Interpolative Formulas for Pion–Nucleon Scattering Phase Shifts*

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INTRODUCTION

A KNOWLEDGE of the energy dependence of the pion–nucleon scattering phase shifts is important for the interpretation of many other interactions involving pions and nucleons. However, since 1956, there has been no over-all examination of the experiments in pion–nucleon scattering which could give interpolative expressions for the phase shifts over the extended energy range now available. A particular necessity for such an examination arose in the course of an investigation[1] of the dispersion relations for the photoproduction of pions on nucleons. Interpolative formulas for the phase shifts were needed in the evaluation of dispersion integrals and in the photoproduction amplitudes for subsequent calculation of cross sections. Such formulas had been developed earlier by several authors and were considered in this investigation, but were found to be inadequate.

Perhaps the best known and most widely used set of such formulas for experimental s- and p-wave pion–nucleon scattering phase shifts is that of Anderson.[2] That set was obtained by analysis of most of the scattering experiments performed before 1956, on the basis of a three parameter (relativistic Breit–Wigner) fit to δ_{33} and polynomial expansions of the tangents of the small phase shifts in powers of q^2, where q is the pion momentum in the center of mass system.[3] All 15 parameters were varied simultaneously to obtain the best fit to the differential cross sections at all energies, and an over-all error matrix was presented along with the optimum values of the parameters. However, the presence of a negative element on the diagonal of the error matrix did indicate that this solution might not be entirely satisfactory. Anderson's presentation suggested that these expressions for the phase shifts should adequately describe pion–nucleon scattering up to 300 MeV.

Other authors have restricted their attention either to low energies where only the s-wave scattering lengths and a simple fit for δ_{33} are required, or to δ_{33} alone as the dominant factor near the resonance.

In the present investigation each s- and p-wave phase shift was treated individually. The Fermi-type phase shifts found in individual experiments at various energies were taken as the experimental points to be fitted by the interpolative expressions. This investigation sought simple expressions for $\tan \delta_l/q^{2l+1}$ which would fit the available data in the energy region up to 600 MeV. This form is suggested by the threshold dependence of partial wave phase shifts for a short range potential ($\tan \delta_l$ proportional to q^{2l+1}) and by the effective range approximation for nuclear forces.[4] For the dominant (3, 3) phase shift, for which the data are most plentiful and precise, polynomial fits with two, three, and six parameters were calculated by a least squares fitting program on ILLIAC, the digital computer of the University of Illinois. For the s-wave phase shifts and the small p-wave phase shifts there were relatively fewer data, and the available data were much more scattered. The fitting polynomials for these small phase shifts were calculated by hand. No fit of d-wave phase shifts was made in this investigation. To indicate the behavior of the phase shift expressions which were obtained, the real parts of the forward scattering amplitudes were calculated and compared to the results of evaluation of the dispersion relations by various authors.

THE (3, 3) PHASE SHIFT

As has been known for some time, the resonant state phase shift δ_{33} can be well represented by a two parameter fit within the energy region between 0 and about 150 MeV. This is the straight line

* This work was supported by the U. S. Office of Naval Research and the General Electric Company.

[1] J. M. McKinley, Technical Report No. 38 University of Illinois, Urbana, 1962.

[2] H. L. Anderson, in *Rochester Conference on High Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1956), Sec. I, p. 20.

[3] The system of units which is used has $\hbar = c = \mu = 1$.

[4] See, for example, J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).

Chew–Low[5] plot: $q^3 \cot \delta_{33}/\omega = 3(1-\omega/\omega_r)/4f^2$, where ω is the reduced total energy in the center of mass system. In some previous analyses, a value for the resonant energy ω_r was assumed, commonly 2.17, and a best value for the coupling parameter f^2 was found by a least squares or other fitting process. In the present analysis, both parameters were allowed to vary and a least squares fit was obtained, including an error matrix. Table I lists various de-

TABLE I. Two parameter Chew–Low fits.

