tation of the spin states follows from the conservation of angular momentum in a similar way.

This connection between particle and antiparticle reactions, though equivalent to the field theoretic one, comes out in a much simpler form for the case of particles with spin. Usually some rather awkward manipulations involving multiplications by appropriate spinors are required. These manipulations, in effect, eliminate the redundant variables associated with the use of, say, four-component Dirac fields to describe spin 1/2 particles. Only two components are really needed, and the S-matrix approach leads directly to a simple covariant two-component formalism for spin 1/2 particles. For higher spins one gets a covariant description involving only the necessary $(2S + 1)$ components.

A principal triumph of axiomatic field theory is the proof of the normal connection between spin and statistics. This connection follows also from the S-matrix postulates, provided, in addition, that the magnitudes of self-conjugate combinations of particle-antiparticle amplitudes are not in principle unobservable. We know experimentally of certain combinations, the K_1 and K_2, that are in fact observable. This added assumption, which is analogous to one needed until recently in field theoretic proofs, can probably be eliminated if the full power of the analyticity postulate is utilized.

SUMMARY

The general properties of the S matrix usually deduced from field theory can be derived from postulates expressing very general physical principles. This provides a basis for the establishment of S-matrix theory as an independent and self-contained framework for describing elementary-particle physics, a framework suited to the modern practical calculations in this field.

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S-Matrix Theory of Strong Interactions without Elementary Particles*†

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1. INTRODUCTION

In this paper I present an indecently optimistic view of strong interaction theory. My belief is that a major breakthrough has occurred and that within a relatively short period we are going to achieve a depth of understanding of strong interactions that a few years ago I, at least, did not expect to see within my lifetime. I know that few of you will be convinced by the arguments given here, but I would be masking my feelings if I were to employ a conventionally cautious attitude in this talk. I am bursting with excitement, as are a number of other theorists in this game.

I present my view of the current situation entirely in terms of the analytically continued S matrix, because there is no other framework that I understand for strong interactions. My oldest and dearest friends tell me that this is a fetish, that field theory is an equally suitable language, but to me the basic strong-interaction concepts, simple and beautiful in a pure S-matrix approach, are weird, if not impossible, for field theory. It must be said, nevertheless, that my own awareness of these concepts was largely achieved through close collaboration with three great experts in field theory, M. L. Goldberger, Francis Low, and Stanley Mandelstam. Each of them has played a major role in the development of the strong interaction theory that I describe,[1] even though the language of my description may be repugnant to them. Murray Gell-Mann, also, although he has not actually published a great deal on the analyticity aspects of strong interactions, has for many years exerted a major positive influence both on the subject and on

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† Invited paper at the New York meeting of the American Physical Society, January 1962.

[1] A brief review of the development of S-matrix theory, with references, may be found in G. F. Chew, *The S-Matrix Theory of Strong Interactions* (W. A. Benjamin and Company, Inc., New York, 1961).

me[2]; his enthusiasm and sharp observations of the past few months have markedly accelerated the course of events as well as my personal sense of excitement.

2. THE POSTULATES

In addition to the postulates discussed by Stapp[3] in the preceding paper, there are three assumptions underlying the S-matrix theory of strong interactions that I discuss here. First of all, Frautschi and I propose extending the maximal analyticity postulate to angular as well as linear momenta, thereby eliminating (as I explain later) the possibility of elementary particles[4]; future developments may well show that such a circumstance is unavoidable and requires no separate assumption. Philosophically speaking, I would motivate both of these maximal analyticity postulates by the principle of "lack of sufficient reason." It seems natural for an S-matrix element to vary smoothly as energies and angles are changed, and a natural mathematical definition of smoothness lies in the concept of analyticity. The fundamental principle therefore is of maximum smoothness. The S matrix has only those irregularities necessary to satisfy unitarity. There is no "reason" for any others. Similarly, as Feynman and Heisenberg have both emphasized, there is no reason why some particles should be on a different footing from others. The elementary particle concept is unnecessary, at least for baryons and mesons.

The second assumption may turn out to be closely related to the first, perhaps even a consequence, but Frautschi and I use it at present as an independent principle.[5] This is the postulate of maximum strength: *Strong interactions saturate the unitarity condition.* Forces in the S-matrix framework are bounded in strength by unitarity, since they are determined by scattering amplitudes in the crossed reactions reached by analytic continuation. It is possible, therefore, to assume that all forces are "as strong as possible" so as to eliminate dimensionless coupling parameters from the theory. I shall explain later how this second postulate is applied in practice, and what are some of its experimental consequences. The final postulate is less satisfying from an aesthetic standpoint but at present seems unavoidable. It is: *The quantities isotopic spin* (I), *strangeness* (S), *and baryon number* (B) *are conserved.* Frautschi and I are hopeful that a connection eventually will be found between this ugly assumption and the preceding two, but at present we have no proposals in such a direction.

