REVIEWS OF MODERN PHYSICS VOLUME 35, NUMBER 2 APRIL 1963

Location of Neutron Single-Particle Levels from Stripping Reactions

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METHODS

THE stripping process,[1] as for example the (d,p) reaction, is an extremely useful tool for studying the single-particle states of the nuclear shell model. Two of the simplest experimental measurements, the cross sections for excitation of the various states of the final nucleus, and the angular distributions of the proton groups corresponding to these states, give essentially all of the pertinent information. The angular distributions give a determination of the orbital angular momentum l of the "stripped" neutron; this assigns the spin and parity to each level except for the ambiguity over whether $j = (l + \frac{1}{2})$ or $(l - \frac{1}{2})$. The cross sections then give a measure of how much of the shell-model state is contained in each level of the proper spin and parity.

The ambiguity between the two members of the spin-orbit doublet is usually easy to resolve near closed shells because of the large energy separation between them. One of them is usually very near the ground state while the other is at rather high excitation energy. In many cases, some spins are known from decay scheme work or direct measurements with atomic beam or hyperfine structure techniques. Additional help is often derived from the fact that the ratio of the (d,p) cross sections (summed over all levels belonging to the shell-model state) for the $(l + \frac{1}{2})$ and $(l - \frac{1}{2})$ states is easily estimated from theory.[2] Another method is to compare (d,p) and (d,t) cross sections for exciting the same states[3] (using different target nuclei, of course). In principle, an unambiguous determination of j could be obtained from measurements of the polarization of the protons, or from their angular correlation with the de-excitation γ rays, but these techniques are very difficult and have not yet been applied for this purpose. The simple techniques described above have generally given rather unambiguous results; there are undoubtedly a few cases where weakly excited levels have been improperly assigned, but these would not change appreciably the results to be presented below.

When a shell is partly filled, residual interactions between the neutrons in that shell have an important effect on the location of the observed nuclear levels. These effects can be taken into account theoretically to obtain locations of the unperturbed shell-model levels,[4] but at best this process is quite uncertain. It is far preferable to locate shell-model levels in nuclei with a closed shell plus one neutron (by bombarding a closed-shell nucleus). In these nuclei, there are no residual interactions to be taken into account, so that the location of the nuclear levels give directly the location of the shell-model states.

It should not be assumed, however, that the entire shell-model level is concentrated in one nuclear level in these nuclei. There are often other levels of the same spin and parity (arising generally from excited proton configurations) in the same energy region, and these mix with the unperturbed shell-model state. As a result, the shell-model state is often distributed among several nuclear levels. The range of this mixing is about W, the depth of the imaginary potential in the optical model. It is expected that W should increase with excitation energy; it is well determined at high excitation energies from elastic scattering studies, and a crude extrapolation of these data to the low excitation energy region gives[5]

$$W \simeq 0.33\ E^* \tag{1}$$

where E^* is the excitation energy. Thus this mixing leads to little difficulty near the ground state, but rapidly becomes important at higher excitations. Of course, if there are no other unperturbed levels of the proper spin and parity in the same energy region, there can be no mixing. This is the situation where the proton shell is closed (e.g., Pb^{207}, Pb^{209}) or sometimes for opposite parity levels (e.g., the $g_{9/2}$ level in the Ni isotopes). When a shell-model level is

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[1] See, for example, S. T. Butler, *Nuclear Stripping Reactions* (John Wiley & Sons, Inc., New York, 1957).

[2] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).

[3] B. L. Cohen, Phys. Rev. **125**, 1358 (1962).

[4] B. L. Cohen and R. E. Price, Phys. Rev. **121**, 1441 (1961).

[5] B. L. Cohen, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. **126**, 698 (1962).

mixed among many nuclear levels, the location is taken as the center of gravity of the group weighting each level in proportion to how strongly it is excited in the (d,p) reaction. While this procedure has not been accurately justified, it seems most expedient and it surely cannot lead to important errors.

One important problem is to be certain that all levels belonging to the shell-model state are included. Experimental energy resolutions are often not sufficient for complete resolutions of every level, especially at high excitation energies. This difficulty is most severe for high l states for which the (d,p) cross section is intrinsically low. Some aid in this problem may be derived from Eq. (1), which gives an estimate of how far one must look for components of a shell-model state. But by far the most useful method is to compare the summed cross section with the predictions of Distorted Wave Born Approximation (DWBA) calculations. While these predictions are often in error by a factor of two or more, the relative cross sections for the various l are generally much more reliable than this. The DWBA calculations are also most useful in the determination of l values from angular distributions, and in correcting cross sections for Q-value dependence. For example, in finding the center of gravity of levels belonging to a given shell-model state, one does not weight each level according to its (d,p) cross section, but rather according to its spectroscopic factor S, which is the ratio of the observed (d,p) cross section to the DWBA cross section for the shell-model state assuming it to be located at that energy. This procedure effectively eliminates the Q-value dependence of the (d,p) cross section—a factor from nuclear reaction theory—from the problem of locating the shell-model state, a concept of nuclear structure theory.