Author	ω_r	f^2
Orear[a]	2.17	0.087
Puppi[b]	2.17	0.088
Barnes *et al.*[c]	2.17	0.0877 ± 0.0014
Present work	2.239	0.0933

[a] J. Orear, Nuovo Cimento **4**, 856 (1956).
[b] G. Puppi, rapporteur in 1958 *Annual International Conference on High Energy Physics at CERN* (European Organization for Nuclear Research, Geneva, 1958), p. 39.
[c] S. W. Barnes, H. Winick, K. Miyake, and K. Kinsey, Phys. Rev. **117**, 238 (1960).

terminations of the two parameters and Table II is the error matrix of the present analysis. The experimental data which were used included most of the scattering experiments below 130 MeV performed since 1953, and are listed as Set A in the Appendix.

A Chew–Low plot of experimental values of δ_{33} at higher energies, Fig. 1, shows that the straight-line fit of low energies no longer describes the experiments, and further that no single straight line can fit all experiments at all energies. The three param-

TABLE II. Error matrix for Chew–Low fit.

	ω_r	f^2
ω_r	0.002655	−0.0002883
f^2		0.00003647
	$\chi^2_{\text{observed}} = 8.18$	
	$\chi^2_{\text{expected}} = 19$	

eter fit of Anderson, which also appears on Fig. 1, is a step toward a better over-all fit, but even it does not "bend" enough to fit well at higher energies. Mukhin *et al.*[6] discovered a three parameter expansion of $q^3 \cot \delta_{33}$ in powers of q^2 which fits rather well at low and at high energies, but as they pointed out, the experiments near the resonance are not fitted so well. Other authors have abandoned altogether the

[5] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).
[6] A. I. Mukhin, E. B. Ozerov, B. Pontecorvo, E. L. Grigoriev, and N. A. Mitin, in *Symposium on High-Energy Accelerators and Pion Physics* (European Organization for Nuclear Research, Geneva, 1956), Vol. II, p. 221.

form suggested by the static Chew–Low theory. Thus Höhler[7] has discovered a two parameter fit for energies above about 170 MeV in the form $\sin^2 \delta_{33} = q^3 A \exp(-\omega/\sigma)$, but it may be noted that this form does not distinguish between a phase shift just

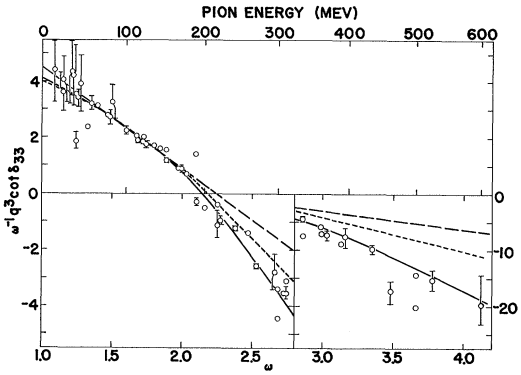

FIG. 1. Chew–Low plot: $q^3 \cot \delta_{33}/\omega$ vs ω. The solid curve is the three parameter fit of this investigation [Table III and Eq. (3)]. The short-dashed curve is the polynomial fit of Anderson (see footnote 2). The long-dashed curve is the low energy two parameter fit of this investigation (Table I).

smaller than 90° and one the same amount larger than 90°. On the other hand, Martin[8] has discovered a two parameter fit to δ_{33} itself.

In order to extend a fit of the Chew–Low type to higher energies, the present investigation included attempts to fit either $q^3 \cot \delta_{33}$ or $q^3 \cot \delta_{33}/\omega$ with a polynomial in either q^2 or ω. In general, it was found that the best least squares fit (smallest value of χ^2) for a given number of parameters was obtained by expanding $q^3 \cot \delta_{33}$ in powers of q^2, as suggested by Mukhin *et al.* The experimental data which were

TABLE III. Coefficients of $q^3 \cot \delta_{33} = a_0 + a_1 q^2 + a_2 q^4$.

