The abbreviation $\Gamma \equiv I_z - 1 \cdot \langle I_z \rangle_0$ is used, and I is the angular momentum in units of $\hbar$.

1. $\langle I_x \rangle_0 = \langle I_y \rangle_0 = 0$

2. $\langle I_z \rangle_0 = (I+\frac{1}{2}) \coth[a(I+\frac{1}{2})] - \frac{1}{2} \coth a/2 = (a/3)I(I+1) + O(a^3)$

3. $\langle I_z I_{x,y} \rangle_0 = \langle I_{x,y} I_z \rangle_0 = 0$

4. $\langle I_z^2 \rangle_0 = I(I+1) + \frac{1}{2} \coth^2 a/2 - (I+\frac{1}{2}) \coth(a/2) \coth[a(I+\frac{1}{2})] = \frac{1}{3}I(I+1) + O(a^2)$

5. $\langle I_z \Gamma \rangle_0 = \partial \langle I_z \rangle_0 / \partial a = \frac{1}{3}I(I+1) + O(a^2)$

6. $$\langle I_z^2 \Gamma \rangle_0 = \frac{\partial \langle I_z^2 \rangle_0}{\partial a} = \frac{aI(I+1)}{45}[4I(I+1) - 3] + O(a^3)$$

7. $$\langle I_{x,y}^2 \Gamma \rangle_0 = -\frac{1}{2}\frac{\partial \langle I_z^2 \rangle_0}{\partial a} = -\frac{aI(I+1)}{90}[4I(I+1) - 3] + O(a^3)$$

8. $\langle I_{x,y} I_z \Gamma \rangle_0 = \langle I_z I_{x,y} \Gamma \rangle_0 = 0$

9. $\langle I_x I_y \Gamma \rangle_0 = -\langle I_y I_x \Gamma \rangle_0 = \frac{1}{2} i\hbar \langle I_z \Gamma \rangle_0$.

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Nuclear Models and Surfaces*

DAVID R. INGLIS

Argonne National Laboratory, Argonne, Illinois

THERE are many schemes going under the name of nuclear models these days and not all of them can be discussed here. Some of them are merely models of models. Among them, the shell model and the distorted-shell model, or collective model, have been very successful in correlating nuclear data. Instead of dwelling on the victories represented by the frequently discussed quantitative agreement, the present discussion emphasizes the relationships between different nuclear models and points out some of the conceptual problems of the models themselves, problems associated with the nature of the nuclear surface, particularly as they appear among some of the light nuclei which are practically all surface.

COMPARISON OF MODELS FOR He^6

Let us first consider the shell model in one of the simplest cases He^6. The ground state is a ^1S state, with the spins of the two p protons antiparallel and the space function symmetric on exchange of the two nucleons. In Fig. 1(a), we see that the orbital planes in which the two nucleons circulate coincide because the angular momenta are exactly oppositely directed. This circumstance, together with the fact that the space function is symmetric in exchange of the two nucleons, maximizes their average proximity to one another and minimizes the energy arising from their attractive interaction. The first excited state ^1D differs from this ground state in having the two orbital angular momenta l_i as nearly parallel as possible [Fig. 1(b)]. The uncertainty principle does not permit them to be exactly parallel, however, as seen by the direction cosines in which l_1^2 is replaced by $l_1(l_1+1)$, etc. The orbits thus are not quite in the same plane and the particles are on the average farther apart, so that the energy is slightly higher than that of the ground state. The higher states of the configuration p^2 comprise the ^3P state, with energy much higher both because the planes of the orbits are approximately normal to one another, as shown in Fig. 1(c), and because the space function is antisymmetric.

In the simple case with two p nucleons, and also in more complicated cases with more nucleons, the main point is that the low state is determined by a maximum angular bunching of these nucleons. If the charge distribution of a p nucleon is roughly represented by a circular orbit, maximum bunching is attained, as a first consideration, by having the orbits as nearly coplanar as possible. But, it is equally important that we should consider the phases involved when we think of the