That the foregoing three postulates, together with those outlined by Stapp, lead to a complete and self-consistent theory of strong interactions has not yet been demonstrated—much less has it been shown that they explain all experimental facts. No inconsistencies have yet become apparent, however, and the sum of the successful experimental predictions is impressive. Predictions both quantitative and qualitative, based on the postulate of maximal analyticity in *linear* momenta together with the conservation laws, have been emerging in a steady stream since 1955, when the relevance of dispersion relations to strong interactions was first recognized. To date, none of these predictions has failed. The current wave of excitement, however, stems from predictions associated with the postulates of maximum strength and maximal analyticity in *angular* momentum. To explain these predictions, I must first tell you about the work of Froissart and of Regge.

3. RESULTS OF FROISSART AND REGGE

It was Regge who drew attention to the possibility of unique analytic continuation in angular momentum, and I shall come quickly to his important results for nonrelativistic potential scattering. Froissart, however, has produced the first rigorous results in this connection for the relativistic S matrix.[6] He considered processes of the type $a+b\rightarrow c+d$, with total barycentric energy squared s, connected by analytic continuation to two other processes, $a+\bar{c}\rightarrow d+\bar{b}$ and $a+\bar{d}\rightarrow c+\bar{b}$, in which the squared energies are, respectively, t and u with $s+t+u=$ constant. Froissart showed on the basis of the Mandelstam representation that an analytic continuation in J (maintaining the unitarity condition for all real J and well-behaved at infinity) can be defined for Re $J>\alpha_{\max}(s)$, if the asymptotic behavior of the amplitude for large t at fixed s is bounded by $t^{\alpha_{\max}(s)}$. He also proved that $\alpha_{\max}(s)\leqslant 1$ for $s\leqslant 0$. In other words, a large domain of analyticity in angular momentum has already been shown to follow from unitarity and maximal analyticity in linear momenta.

[2] See, for example, M. Gell-Mann, in *Proceedings of the Sixth Annual Rochester Conference on High-Energy Nuclear Pyysics, April 1956* (Interscience Publishers, Inc., New York, 1956), Sec. III, p. 30.

[3] H. P. Stapp, Phys. Rev. **125**, 2139 (1962); Revs. Modern Phys. **34**, 390 (1962), preceding paper.

[4] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **7**, 394 (1961).

[5] G. F. Chew and S. C. Frautschi, Phys. Rev. **123**, 1478 (1961); Phys. Rev. Letters **8**, 41 (1962).

[6] M. Froissart, Phys. Rev. **123**, 1053 (1961); also unpublished report to the La Jolla Conference on Weak and Strong Interactions, June 1961. The unpublished work of Froissart has recently been reviewed by E. J. Squires, On the Continuation of Partial Wave Amplitudes to Complex l, Lawrence Radiation Laboratory Report UCRL-10033, January 1962 (unpublished).

Now Regge has proved for potential scattering that there is an even larger region of analyticity in J—at least as large as Re $J > -1/2$—if simple poles are allowed.[7] It is exceedingly tempting to conjecture that the same circumstance will hold relativistically, and that the characteristics of the poles in the general case are essentially those deduced by Regge. This conjecture has proved irresistible to at least three independent teams of theorists, Blankenbecler and Goldberger at Princeton[8]; Gell-Mann and Zachariasen at Cal Tech[9]; and Chew, Frautschi, and Mandelstam at Berkeley[10] (Frautschi is now at Cornell, Mandelstam at Birmingham). Chronologically, I believe that it was Mandelstam who first noticed the possible importance of Regge poles in the relativistic S matrix.

I am confident that there will soon be a proof of J analyticity, except for poles, throughout the region Re $J > 1$ for all s, but an extension to the entire J plane (or even to Re $J \geqslant 0$) may not follow purely from analyticity in linear momenta. The further extension may require a separate postulate because physically it amounts to a denial of the existence of *any* elementary particles. In order for you to understand this last statement I must explain certain properties of Regge poles.