Author	a_0	a_1	a_2
Mukhin *et al.*[a]	4.3	0.6	−0.8
Present work	4.108	0.7987	−0.8337

[a] A. I. Mukhin, E. B. Ozerov, B. Pontecorvo, E. L. Grigoriev, and N. A. Mitin, in *Symposium on High-Energy Accelerators and Pion Physics* (European Organization for Nuclear Research, Geneva, 1956), Vol. II, p. 221.

considered included most of the determinations of δ_{33} up to 525 MeV (Set A plus Set B as listed in the Appendix). Whenever both *s-p* and *s-p-d* analyses were available, the *s-p-d* analysis was used. In order to include even those experiments where no standard deviation for δ_{33} was originally quoted, a standard deviation was estimated by considering the standard deviations quoted for each measured differential

[7] G. Höhler, Nuovo Cimento **16**, 585 (1960).
[8] A. Martin, Nuovo Cimento **13**, 241 (1959).

cross section in such experiments and comparing to other nearby experiments with over-all standard deviations. The values obtained for the polynomial coefficients were changed very little by halving or doubling these estimated standard deviations.

A three parameter fit which was obtained is compared with the fit of Mukhin *et al.* in Table III, and the corresponding error matrix is given in Table IV. The curve for this fit also appears in Fig. 1. There is a probability of less than 0.03% that such a large χ^2 is statistical only. Most of the contribution to χ^2 comes from a few experiments in the range 200 MeV to 300 MeV, for which the quoted standard deviations are quite small. These experimental points nearly alternate above and below any smooth curve which fits at higher and lower energies. It may be that the quoted standard deviations are smaller than the experiments actually warrant.

TABLE IV. Error matrix for three parameter fit.

	a_0	a_1	a_2
a_0	0.04290	-0.03478	0.006167
a_1		0.03065	-0.005828
a_2			0.001206
		$\chi^2_{\text{observed}} = 59.9$	
		$\chi^2_{\text{expected}} = 41$	

A six parameter fit for δ_{33} was also found, based upon the same experimental data as the three parameter fit above. The coefficients and the associated error matrix for this fit are given in Table V. This fit is no better statistically than the three parameter fit, mainly because of the same intermediate energy experiments mentioned above. The large covariances seen in the error matrix, and the absence of relative constancy of a given coefficient as the number of coefficients is changed, are indicative that a polynomial expansion is not the best direction to seek for an analytic approximation for δ_{33}. The only advantage of such an expansion is simplicity in programming an electronic digital computer for subsequent calculations.

THE SMALL PHASE SHIFTS

For the remaining s- and p-wave phase shifts, it was felt that the experimental data presently available were scattered so widely that a complete least squares analysis was not called for. Accordingly, the polynomial expressions indicated below were obtained by passing a straight line or parabola through the apparent center of the distribution of experimental points. Any of the coefficients quoted can be changed by ten percent or more without seriously degrading the quality of the fit. Consequently, there are no error matrices or standard deviations given for these coefficients. The experiments which were considered are all of those listed in Sets A, B, and C of the Appendix.

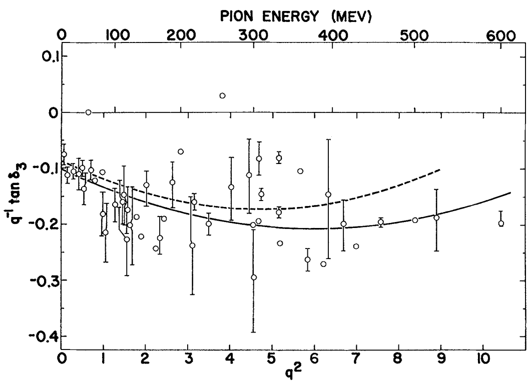

FIG. 2. Tan δ_3/q vs q^2. The solid curve is the polynomial fit of this investigation [Eq. (1)]. The short-dashed curve is the polynomial fit of Anderson (see footnote 2).