Everything that has been said above on the location of particle states with (d,p) reactions applies equally well to the location of neutron hole states with (p,d) or (d,t) reactions. Here, the most advantageous reactions are those in which closed-shell nuclei are bombarded so that the residual nucleus has a closed shell minus one neutron. Here again there can be no residual interactions, so that the location of the nuclear levels gives directly the location of the shell-model states, but with the sign of the energies reversed. As of now, the work on pick-up reactions is in a more primitive state than that on stripping. There have been almost no DWBA calculations on (d,t) reactions, partly because there have been no data on elastic triton scattering from which to derive optical model parameters for tritons. Furthermore, tritons from 15-MeV deuteron induced (d,t) reactions in heavy nuclei are of too low an energy to exhibit sharply different angular distributions for different l values. Consequently, much of the work to date has depended on systematics of relative cross sections among neighboring elements where the spin and parity of some levels is known from other sources such as decay scheme work or stripping experiments. However, work on (p,d) reactions is increasing rapidly, so that better information should be forthcoming soon.

In general, the job of locating single-particle levels with stripping reactions should be considered to be fairly well advanced, but still very far from completion. While almost all assignments among low-lying levels are probably correct, there is much room for misinterpretation of the data for more highly excited states. For these latter, the results to be presented are in some cases little more than preliminary estimates.

RESULTS

The excitation energies of various single-particle states are listed in Table I. A few comments on these are in order. In Ni^{59}, the $p_{1/2}$ state is slightly filled so that a correction for this was applied by use of pairing theory. It lowered the energy from 1.7 to 1.4 MeV. In both Ni^{59} and Fe^{54} the position of the $d_{5/2}$ state is somewhat uncertain because there is no method for distinguishing between $d_{5/2}$ and $d_{3/2}$ levels. The location of the $s_{1/2}$ state in these nuclei is also somewhat uncertain as some of its components lie in an energy region where all levels are not resolved. In Zr^{89}, two p states were observed experimentally,[6] at 0.6 and

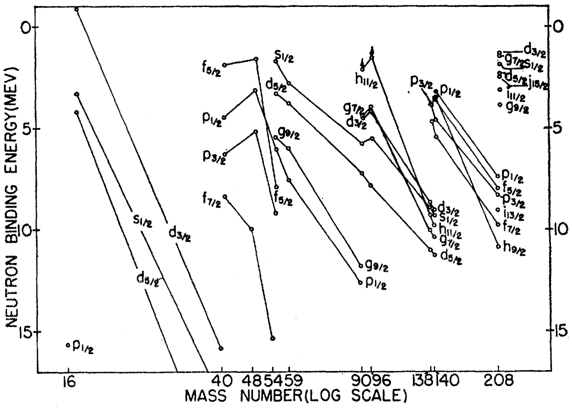

FIG. 1. Binding energy of single-particle and single-hole states in various nuclei. Data are from reference 7 and Table I. Some states in A = 16 and A = 40 are off-scale; their locations may be obtained from Table I.

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[6] C. D. Goodman (to be published).

TABLE I. Excitation energies of various single particle and single hole-states. All energies are in MeV above the ground state.

O^{15}—reference a		O^{17}—reference a		Ca^{39}—reference b		Ca^{41}—references a and c		Ca^{49}—reference d		Fe^{53}—reference e	
$p_{1/2}$	0	$d_{5/2}$	0	$d_{3/2}$	0	$f_{7/2}$	0	$p_{3/2}$	0	$f_{7/2}$	~ 2.0
$p_{3/2}$	6.16	$s_{1/2}$	0.87	$s_{1/2}$	2.6	$p_{3/2}$	2.1	$p_{1/2}$	2.03		
		$d_{3/2}$	5.08	$d_{5/2}$	~ 7	$p_{1/2}$	3.9	$f_{5/2}$	3.6		
						$f_{5/2}$	6.5				