4. PROPERTIES OF REGGE POLES

The most illuminating way to discuss Regge poles is in terms of the union of the two complex variables s and J. These two variables characterize systems of arbitrary multiplicity, so it seems almost certain that analyticity properties in s and J are common to *all* S-matrix elements of the same internal quantum numbers (not just elements for $a + b \to c + d$), since this entire subset of elements is coupled by unitarity. In particular a pole in one must be accompanied by a pole in all at the same values of s and J, although the residues of corresponding poles in different elements will differ. According to Regge's analysis of scattering by a superposition of Yukawa potentials, all poles in the right-half J plane are at least incipiently connected with bound states and resonances. Any pole may be viewed in the s plane (perhaps on an unphysical sheet), where its position depends on J, or in the J plane, where its position α_i depends on s. In fact, $\alpha_i(s)$ is a real analytic function with a positive definite imaginary part along the upper side of the physical s cut. The position of a Regge pole necessarily, therefore, varies with s—a circumstance of the utmost importance.

Regge has had nothing to say about the left-half J plane but he showed that, for a sufficiently attractive potential, as s is increased from $-\infty$ along the real axis a succession of poles in the J plane passes through the point $J = -1/2$ and moves to the right along the real J axis. For those values of s below the threshold of the physical scattering region (i.e., the beginning of the right-hand cut in the s plane) for which a particular pole crosses a real positive integer value of angular momentum, $J = 0,1,2,\cdots$, one has a bound state with this spin. At the threshold energy each pole moves into the upper-half J plane, but with short range forces continues its rightward excursion for some range of physical s. If Re $\alpha_i(s)$ crosses any further positive integer, i.e., Re $\alpha_i(s_i^M) = M$, one has here a resonance with half width given by

$$\tfrac{1}{2}\,\Gamma^M_{i/2} = \mathrm{Im}\,\alpha_i(s_i^M)/(d\,\mathrm{Re}\,\alpha_i/d\sqrt{s})_{s_i^M}\,. \qquad (1)$$

In the region of sharp resonances Im α_i is small compared to unity. Where there occur either bound states (stable particles) or sharp resonances (metastable particles) one may use the formula

$$d(\alpha + \tfrac{1}{2})^2/dp^2 = R^2\,, \qquad (2)$$

where p is the momentum and R some average "radius" of the particle. For sufficiently large energy the trajectory of each Regge pole is presumed to turn around and retreat to the left-half J plane. The crossing of integer Re J on the return trip does not produce further resonances, since here the phase shift is decreasing.

It is clear that if $J_{\max}$ is the maximum (integer) angular momentum of a bound state or resonance produced by a given Regge trajectory, then there will be resonances or bound states for all integer $J \leqslant J_{\max}$, and one has a whole family of particles for each trajectory. Because of the general occurrence of exchange forces, one must ascribe different potentials to odd and even J, so that a particular trajectory is relevant only to physical odd or to physical even values of angular momentum. Nevertheless, for attractive forces of sufficient strength one may expect to find families of particles with a common set of internal quantum numbers (B, S, I, etc.), and definite J parity. The number of family members—or, equivalently, the extent of the rightward excursion of the Regge trajectory in the J plane—will increase with

[7] T. Regge, Nuovo cimento **14**, 951 (1959); **18**, 947 (1960). Also see A. Bottino, A. M. Longoni, and T. Regge, Nuovo cimento **23**, 954 (1962).

[8] M. L. Goldberger, report to the La Jolla Conference on Weak and Strong Interactions, June 1961; R. Blankenbecler and M. L. Goldberger, Phys. Rev. **126**, 766 (1962).

[9] M. Gell-Mann, invited paper at the Los Angeles meeting of the American Physical Society, December, 1961; S. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962).

[10] G. F. Chew, S. C. Frautschi, and S. Mandelstam, Phys. Rev. **126**, 1204 (1962).

the strength of the attractive force. Also directly correlated with the force strength is the number of *different* trajectories that lead to particles, i.e., whose rightward excursion reaches as far as Re $J = 0$. It seems plausible that for each set of internal quantum numbers there are an infinite number of Regge trajectories beginning and ending in the left-half J plane, but that only a few manage to reach the right-half plane for the short range forces actually occurring in nature.