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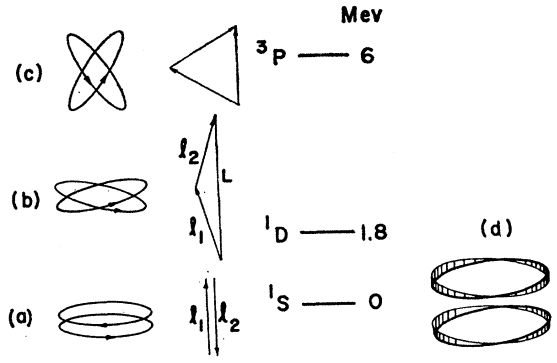

FIG. 1. Features determining the average proximity and interaction energy of the two p nucleons in the low states of the shell model of He^6. This pictorial representation, of course, depicts merely the main features of nuclear states which are more-precisely described by wave functions. The corresponding density functions are more diffuse than the line orbits so as to satisfy the uncertainty principle. The wave functions for the S and D states, for example, in an oscillator potential are

$$^1D\sim(x_1x_2+y_1y_2-2z_1z_2)\exp[-\tfrac{1}{2}(r_1^2+r_2^2)]$$
$$^1S\sim(x_1x_2+y_1y_2+z_1z_2)\exp[-\tfrac{1}{2}(r_1^2+r_2^2)].$$

In terms of the relative and c.m. coordinates of the two p nucleons

$$r=2^{-\frac{1}{2}}|\mathbf{r}_2-\mathbf{r}_1|,\quad R=2^{-\frac{1}{2}}|\mathbf{r}_1+\mathbf{r}_2|,$$

we have in the exponent the identity $r_1^2+r_2^2=R^2+r^2$ and these wave functions may be written instead in terms of the $2s$ and $1d$ wave functions of these coordinates (containing the exponentials $e^{-R^2/2}$ and $e^{-r^2/2}$) so as to emphasize the dependence of the probability distribution on the relative coordinate, thus:

$$^1D=1d(R)1s(r)-1s(R)1d(r),$$
$$^1S=2s(R)1s(r)-1s(R)2s(r).$$

The corresponding matrix elements of the interaction $V(r)$ are then

$$\langle^1D|V|^1D\rangle=\langle1s|V|1s\rangle+\langle1d|V|1d\rangle,$$
$$\langle^1S|V|^1S\rangle=\langle1s|V|1s\rangle+\langle2s|V|2s\rangle.$$

The relative $2s$ function has a maximum at $r=0$, when the two nucleons coincide, whereas $1d(r)$ has a node there. For a short-range attractive potential, the last term in the matrix element thus gives the ^1S a lower energy than ^1D, as may also be inferred by comparing the angles between the orbital planes in sketches (a) and (b).

motion of the nucleon as given by a wave drawn around the circular orbit, as in Fig. 1(d), to represent the angular factor of the wave function. In the simple example of He^6, the symmetric wave function with maximum bunching corresponds to having the phases arranged so that the crests of the waves for the two nucleons coincide. This means that, in this case, the bunching is not a flattened distribution in a plane, as the orbits alone would suggest, but a bunching about an axis of unknown orientation.

This result was obtained after starting from a spherical potential well. Much the same result can be obtained by introducing the ellipsoidal potential well of the distorted-shell model in the first place (a prolate well for He^6), and by letting much of the angular dependence of the interaction between nucleons be represented by the interaction of the individual nucleons with the shell.

There is thus a close correspondence between the shell model and the distorted-shell model. The shell-model calculations become very extensive when many nucleons are involved, and the use of the distorted-shell model constitutes a distinct simplification. The collective rotation in the distorted-shell model provides an explanation for the widely observed rotational bands. The fact that the shell model explains similar excited states in the simple cases in which comparison has been possible seems to reveal the nature of some of the low excited states in the shell model, suggesting that although the relative orientation of the nucleon orbits is changed to make higher values of the total angular momentum, it is done in such a way that the wave function has almost as high a degree of internal symmetry as in the ground state.

The symmetry is not quite as high in the rotational state as in the ground state. In the rotating distorted-shell model (called the cranked model), this is expressed by the admixture of higher single particle states through the action of the Coriolis force. The Coriolis force acting on the free nucleons gives too small a spacing between the rotational states (the spacing corresponding to the rigid moment of inertia). This excessive freedom of the individual nucleons to react to the Coriolis force is suppressed by a residual interaction between pairs of nucleons (the part not already included in the distorted potential well). This, too is most effective between pairs of nucleons with oppositely-directed momentum components, for a reason similar to that in the ground state of He^6. An important mathematical simplification, which goes under the name of "pairing model" or "superconducting model," assumes all such interactions between pairs to have the same value and neglects all other matrix elements. The methods used are rather formal, being derived from field theory and treating closed shells as the "vacuum." These useful methods are thus not really nuclear models, but model calculations within a nuclear model, or models of a model. They make possible rough, approximate calculations in cases too complicated for direct application of normal shell-model methods. They lead to the concept of a quasi-particle, which involves the inclusion of certain important parts of the configuration mixing, so that a nucleon seems to sweep along with it some of the medium in which it moves.