Among these small phase shifts, only δ_3 is at all well represented above 200 MeV by Anderson's formulas. Figure 2 shows the experimental data, together with Anderson's fit and the parabola obtained in the present work, which is given by $\tan \delta_3/q = -0.10 - 0.036q^2 + 0.003q^4$. This expression agrees rather well with the determinations of the

TABLE V. Coefficients and error matrix of $q^3 \cot \delta_{33} = \sum_{i=0}^{5} a_i q^{2i}$.

	Coefficients	Error Matrix a_0	a_1	a_2	a_3	a_4	a_5
a_0	4.968	0.3869	-0.8791	0.6787	-0.2276	0.03351	-0.001729
a_1	-1.174		2.176	-1.766	0.6122	-0.09216	0.004825
a_2	1.474			1.481	-0.5262	0.08065	-0.004275
a_3	-0.8859				0.1907	-0.02968	0.001591
a_4	0.1448					0.004681	-0.0002534
a_5	-0.008082						0.00001384
				$\chi^2_{\text{observed}} = 55.0$			
				$\chi^2_{\text{expected}} = 38$			

threshold scattering lengths by Orear[9] ($a_3 = -0.105 \pm 0.010$), and by Hamilton and Woolcock[10] ($a_3 = -0.087$).

Figure 3 shows the experimental data for δ_1 and the fit given by Anderson. It is evident that Anderson's fit is not representative of experiments above 200 MeV. However, the attempt to obtain a better fit is beset by a difficulty which also arises in the cases of the other phase shifts for total isobaric spin $\frac{1}{2}$. A straight line can fit the low energy (<50 MeV) and the high energy (>250 MeV) points, but does not represent the intermediate points. Such a straight line given by $\tan \delta_1/q = 0.17 + 0.02q^2$ is shown as the long-dashed curve in Fig. 3. If the

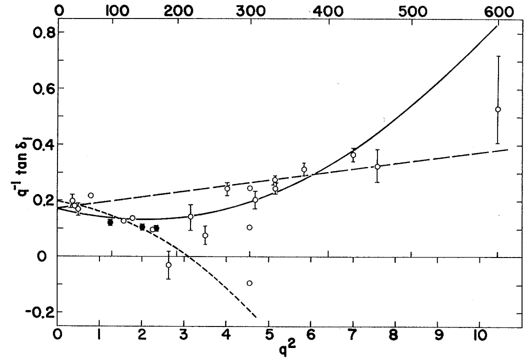

FIG. 3. Tan δ_1/q vs. q^2. The solid curve is the polynomial fit of Set X, which attempts to include all experiments [Eq. (4)]. The long-dashed curve is the polynomial fit of Set Y, which ignores three intermediate energy experiments [Eq. (7)]. The short-dashed curve is the polynomial fit of Anderson (see footnote 2). The solid circles indicate the three particular experiments.

intermediate points are included, particularly those at 98 MeV,[11] 150 MeV,[12] and 170 MeV,[12] a parabola is required. Such a parabola, given by $\tan \delta_1/q = 0.17 - 0.04q^2 + 0.01q^4$, is shown as the solid curve in Fig. 3. Again either of these expressions agrees well enough with the scattering lengths as found by Orear ($a_1 = 0.167 \pm 0.012$) and by Hamilton and Woolcock ($a_1 = 0.178$).

Figure 4 shows the experimental data for δ_{11} and the fit given by Anderson. Again Anderson's fit is not applicable above 200 MeV, although it fits the lower energy points quite well. The intermediate energy experiments, particularly the three mentioned above, again have a considerable effect upon the analysis. When they are included, the straight

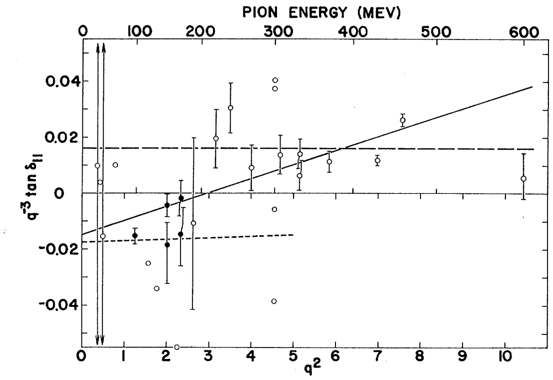

FIG. 4. Tan δ_{11}/q^3 vs q^2. The solid curve is the polynomial fit of Set X, which attempts to include all experiments [Eq. (5)]. The long-dashed curve is the polynomial fit of Set Y, which ignores three intermediate energy experiments [Eq. (8)]. The short-dashed curve is the polynomial fit of Anderson (see footnote 2). The solid circles indicate the three particular experiments. (Two different evaluations are shown for two of these experiments.)