Frautschi and I consider it obvious that any particle associated with a Regge trajectory is not elementary in the conventional sense, because its spin (as well as its mass) is a dynamical consequence of the forces.[4] To avoid semantic arguments, however, it would be better to say that all particles associated with Regge trajectories are on a dynamically equivalent footing. None is more fundamental than any other. [Incidentally, none of the above-mentioned theorists,[8–10] who have fallen in love with Regge poles, hesitates to apply the notion to baryons, where half odd-integer spins occur and where J parity should be defined as $(-1)^{J-\frac{1}{2}B}$.] If one asks what kind of a pole in the S matrix would be associated with the conventional elementary-particle concept, it appears to be a pole in s for a definite physical value of J that has no analytic continuation in J. The results of Froissart show that such a singularity for $J > 1$ is inconsistent with the postulates of unitarity and maximal analyticity in linear momenta,[6] and further study may show that such poles are mathematically inconsistent even for $J = 0, 1/2$, and 1. If one is willing to *assume* maximal analyticity in J, as Frautschi and I are doing,[4] then elementary-particle poles are automatically eliminated.

A further crucial property of Regge poles is that each contributes a term $\propto t^{\alpha_i(s)}$ to the asymptotic behavior of the amplitude for large t (the negative square of the momentum transfer in the channel where s is the square of the energy). This circumstance follows from the Sommerfeld-Watson contour representation in the complex J plane for the amplitude $A(s,t)$.[11] This representation divides $A(s,t)$ into two parts with different asymptotic behavior in $\cos\theta$ (or, equivalently, in t since $t \propto \cos\theta$). The first part is an integral along the vertical line Re $J = -1/2$ that vanishes as $\cos\theta \to \infty$. The second part consists of pole contributions that generally do not vanish at infinity, these being of the form

$$\sum_i \frac{\beta_i(s)}{\sin \pi\,\alpha_i(s)}\, P_{\alpha_i(s)}(-\cos\theta)\,, \tag{3}$$

where α_i is the position of the ith pole in the complex J plane, and β_i is the residue. Since $P_\alpha(z) \propto z^\alpha$ for large z, those poles that at any particular energy stand farthest to the right in the J plane control the asymptotic behavior in t. Now large t at finite positive s is always an unphysical region, but for s negative one is in the region of forward or backward high-energy scattering of a crossed reaction (because s and t have switched roles). Thus, if Regge's analyticity—except for poles—is maintained in the J plane for negative as well as positive s, then there follows a magnificently simple theory of high-energy scattering. Conversely, one has here an elegant experimental tool to trace out Regge trajectories for $s \leqslant 0$.[4]

5. EXPERIMENTAL STATUS OF THE PRINCIPLE OF MAXIMUM STRENGTH

In Fig. 1 is plotted the angular momentum of all particles of baryon number less than two, for which spin evidence exists, as a function of the square of the mass. [I am indebted to Arthur Rosenfeld and Duane Carmony for preparing this plot.] Each point is supposed to lie on a Regge trajectory, but according to the rule of J parity only a few pairs could belong to the same trajectory. These pairs have been connected with straight lines even though a strict linear behavior of the trajectories is not expected. (In particular there are singularities at the various physical

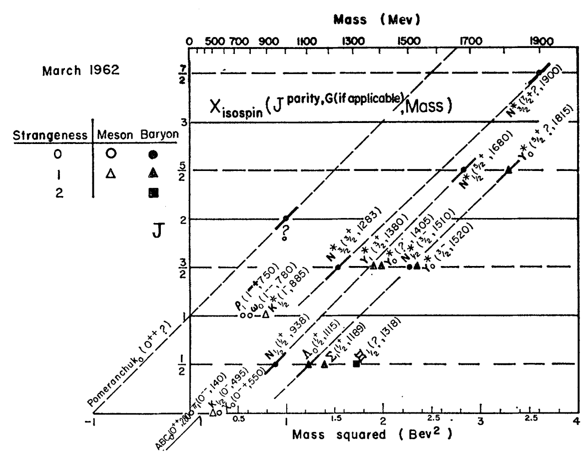

FIG. 1. The spin of strongly-interacting particles of baryon number less than two plotted against the square of the mass. Points conjectured to lie on the same Regge trajectories are connected by straight lines, but a strictly linear behavior of the trajectories is not to be inferred.

thresholds, but Barut and Zwanziger have shown that the slope of a trajectory is continuous in crossing a threshold if at this point Re $\alpha_i > 1/2$.)[12] The uni-

[11] A. Sommerfeld, *Partial Differential Equations in Physics* (Academic Press Inc., New York, 1949), p. 279.

