the total heating a theoretical euv heating whose phase must depend on the assumed boundary conditions and, to a degree, on the simplifications introduced to reduce the independent variables to height and time only.

16. SUMMARY OF TEMPERATURE VARIATIONS IN THE UPPER ATMOSPHERE

The variations described in Secs. 11 through 14 can be collected in one formula to compute the smoothed nighttime minimum temperature $\overline{T}_N$ of the upper atmosphere, from which the temperature T at any other point of the earth at any time can then be derived; this formula is a modification of the one prepared by the author for the U.S. Standard Atmosphere (1962); the temperatures are based on Nicolet's model.

We can write:

$$\overline{T}_N = 635^\circ + 0\overset{\circ}{.}3\,\overline{F}_{10.7} + 0\overset{\circ}{.}012\,\overline{F}^2_{10.7} \qquad \text{(11-yr cycle)}$$

$$T = \overline{T}_N(1 + 0.4\cos^4 \tfrac{1}{2}\psi') \qquad \text{(diurnal variation)}$$

$$+\, 2\overset{\circ}{.}5(F_{10.7} - \overline{F}_{10.7}) \qquad \text{(“27-day” oscillations)}$$

$$+\, 0\overset{\circ}{.}5\,\overline{F}_{10.7}\cos 4\pi \frac{t - \text{Apr. 7}}{365} \qquad \text{(semiannual variation)}$$

$$+\, 1\overset{\circ}{.}2a_p \qquad \text{(geomagnetic effect)}$$

Here $F_{10.7}$ is the daily value of the 10.7-cm solar flux in units of 10^{-22} W m^{-2} cps^{-1} bandwidth, and $\overline{F}_{10.7}$ its monthly average; t is the time in days, ψ' the geocentric angular distance from the center of the diurnal bulge, and a_p the 3-h geomagnetic index.

During the last minimum of solar activity, in 1954, $F_{10.7}$ remained practically constant at 70 for many months in a row. Assuming that this is the normal condition at any sunspot minimum, we obtain from the formula an absolute minimum temperature of 680°K at night and of 952°K at the center of the bulge.

During the month of December 1957 at the time of the greatest sunspot activity ever observed, $F_{10.7}$ reached a peak value of 377, while $\overline{F}_{10.7}$ was 282. This would place the highest value of the nighttime minimum at about 2000° and the corresponding daytime maximum at about 2700°, in the absence of any geomagnetic activity. If a first-class magnetic storm had occurred just during that brief spell of exceptional solar activity (there was none in December 1957), the temperature might presumably have risen to the vicinity of 3000° even in low geomagnetic latitudes.

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