line given by $\tan \delta_{11}/q^3 = -0.015 + 0.005q^2$ is appropriate. If they are ignored, all of the remaining low- and high-energy experiments are adequately described by a single parameter, namely, $\tan \delta_{11}/q^3 = 0.016$. Both of these curves are also shown on Fig. 4.

The experimental points for δ_{31}, as shown on Fig. 5, are scattered quite widely, but after attempts at fits of various types it was discovered that a modified Chew–Low formula gave the most reasonable fit for a given number of parameters. The curve shown is given by $\tan \delta_{31}/q^3 = (-0.13 + 0.072\omega - 0.012\omega^2)/\omega$. The Anderson fit also shown is completely inappropriate even at 150 MeV.

The last of the s- and p-wave phase shifts, δ_{13}, appears on Fig. 6. Again the more recent high-energy experimental points do not coincide with Anderson's fit. In fact, only the intermediate experiments (from 120 MeV to 170 MeV) do agree with Ander-

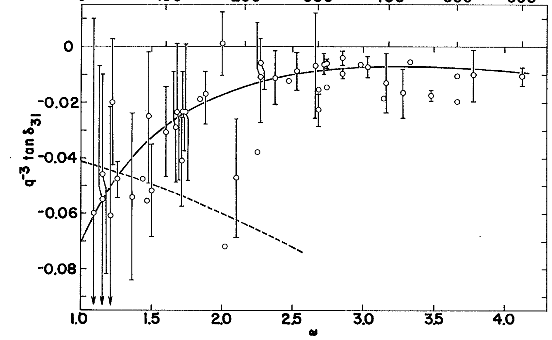

FIG. 5. Tan δ_{31}/q^3 vs ω. The solid curve is the polynomial fit of this investigation [Eq. (2)]. The short-dashed curve is the polynomial fit of Anderson (see footnote 2).

[9] J. Orear, Nuovo Cimento **4**, 856 (1956).
[10] J. Hamilton and W. S. Woolcock, Phys. Rev. **118**, 291 (1960).
[11] D. N. Edwards, S. G. F. Frank, and J. R. Holt, Proc. Phys. Soc. (London) **73**, 856 (1959).
[12] J. Ashkin, J. P. Blaser, F. Feiner, and M. O. Stern, Phys. Rev. **101**, 1149 (1959).

son's expression, and there is no obvious smooth curve which can adequately represent all energies. If all experiments are considered, all that can be said is that δ_{13} is quite small. A single parameter expression, namely, $\tan\delta_{13}/q^3 = -0.0035$, is shown on Fig. 6, but the parameter is not really determined within 100%. However, here as with δ_1 and δ_{11} a

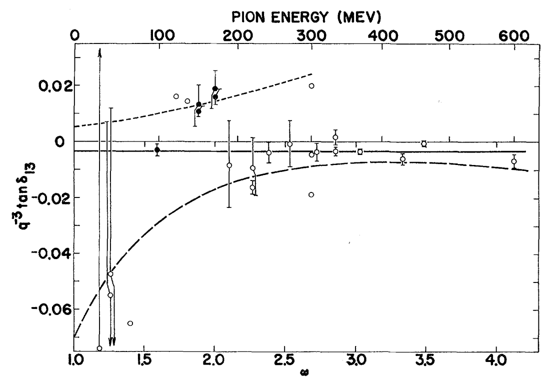

FIG. 6. Tan δ_{13}/q^3 vs. ω. The solid curve is the polynomial fit of Set X, which attempts to include all experiments [Eq. (6)]. The long-dashed curve is the polynomial fit of Set Y, which ignores three intermediate energy experiments. (This curve is the same polynomial fit which was found for δ_{31} and shown in Fig. 5.) The short-dashed curve is the polynomial fit of Anderson (see footnote 2). The solid circles indicate the three particular experiments. (Two different evaluations are shown for two of these experiments.)