[12] A. Barut and D. Zwanziger, "Threshold Behavior and Analyticity of Regge Poles and Their Residues," University of California Physics Department preprint (1962).

formity of the slopes in Fig. 1 is striking, and is perhaps to be understood in terms of formula (2) and the plausible notion that all the particles in Fig. 1 are of about the same "size." The slopes shown are of the order $d\alpha/ds \sim 1$ BeV^{-2}, implying through formula (2) a particle "radius" of the order $(2m_\pi)^{-1}$—a result that will surprise no one. Note that with the family interval rule $\Delta J = 2$, the spacing in m^2 between two members of the same family is ~ 2 BeV^2, so in each family only the member of lowest spin has a chance of being stable. Frequently all members are unstable, and the ABC trajectory tentatively shown at the bottom may be an example where the maximum of Re α is achieved before any particles are generated.[13] (In this case the trajectory almost reaches $J = 0$ and one has what is usually called a "virtual particle.")

The principle of maximum strength depends on the assumption that Regge trajectories can be continued to the region $s \leqslant 0$, where Froissart has proved for all trajectories that $\alpha_i(s) \leqslant 1$.[6] His proof depends on the circumstance, noted above, that in this region $t \propto E_{\text{lab}}$ for a crossed reaction, while at the same time $s = -\Delta^2$, the negative square of momentum transfer. The high-energy amplitude for the crossed reaction thus contains a contribution

$$\propto (E_{\text{lab}})^{\alpha_i(-\Delta^2)}$$

from a Regge pole located at $\alpha_i(s)$, and Froissart was able to establish that physical amplitudes violate unitarity if they increase asymptotically as a power of E_{lab} greater than 1. A glance at Fig. 1 shows that none of the trajectories associated with known particles is likely to reach the Froissart limit if all slopes are of the order of magnitude 1 BeV^{-2}. What then do Frautschi and I mean when we speak of a saturation of the unitarity condition?

Our motivation lies in the fact that total cross sections appear to approach constants at high energy, implying an imaginary part of forward amplitudes $\propto E_{\text{lab}}$. We have been emboldened to conjecture that a Regge pole for the quantum numbers of the vacuum is responsible—with a trajectory such that $\alpha(s = 0) = 1$. If $\alpha_i(0) < 1$ for all *other* trajectories, then the various Pomeranchuk conditions for total cross section limits are all automatically satisfied. I shall refer to this top-level trajectory as the Pomeranchuk trajectory. Only even integer J are physical for the Pomeranchuk trajectory, but if its slope is of the order 1 BeV^{-2} then we may expect it to produce a spin 2 meson with a mass ~ 1 BeV. Sharp eyes in the audience will be wondering what happens at $s \approx -1$ BeV^2 where the Pomerncahuk trajectory cuts $J = 0$. I shall return to this question presently. First I must give our tentative explanation of why it is only the vacuum quantum numbers that saturate Froissart.

In a detailed study of the $\pi\pi$ system—where the quantum numbers can be $I = 0,1,2$ (together with $B = 0$, $S = 0$, and $G = 0$)—Frautschi, Mandelstam, and I have shown that the long-range forces are most attractive for $I = 0$, less attractive for $I = 1$, and perhaps even repulsive for $I = 2$.[10] This circumstance follows entirely from the crossing matrix, and we are consequently inclined to make the conjecture that generally forces are strongest in systems with the simplest quantum numbers. The more complicated quantum numbers, we believe, will usually lead to elements of the crossing matrix whose relative signs lack coherence. In systems with the quantum numbers of the vacuum ($I = 0$, $B = 0$, $S = 0$, $G = 0$), all the partial contributors to the force add with the same sign, and one gets the maximum possible force. For all other sets of quantum numbers the net forces are weaker. Since the level of each Regge trajectory in Fig. 1 is correlated with the strength of the forces acting, such a hypothesis would immediately explain why only the Pomeranchuk trajectory reaches Froissart's limit.

An examination of the order of trajectory levels in Fig. 1 bears out our hypothesis in a satisfactory fashion. There is a clear tendency for height of trajectory to be correlated with simplicity of quantum numbers. (The top three trajectories, for example, have $S = 0$, $B = 0$, and I zero or one.) Further support is given by the circumstance that the only set of quantum numbers for which two separate trajectories manifest themselves through particles is the quantum numbers of the vacuum (Pomeranchuk and ABC trajectories).

Another qualitative feature of Fig. 1 worth noting is that high-level trajectories rarely occur for *both* values of J parity if all other quantum numbers are the same. Such a circumstance is understandable if we remember that corresponding systems of opposite J parity have the same ordinary force but exchange forces of opposite sign. Since exchange and ordinary forces are normally of comparable strength, the *net* attractive force for *one* value of J parity will then usually be substantially greater than for the other.