POTENTIAL WELL FOR HEAVY NUCLEI

When shell-model calculations are made for heavy nuclei and some of the integrals encountered are determined by comparison with experimental data before predicting other experimental data, these should be considered as shell-model calculations for quasi-particles, since the configuration interaction associated with breaking the closed shells to make quasi-particles must affect the data used to fix parameters. An attempt to calculate the total binding energy of light nuclei must deal with interactions between actual nucleons, and such calculations are usually not very successful—partly because some of this configuration interaction is neglected.

In constructing shell-model wave functions, the three-dimensional harmonic-oscillator potential is commonly employed. This goes up rapidly toward infinity outside the nucleus, and thus would seem to confine the wave functions in an unrealistic manner. One usually thinks of the central part of this oscillator potential as the important part, and considers that the shoulder should be cut off squarely at the separation energy for a neutron (or with a Coulomb barrier above this for a proton), so as to make a potential resembling the simple "finite square well," but with a rounded bottom. However, with such a cutoff potential, the tails of the wave functions of the least-bound nucleons would penetrate into the classically forbidden region outside the nucleus considerably more than do the artificially confined, oscillator wave functions.

When a neutron is separated "adiabatically" from the nucleus (as in the usual low-energy, nuclear reactions), the energy of the neutron threshold is calculated in terms of the energy of the residual nucleus after the remaining nucleons have rearranged the symmetry of their wave function to take advantage of the absence of the departed nucleon. The energy of the neutron threshold is lowered by taking advantage of this rearrangement energy. This is not the case in the determination of the penetration of the tail of the single-neutron wave function into the classically forbidden region outside of the nucleus.[1] Instead, a considerable part of the wave function still exists inside the nucleus and has its influence on the symmetries of the other nucleons, forbidding "rearrangement." Thus, the shoulder of the potential well confining the single-particle wave function is higher than the threshold for separation by an amount equal to the rearrangement energy. The confining effect of this higher potential is more nearly like that of the parabolic potential on which the oscillator wave functions are based. The customary use of these simple functions is thus better justified if rearrangement is taken into account.

THRESHOLD STATES

If we think of changing some such parameter as the radius of the nucleus in order to raise the energy of a single-neutron state toward the threshold energy, and if we cut off the potential well at the threshold, then the tail of the wave function penetrates more and more outside the nucleus as the threshold is approached. The neutron distribution refuses to be squeezed as much as it otherwise would, and the energy of the state correspondingly refuses to be raised so rapidly. At the very last, the curve of energy plotted against radius bends over to become horizontal, and thus a relatively large increment of the arbitrary parameter corresponds to a very small change of energy. This is illustrated by the solid line in the magnified portion of Fig. 2. On the basis of this type of effect, it has been suggested that,

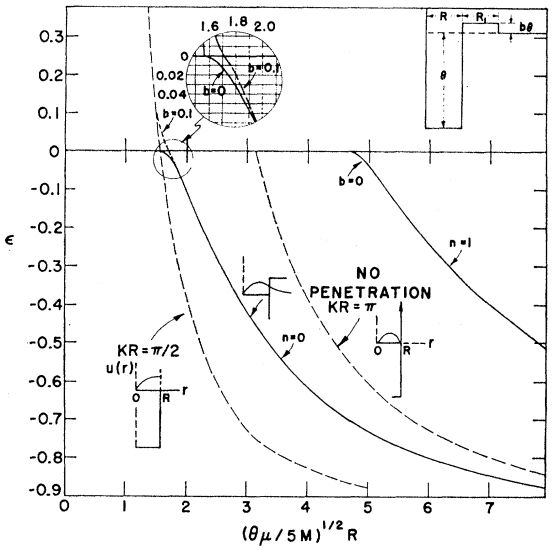

FIG. 2. With a potential well cutoff at threshold, the energy as a function of well radius bends over to approach the threshold asymptotically.

because of the statistical distribution of such parameters, one should find nuclear-energy states more frequently in an energy interval close to such thresholds than elsewhere.