Fe^{55}—reference f		Ni^{59}—reference f		Zr^{89}—reference g		Zr^{91}—reference h		Zr^{97}—reference h		Ba^{137}—reference i	
$p_{3/2}$	0.11	$p_{1/2}$	1.4[k]	$g_{9/2}$	0	$d_{5/2}$	0	$s_{1/2}$	0	$d_{3/2}$	0
$f_{5/2}$	1.4	$g_{9/2}$	3.0	$p_{1/2}$	0.8	$s_{1/2}$	1.55	$d_{3/2}$	1.37	$s_{1/2}$	0.29
$p_{1/2}$	3.26	$d_{5/2}$	~ 5.2			$g_{7/2}$	2.70	$g_{7/2}$	1.64	$h_{11/2}$	0.66
$g_{9/2}$	3.86	$s_{1/2}$	~ 6.2			$d_{3/2}$	2.70	$h_{11/2}$	>4.0	$g_{7/2}$	1.40
$d_{5/2}$	~ 6.0					$h_{11/2}$	>5.1			$d_{5/2}$	2.4
$s_{1/2}$	~ 7.6										

Ba^{139}—reference i		Ce^{139}—reference i		Ce^{141}—reference i		Pb^{207}—reference j		Pb^{209}—reference j	
$f_{7/2}$	0	$d_{3/2}$	0	$f_{7/2}$	0	$p_{1/2}$	0	$g_{9/2}$	0
$p_{3/2}$	0.78	$s_{1/2}$	0.25	$p_{3/2}$	0.88	$f_{5/2}$	0.57	$i_{11/2}$	0.77
		$h_{11/2}$	0.75	$f_{5/2}$	1.88	$p_{3/2}$	0.90	$j_{15/2}$	1.41
		$g_{7/2}$	1.34	$h_{9/2}$	~ 1.9	$i_{13/2}$	1.64	$d_{5/2}$	1.56
		$d_{5/2}$	2.2	$p_{1/2}$	2.25	$f_{7/2}$	2.35	$s_{1/2}$	2.03
						$h_{9/2}$	3.47	$g_{7/2}$	2.47
								$d_{3/2}$	2.52

[a] *Nuclear Data Sheets,* National Academy of Sciences National Research Council (U. S. Government Printing Office, Washington, D.C., 1960, 1962).

[b] C. D. Kavaloski, G. Bassani, and N. Hints, Univ. of Minnesota Linear Accelerator Laboratory Progress Report, 1962; P. E. Cavanaugh *et al.* (private communication). We are greatly indebted to these authors for making their results available in advance of publication.

[c] K. Ramavataram, Bull. Am. Phys. Soc. **7**, 302 (1962).

[d] E. Kashy, A. Sperduto, H. A. Enge, and W. W. Buechner, Bull. Am. Phys. Soc. **7**, 315 (1962).

[e] B. Zeidman and T. H. Braid, Bull. Am. Phys. Soc. **7**, 315 (1962); R. D. Lawson and B. Zeidman (to be published).

[f] R. H. Fulmer, A. L. McCarthy, and B. L. Cohen (to be published).

[g] C. D. Goodman (to be published). We are greatly indebted to Dr. Goodman for making his results available in advance of publication.

[h] B. L. Cohen (to be published); see reference 3 for preliminary results.

[i] R. H. Fulmer, A. L. McCarthy, and B. L. Cohen, Phys. Rev. **128**, 1302 (1962).

[j] P. Mukherjee and B. L. Cohen, Phys. Rev. **127**, 1284 (1962).

[k] This level is slightly filled; a 0.3-MeV correction (from pairing theory) has been applied to correct to the situations where the level is empty.

1.0 MeV; we assign both of these as $p_{1/2}$ states. In Ba^{137} and Ce^{139}, the experimental distinction between the $g_{7/2}$ and $d_{5/2}$ states is somewhat uncertain; the evidence is rather good, however, that the average energy of these two states is about equal to the average of the values listed in Table I.

Figure 1 shows the binding energy of a neutron in all single-particle levels plotted vs. mass number. Absolute binding energies of ground states are from reference 7. Points belonging to a given single-particle state are connected by straight lines. Some interesting conclusions that may be drawn from Fig. 1 will be discussed in a forthcoming paper.[8]

ACKNOWLEDGMENTS

The authors are greatly indebted to N. Hintz and C. D. Goodman for making their data available prior to publication. They would also like to acknowledge useful conversations with N. Austern, E. Baranger, and J. P. Schiffer.

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[7] University of California Report, UCRL 5419 (unpublished).

[8] B. L. Cohen, Phys. Rev. **130**, 227 (1963).