quite different fit can be made if the experiments at 98 MeV, 150 MeV, and 170 MeV are ignored. In this event a reasonable fit is obtained by setting $\delta_{13} = \delta_{31}$, which is exactly the prescription of the static theory of Chew and Low.

Only a relatively small number of experiments, mainly above 300 MeV, have required d-wave phase shifts in their analysis. Below that energy the above considered s- and p-wave phase shifts are generally adequate to represent the scattering differential cross sections, so that the expressions found here can be expected to provide a more reasonable over-all description of low energy pion–nucleon interactions than has been available before.

THE COMPLETE SET OF PHASE SHIFTS

Because all of the isobaric spin-$\frac{1}{2}$ phase shifts appeared to indicate that the experiments at 98 MeV, 150 MeV, and 170 MeV might not follow the trend of the other experiments, two different sets of expressions were found for the scattering phase shifts. Both sets use the same expressions for the isobaric spin-$\frac{3}{2}$ phase shifts, which are assembled here for convenience in reference.

$$\tan\delta_3/q = -0.10 - 0.036q^2 + 0.003q^4, \quad (1)$$

$$\tan\delta_{31}/q^3 = (-0.13 + 0.072\omega - 0.012\omega^2)/\omega, \quad (2)$$

$$q^3\cot\delta_{33} = 4.108 + 0.7987q^2 - 0.8337q^4. \quad (3)$$

One set of phase shifts, hereafter labeled Set X, incorporated the three particular experiments along with the others. The expressions for the remaining phase shifts of Set X are

$$\tan\delta_1/q = 0.17 - 0.04q^2 + 0.01q^4, \quad (4)$$

$$\tan\delta_{11}/q^3 = -0.015 + 0.005q^2, \quad (5)$$

$$\tan\delta_{13}/q^3 = -0.0035. \quad (6)$$

The other set, hereafter labeled Set Y, ignored these three experiments. The expressions for the isobaric spin-$\frac{1}{2}$ phase shifts of Set Y are

$$\tan\delta_1/q = 0.17 + 0.02q^2, \quad (7)$$

$$\tan\delta_{11}/q^3 = 0.016, \quad (8)$$

$$\tan\delta_{13}/q^3 = \tan\delta_{31}/q^3. \quad (9)$$

The difficulty in reconciling the experiments at 98 MeV, 150 MeV, and 170 MeV with other experiments has been mentioned by Barnes *et al.*[13]

As a first indication of the over-all behavior of the phase shift expressions thus obtained, the real parts of the amplitudes for forward scattering of positive and negative pions on protons in the center of mass system were calculated according to the formulas:

$$D_+^B(0) = (\sin 2\delta_3 + \sin 2\delta_{31} + 2\sin 2\delta_{33})/2q, \quad (10)$$

$$D_-^B(0) = (2\sin 2\delta_1 + \sin 2\delta_3 + 2\sin 2\delta_{11} + \sin 2\delta_{31} + 4\sin 2\delta_{13} + 2\sin 2\delta_{33})/6q. \quad (11)$$

Figure 7 for $D_+^B(0)$ and Fig. 8 for $D_-^B(0)$ show the curves calculated with phase shift sets X and Y and for comparison the curves calculated by integration of the forward scattering dispersion relations by various authors.[14–18]

For positive pions the curve found here agrees quite well with the integrated dispersion relations out to 400 MeV. For negative pions it must be noted that the various evaluations of dispersion relations do not agree among themselves within 20% anywhere between 0 and 400 MeV. While neither of the curves found here agrees completely with any one of the integrated curves, they do generally follow the consensus of the integrated curves,