The existence of a confining barrier which is higher than this by an amount equal to the rearrangement energy of course alters this expectation. However, the greater the penetration by the tail, the smaller is the amplitude of the wave function inside the nucleus; one may assume that this reduces the height of the rearrangement barrier in direct proportion to the lowered probability that the neutron is in the nucleus. Thus, as the threshold is approached from below, the penetration reduces the barrier height and there is a feedback by which this increases the penetration, and at a certain point above the threshold the feedback mechanism becomes unstable and the rearrangement barrier disappears. If the rearrangement energy is "intrinsically" rather small (that is, small even for a neutron wave function concentration in the nucleus), then this approach to instability occurs quite close to the threshold, and there is an enhanced probability of finding a state as near to the threshold as this. For a realistic, intrinsic, rearrangement energy of several Mev, however, the probability of finding a state within a few tens of kev

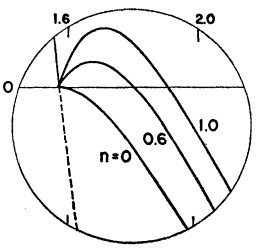

FIG. 3. The energy function of Fig. 2 is replaced by one with finite slope at the threshold if the height of the potential barrier is augmented by a rearrangement energy proportional to the density of the nucleon state inside the nucleus.

[1] D. R. Inglis, Nuclear Phys. **30**, 1 (1962).

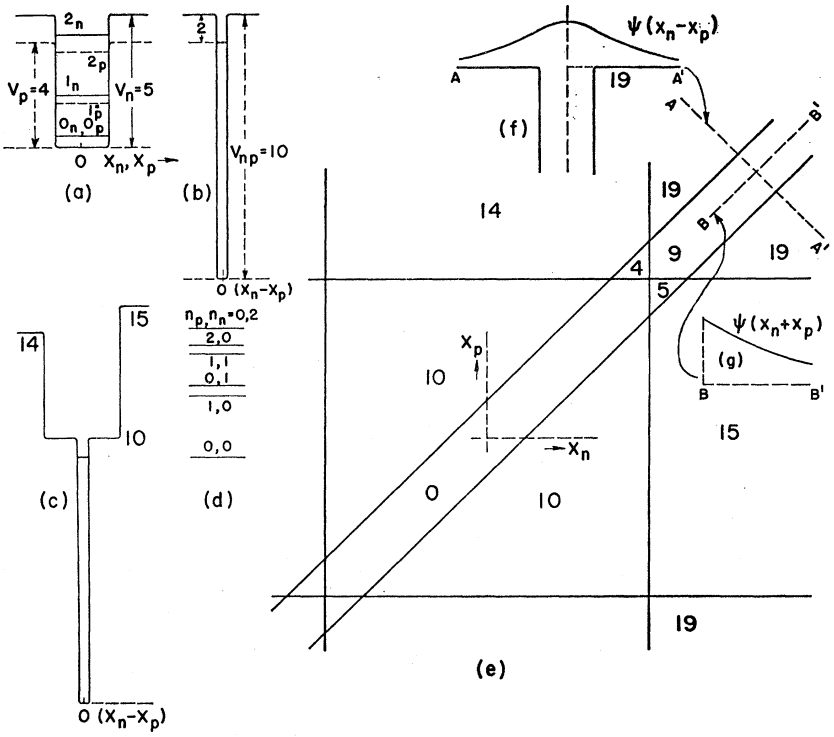

FIG. 4. Typical one-dimensional potentials in the two-dimensional configuration space of two interacting nucleons in a one-dimensional square well.

of the threshold is not appreciably greater than at other energies. This effect is illustrated in Fig. 3, where the various curves give the energy of a single-neutron state in a square well with its barrier height augmented on the basis of various values of the intrinsic rearrangement energy.

