

other two Murray-von Neumann types of infinite unitary representations in that it alone would retain (like the finite-dimensional case which it includes) a certain uniqueness of decomposition into reps—uniqueness but not necessarily discreteness since it may involve either a continuous-discrete sum or a discrete-continuous sum.

“For physics we may add the following comments: The concept of multiplicity-freeness enables us to define quite generally the conditions on a set of observables such that, when the observables are measured, the quantum mechanical state is completely specified. In other words the concept of ‘multiplicity freeness’ permits us to state more concisely a general definition which Jauch (1960, 1961) has given of ‘a complete set of commuting observables.’ This definition applies equally well to operators with continuous as with discrete spectra—in contrast with the usual definition of a complete commuting set, requiring that there occur only nondegenerate common eigenvalues. Suppose L is the set of all bounded functions generated by a set of commuting observables. Then L is the smallest ‘weakly closed’ algebra² containing the original set. By a theorem of von Neumann (1929) any such ‘smallest weakly closed algebra’ is equal to its *double commutant*, i.e., the commutant of its commutant:

$$L' = L.$$

In the particular case we are considering, since L is generated by commuting observables it is Abelian. If L is also multiplicity free, i.e., its commutant L' Abelian, then

$$L \subseteq L' \quad L' \subseteq L' \quad \therefore L' = L$$

and L is said to be *maximal Abelian*. Under these conditions the original set of generators of L is said to form a *complete set*. Restating: *A commuting set of*

operators generates with all its bounded functions an Abelian algebra. If this algebra is multiplicity free (i.e., its commutant is Abelian) the original set is complete. One sees that multiplicity freeness, or type I-ness, guarantees the existence of a complete set of commuting observables. Jauch (1960, 1961) has shown that the converse is also true: Type I-ness of the algebra of observables is a necessary and sufficient condition that a complete set of commuting observables exist, i.e., that the theory not exhibit innumerable and irremovable ‘hidden variable’ features.”

Correspondingly the last sentence of the second paragraph, p. 497, should be replaced by:

“Thus for example, the Poincaré group is of type I. This follows from the relation of the Poincaré group to the Lorentz group and from a general theorem of Mackey (1952) about unitary representations of group extensions.”

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Erratum: Simple Groups and Strong Interaction Symmetries

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THE fourth author's name should be B. W. Lee as above instead of W. Lee as printed.