[13] S. W. Barnes, H. Winick, K. Miyake, and K. Kinsey, Phys. Rev. **117**, 238 (1960).
[14] G. Puppi and A. Stanghellini, Nuovo Cimento **5**, 1305 (1957).
[15] H. J. Schnitzer and G. Salzman, Phys. Rev. **112**, 1802 (1958).
[16] J. W. Cronin, Phys. Rev. **118**, 824 (1960).
[17] T. D. Spearman, Nuovo Cimento **15**, 147 (1960).
[18] N. P. Klepikov, V. A. Meshcheryakov, and S. N. Sokolov, D-584 (Joint Institute for Nuclear Studies, Dubna, 1960).

at least out to about 300 MeV. At the higher energies one can expect that an s- and p-wave fit would describe π^- scattering less well than π^+, since only π^- scattering is affected by the higher partial waves with total isobaric spin $\frac{1}{2}$ which are involved in the second and third pion–nucleon resonances. However, neither set of phase shifts can be considered to eliminate completely the Puppi–Stanghellini[14] discrepancy, in that $D_-^B(0)$ as found here is still apparently too large below the resonance and too small immediately above the resonance.

These sets of phase shifts must be considered as only an interim improvement in the representation of the experimental results. A more detailed investigation of the energy dependence of the scattering phase shifts would be worth while. This detailed investigation could take the form of a simultaneous determination of coefficients of a set of polynomials for s-, p-, and d-wave phase shifts to give the best fit to all of the experimental scattering differential cross sections, rather than to the phase shifts determined from individual experiments. This is the same technique which was employed by Anderson,[2] but now there are available more accurate experimental measurements, extending over a wider selection and range of energies. In particular, this technique would allow incorporation of π^- scattering experiments which cannot be individually analyzed into phase shifts because no corresponding π^+ experiment was performed.

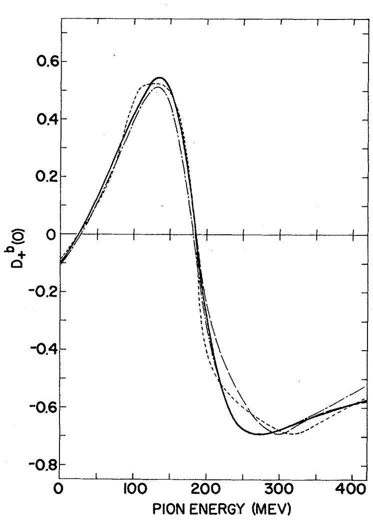

FIG. 7. Real part of barycentric forward scattering amplitude for π^+ on protons vs pion laboratory energy. The solid curve is calculated from the polynomial fits for phase shifts. The broken curves are results of evaluation of the dispersion relations by various authors as follows: long-dashed, Cronin (see footnote 16); short-dashed, Spearman, (see footnote 17); dash-dot, Klepikov, Meshcheryakov, and Sokolov (see footnote 18).

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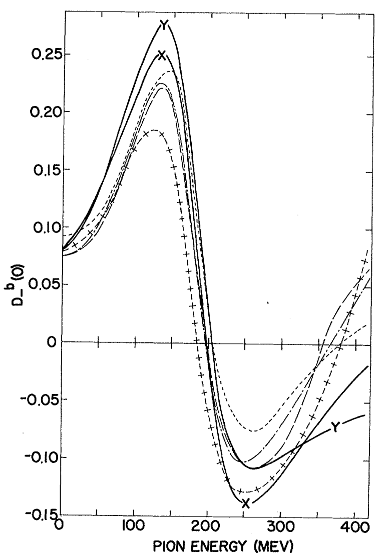

FIG. 8. Real part of barycentric forward scattering amplitude for π^- on protons vs pion laboratory energy. The solid curves are calculated from Sets X and Y of the polynomial fits for phase shifts, as labeled. The broken curves are results of evaluation of the dispersion relations by various authors as follows: long-dashed, Schnitzer and Salzman (see footnote 15); short-dashed, Spearman (see footnote 17); dash-dot, Klepikov, Meshcheryakov and Sokolov (see footnote 18); $-+-+$, Puppi and Stanghellini (see footnote 14).