Quantitatively reliable methods for calculating short-range forces have yet to be developed, but a preliminary investigation by Frautschi, Mandelstam, and myself leads us to believe that although such forces are crucial they are mainly repulsive,[10] so that a less detailed knowledge should be needed

[13] A. Barut, "Virtual Particles," Lawrence Radiation Laboratory Report UCRL-9993, December 1961 (unpublished).

than for the long-range forces—where quantitative formulas in a number of cases are already available. The current scheme of calculation is what Mandelstam and I like to call the "bootstrap"[14]: Given a generalized potential for one channel—in terms of analytic continuation from crossed channels—one solves integral equations to find the S matrix. With the assumptions listed above there are no arbitrary parameters so far as we can see, except for one mass to be added to $\hbar$ and c in order to complete the dimensional structure. Gell-Mann has predicted that calculational procedures based on analyticity in linear momenta (e.g., the Mandelstam representation) will soon be superseded by methods that treat Regge poles as primary rather than derived aspects of the theory. I am inclined to agree, but no such methods have yet been formulated.

6. THEORY OF HIGH-ENERGY SCATTERING

Whether primary or derived in a theoretical sense, Regge trajectories, if they have the properties conjectured, henceforth will dominate the subject of strong interactions because of the direct light they shed on experiment. Experimenters are going to determine these trajectories just as they have determined phase shifts, the quantities that heretofore have constituted the meeting ground between strong interaction theory and experiment. The details of Fig. 1 will be filled in, step by step, for the imaginary as well as the real part of $\alpha_i(s)$. The technique for $s > 0$ is obvious and would be followed even if Regge were not so smart: One looks for both stable particles and resonances and tries to determine the masses, widths, spins, and internal quantum numbers. Figure 1 will be of help, of course, in suggesting *where* to look. The spin 2 meson of mass $\approx 7m_\pi$, belonging to the Pomeranchuk trajectory, is a good example.[15] Equally potent experimental tools, however, are high-energy total and differential scattering cross sections. These will determine trajectories for $s \leqslant 0$.

The essential ingredient for a theory of high-energy scattering has already been stated above in formula (3): Each Regge pole contributes in the crossed amplitude a term that asymptotically is of the form

$$A_i(E_{\text{lab}},\triangle^2) \approx \frac{\beta_i(-\triangle^2)}{\sin \pi\alpha_i(-\triangle^2)} \times \left[(-E_{\text{lab}})^{\alpha_i(-\triangle^2)} \pm (E_{\text{lab}})^{\alpha_i(-\triangle^2)}\right], \quad (4)$$

where α_i is the position of the pole and β_i the residue. Both α_i and β_i are real for $\Delta^2 > 0$. The plus or minus sign in (4) depends on the J parity of the trajectory.[9] The simplest experimental application of formula (3) is to total cross sections, which are proportional to $E_{\text{lab}}^{-1}\,\text{Im}\,A_i(E_{\text{lab}}, \Delta^2 = 0)$, and which will get contributions from trajectories with $B = 0$, $S = 0$, and $I_z = 0$. The highest of these are the Pomeranchuk, ρ, and ω—probably in that order. So one should be able to represent any high-energy total cross section by the formula

$$\sigma_n^{\text{tot}}(E_{\text{lab}}) = P_n + \rho_n E_{\text{lab}}^{-[1-\alpha_\rho(0)]} + \omega_n E_{\text{lab}}^{-[1-\alpha_\omega(0)]} + \cdots, \quad (5)$$

where the series converges more rapidly the higher the energy. If further relevant trajectories exist with $\alpha(0) \geqslant 0$ they should of course be included (e.g., the ABC trajectory). There are many relations between the residues in different amplitudes that are now being studied. Udgaonkar has looked at the NN, πN, and KN combinations.[16] As an illustration of the power of this approach, he finds for the NN cross sections the following connections:

$$\begin{aligned}
\sigma_{pp}^{\text{tot}}(E_{\text{lab}}) &= P_{NN} - \rho_{NN} E_{\text{lab}}^{-[1-\alpha_\rho(0)]} - \omega_{NN} E_{\text{lab}}^{-[1-\alpha_\omega(0)]} + \cdots \\
\sigma_{np}^{\text{tot}}(E_{\text{lab}}) &= P_{NN} + \rho_{NN} E_{\text{lab}}^{-[1-\alpha_\rho(0)]} - \omega_{NN} E_{\text{lab}}^{-[1-\alpha_\omega(0)]} + \cdots \\
\sigma_{\bar{p}p}^{\text{tot}}(E_{\text{lab}}) &= P_{NN} + \rho_{NN} E_{\text{lab}}^{-[1-\alpha_\rho(0)]} + \omega_{NN} E_{\text{lab}}^{-[1-\alpha_\omega(0)]} + \cdots \\
\sigma_{\bar{p}n}^{\text{tot}}(E_{\text{lab}}) &= P_{NN} - \rho_{NN} E_{\text{lab}}^{-[1-\alpha_\rho(0)]} + \omega_{NN} E_{\text{lab}}^{-[1-\alpha_\omega(0)]} + \cdots.
\end{aligned} \quad (6)$$

(The signs of all the coefficients here are probably positive.) Experimenters may use formulas of this type just as they use phase shift expansions and determine empirically the real parameters appearing therein. There is reason to believe that such asymptotic expressions should be usable almost as soon as one gets beyond the resonance region, i.e., above about 2 BeV. Udgaonkar is currently analyzing existing total cross-section data on this basis, but more particle combinations and much greater experimental accuracy must be achieved before the potentialities of such formulas are fulfilled.

For example, one especially simple application isolates $\alpha_\rho(0)$:

$$\begin{aligned}
\sigma_{np}^{\text{tot}}(E_{\text{lab}}) - \sigma_{pp}^{\text{tot}}(E_{\text{lab}}) &\approx 2\rho_{NN} E_{\text{lab}}^{-[1-\alpha_\rho(0)]} \,. \\
\sigma_{\pi-p}^{\text{tot}}(E_{\text{lab}}) - \sigma_{\pi+p}^{\text{tot}}(E_{\text{lab}}) &\approx 2\rho_{\pi N} E_{\text{lab}}^{-[1-\alpha_\rho(0)]}\,,
\end{aligned} \quad (7)$$

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[14] G. F. Chew and S. Mandelstam, Nuovo cimento **19**, 752 (1961).

[15] C. Lovelace, Imperial College Physics Department preprint, London (1961).

[16] B. Udgaonkar, Phys. Rev. Letters **8**, 142 (1962).

Udgaonkar finds that existing data are consistent with $\alpha_\rho(0) \approx 1/2$, a result that would follow if the slope of the ρ trajectory is of the order 1 BeV^{-2}; but experiments need drastic improvement before the energy dependence (7) can be definitely established and a value for $\alpha_\rho(0)$ determined.

A second application of Formula (4) is to high-energy differential cross sections for processes of the type $a + b \rightarrow c + d$, which will have peaks in the forward direction due to Regge poles in the channel $a + \bar{c} \rightarrow \bar{b} + d$, and in the backward direction due to poles in the channel $a + \bar{d} \rightarrow \bar{b} + c$. These applications have been studied in some detail by Frautschi, Gell-Mann, and Zachariasen.[9] A careful measurement of the shape and energy dependence of these peaks evidently will yield both the residues and the positions of the relevant Regge poles for a continuous range of negative s. The most prominent peak is for elastic scattering in the forward direction, and is dominated by the Pomeranchuk trajectory. Keeping only this contribution one finds

$$\frac{d\sigma_n^{el}}{d\triangle^2} \approx \frac{1}{16\pi} \frac{P_n^2(-\triangle^2)}{\sin^2[\pi\alpha_P(-\triangle^2)/2]} E_{\text{lab}}^{-2[1-\alpha_P(-\triangle^2)]} . \quad (8)$$

Fitting to recent CERN data on pp elastic scattering, Frautschi *et al.* find that $\alpha_P(-\Delta^2)$ decreases smoothly with increasing Δ^2, and probably goes negative at $\Delta^2 \approx 1$ BeV^2, as indicated in Fig. 1.[9] Now formula (8) blows up when $\alpha_P(-\Delta^2) = 0$ unless at the same time the residue P_n vanishes linearly with α_P. Presumably such a vanishing occurs, since an infinite cross section is neither tolerated by unitarity nor observed experimentally. Gell-Mann likes to interpret the simultaneous vanishing at negative s of $\alpha_P(s)$ and all the residues of this Pomeranchuk Regge-pole as meaning that there is a fundamental spin-zero ghost particle of imaginary mass (≈ 1 BeV i), with the quantum numbers of the vacuum but zero coupling to all physical systems. I prefer not to use such language, but there is no difference in our attitudes toward the physics. The Pomeranchuk trajectory manages to cross $J = 0$ at negative s without disaster for the S matrix.