APPENDIX: PION-NUCLEON SCATTERING EXPERIMENTS

Set A. The following experiments were used alone to find the two parameter Chew–Low fit to δ_{33} at low energies. In combination with the experiments of Set B below, they were used to find the three and six parameter fits to δ_{33}. In combination with the experiments of Sets B and C below, they were used to find the polynomial fits for the other s- and p-wave phase shifts.

15, 25, 35 MeV	G. E. Fischer and E. W. Jenkins, Phys. Rev. **116**, 749 (1959).
24.8	D. Miller and J. Ring, Phys. Rev. **117**, 582 (1960). See also D. Miller, Nucl. Phys. **14**, 238 (1959).
37	A. M. Sachs, H. Winick, and B. A. Wooten, Phys. Rev. **109**, 1750 (1958).
40	J. P. Perry and C. E. Angell, Phys. Rev. **91**, 1289 (1953).
41.5	S. W. Barnes, B. Rose, G. Giacomelli, J. Ring, K. Miyake, and K. Kinsey, Phys. Rev. **117**, 226 (1960). See also D. Miller, Nucl. Phys. **14**, 238 (1959).
45	J. Orear, J. J. Lord, and A. B. Weaver, Phys. Rev. **93**, 575 (1954).
53, 78, 120	H. L. Anderson, E. Fermi, R. Martin, and D. E. Nagle, Phys. Rev. **91**, 155 (1953).
58	D. Bodansky, A. M. Sachs, and J. Steinberger, Phys. Rev. **90**, 997 (1953).
65	D. Bodansky, A. M. Sachs, and J. Steinberger, Phys. Rev. **93**, 1367 (1954).
80	G. Ferrari, L. Ferretti, R. Gessaroli, E. Manaresi, G. Puppi, G. Quareni, A. Ranza, and A. Stanghellini, in *Symposium on High-Energy Accelerators and Pion Physics* (Euro-

	pean Organization for Nuclear Research, Geneva, 1956), Vol. II, p. 230. Analyzed by Anderson and Davidon.[19]
83	L. Ferretti, G. Quareni, M. Della Corte, and T. Fazzini, Nuovo Cimento **5**, 1660 (1957).
100	R. Gessaroli, G. Quareni, G. Dascola, S. Mora, and G. Todesco, Nuovo Cimento **5**, 1658 (1957). Analyzed by Anderson and Davidon.[19]
113	J. Orear, Phys. Rev. **96**, 1417 (1954). Analyzed by Anderson and Davidon,[19] and by Mukhin, Popova, and Tentiukova.[20]
114, 124	L. Ferretti, E. Manaresi, G. Puppi, G. Quareni, and A. Ranza, Nuovo Cimento **1**, 1238 (1955). Analyzed by Anderson and Davidon.[19]
119	A. Loria, P. Mittner, R. Santangelo, G. Zago, A. E. Brenner, and L. Montanet, in *Ninth International Annual Conference on High Energy Physics*, Kiev, July 1959 (Academy of Science USSR, Moscow, 1960), Plenary Sessions I–V, p. 102.
120	L. Ferretti, E. Manaresi, G. Puppi, G. Quareni, and A. Ranza, Nuovo Cimento **1**, 1238 (1955). G. Ferrari, E. Manaresi, and G. Quareni, Nuovo Cimento **5**, 1651 (1957). Analyzed by Anderson and Davidon,[19] and by Mukhin, Popova, and Tentiukova.[20]

Set B. The following experiments were used in combination with the experiments of Set A above to find the three and six parameter fits to δ_{33}. In combination with the experiments of Sets A and C, they were used to find the polynomial fits for the other *s*- and *p*-wave phase shifts.

135 MeV	H. L. Anderson, E. Fermi, R. Martin, and D. E. Nagle, Phys. Rev. **91**, 155 (1953).
142	J. J. Lord and A. B. Weaver, reported in Anderson and Davidon.[19]
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