Formula (8) is remarkable in that it predicts an indefinite logarithmic decrease with energy in the width of the diffraction peak, and thus in the ratio $\sigma_n^{el}/\sigma_n^{tot}$. For many months Frautschi and I felt this circumstance to be so unreasonable that we were unwilling to ascribe diffraction scattering to a Regge trajectory. However, a related property of (8) is that at fixed E_{lab} there is an exponential decrease with Δ^2 so long as $\alpha_P(-\Delta^2)$ continues to fall. This feature also is contrary to a classical picture of diffraction scattering, but it is clearly observed in elastic pp experiments.[9] Lovelace has pointed out that such exponential behavior also occurs for πp elastic scattering,[15] so the nonclassical aspects of formula (8) have to be taken seriously. No one yet is sure as to what interpretation should be given to an asymptotic logarithmic vanishing of the elastic cross section, but we believe that it is going to be observed experimentally.

An important remark about formula (4) is due to Frautschi.[4] He pointed out that high-energy forward and backward peaks must be a consequence of coherence in the scattering, and the strength of a peak, i.e., the value of $\alpha_i(0)$, should increase with the degree of coherence. This notion fits perfectly with the earlier hypothesis that height of trajectory is correlated with simplicity of quantum numbers. Maximum coherence should be and is achieved by scattering with exchange of the vacuum quantum numbers. As the exchanged quantum numbers become more complicated the degree of coherence decreases and $\alpha_i(0)$ becomes smaller.

7. CONCLUSION

One of the most attractive aspects of S-matrix theory is that checks with experiment are possible at many different levels, and do not require a complete solution of the dynamical equations. We shall, in fact, never have a complete solution; it would be far too complicated, since *all* particles would have to be considered simultaneously. It *may* be that an approximation isolating a few of the top-level Regge trajectories will make sense, so that a few mass ratios can be roughly calculated; that remains to be seen. One need not wait for such a development, however, to join the fun. This report has only scratched the surface of possible contacts between theory and experiment, and I am convinced that a wild period of merrymaking lies before us. All the physicists who never learned field theory can get in the game, and experimenters are as likely to come up with important ideas as are theorists. They may even have an advantage over us.

An inevitable question at this point is, "What about electromagnetism and weak interactions?" I personally have not developed strong convictions on this question, but I do not see how leptons and the photon can emerge from the principles enunciated here. One may imagine, in fact, that leptons and weak interactions represent a deficiency in one or more of these principles that will become a major effect in some experimental domain of the future.

Had we known of hyperfine structure in the early days of atomic physics, however, it would have been a mistake to insist that any theory should explain the effect. Historically, *all* dynamical theories in physics have had limitations on their domain of validity, no matter how general they seemed when they were proposed. We must not be too greedy.

Note added: After preparation of this manuscript I became aware of an article by V. N. Gribov, J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 667 (1961), which discusses the importance of the Pomeranchuk trajectory in high energy scattering.

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Field Theories with Persistent One Particle States. I. General Formalism*

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1. MOTIVATION AND PURPOSE

In the last few years considerable attention has been devoted to the study of the general aspects of field theory. Currently there are actually several rather different approaches. One approach is based on the notion of local field operators; general principles such as relativistic invariance and local commutativity are assumed. On this basis an extensive, mathematically precise theory may be developed. Initiated by Wightman [1],[1] this approach through the work of Jost [2], Haag [3], Lehman, Symanzik, and Zimmerman [4], and many others (see the extensive bibliography in Schweber [5], has led to a considerable deepening of the mathematical basis of field theory and to specific results of physical interest, such as proofs of the *TCP* theorem (Jost [6]) and the connection between spin and statistics (Burgoyne [7]). Another approach described most explicitly by Chew [8], also utilizes the principle of relativistic invariance. The notions of the "local field" and field operators, so essential in the abstract approach, are eliminated as far as possible in this approach. In fact the ultimate hope of this general philosophy is that the relevant physical principles may be expressed exclusively in terms of the analytic properties of S-matrix elements. If, according to this attitude, a particular assumption about the analytic character of the S matrix, such as the Mandelstam representation, cannot be proven, using the general principles of field theory—this is yet another indication that field theory is incorrect, inconsistent, and irrelevant. Thus using the *assumed* analytic properties of S-matrix elements as given by the Mandelstam representation (for two incoming and two outgoing particles), together with the requirements of unitarity and relativistic invariance, a detailed theoretical framework has been constructed, which has been successful

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[1] The references are to be found at the end of this paper.