A similar argument can be applied to the separation of a cluster of nucleons, such as a deuteron or alpha particle, from a nuclear system. An interesting case is to be found in the fact that the first excited state of He^5 (above the very diffuse ground ^2P) as a $(3/2)^+$ state at 16.69 Mev, only 0.07 Mev from the threshold for breakup into H^3+d. The shell-model description of this excited state of He^5 is similar to that of the ground state of He^6 (suggested in Fig. 1) plus a proton hole in the s shell. The two p nucleons are similar to a deuteron in that they form a ^3S state with parallel spins and a symmetric space function. The s shell and p shell presumably have rather little influence on one another's symmetries. Thus, there probably is only a very small rearrangement barrier because the relative symmetries of the nucleons in the shell model of the compact system are very similar to those in the cluster model in which the deuteron and triton are slightly separated in space. In this case, the suggested penetration effect on the probability of finding a state near threshold probably helps to explain the occurrence of the threshold state. This may become clearer in the following discussion.

The cluster model is not sufficient in itself to explain the proximity of the energy level to the threshold, since it is difficult to calculate whether the two clusters should almost or just barely stick together, or stick together with a fairly strong binding.

Figure 4 presents a one-dimensional shell model of Li^6 or of F^{18}, whichever you prefer. It consists of a one-dimensional potential well (a), which we imagine to be provided by an unspecified filled shell, containing a neutron and a proton interacting with each other by a short-range potential (b). Figure 4(e) is a map of the energy surface as a function of the proton coordinate x_p and the neutron coordinate x_n. The heights of the various plateaus are given in the same arbitrary units as were used to express the depth of the one-dimensional well for protons ($V_p=4$ arbitrary units) and for neutrons ($V_n=5$ units). The interaction V_{np} makes a deep trench near the main diagonal where $x_p=x_n$. The potential energy is 19 if the proton and neutron leave the potential well separately, but, if the energy of binding of the one-dimensional deuteron [whose wave function is shown in (f)] is $E_d=-2$, then the threshold energy for deuteron emission is 17.

If the energy of the system is slightly lower than this threshold, the penetration of the deuteron into the barrier will be represented by a decaying exponential as in Fig. 4(g). One can imagine using a net of points to solve the differential (or difference) equation over this surface, and finding that a considerable part of the wave function corresponds to a shell-model-like existence of proton and neutron inside the nucleus, and part to a cluster-model-like penetration of the barrier in the deuteron channel in the upper right-hand part of Fig. 4(e).

The numbers in Fig. 4, which have been chosen to place the single-nucleon thresholds below the deuteron threshold, correspond to the situation in F^{18} but not in Li^6. In the situation shown, the rearrangement barrier against single-nucleon emission may be sufficiently high to contain the wave function of a fairly sharp nuclear state near the deuteron threshold.

NATURE OF THE NUCLEAR SURFACE

Because a nucleon wave function contains four components associated with the specification of spin and isobaric spin, any type of nuclear state displays a special stability when occupied by four nucleons. In calculations of the stability of nuclear matter, it is a fairly close decision whether the more stable configuration consists of four interpenetrating, uniform distributions of the four types of nucleons, or a nonuniform distribution in which four groups of closely interacting nucleons cluster together. Calculations by Brueckner and others indicate that at the normal density of nuclear matter such clustering does not occur. In these calculations, the interaction between the pair of nucleons is represented as a very short-range repulsion, which helps to prevent collapse of the nucleus, superimposed on a fairly short-range attraction. At lower densities, however, it seems that some degree of clustering is possible.

Wilkinson has discussed evidence that the nucleons near the nuclear surface (where the density is small) are to a considerable extent fleetingly clustered.[2] The evidence is based on such phenomena as natural alpha decay and the relative frequency of different modes of K^--meson capture in the "nuclear stratosphere." It is easy to imagine a process similar to that suggested by Fig. 4(e) permitting clusters like alpha particles to penetrate into the Coulomb barrier, to make the surface consist largely of such clusters. [Alpha decay through the Coulomb barrier in many-dimensional space is expected to take place where the barrier is lowest, namely, through the channel in which the nucleons are clustered together to form an alpha particle as they go through. The tail of the cluster wave function penetrating into this channel, as in the corner of Fig. 4(e), corresponds to some preformation of the alpha and facilitates the decay to make it as rapid as observed.]

In a few special nuclei and particularly in O^{16}, this clustering at the surface may have a more nearly permanent character; i.e., there may be a collective deformation similar to that in the more commonly discussed, ellipsoidally deformed, shell model. This could give rise to a surface with a tetrahedral symmetry, corresponding to the partial formation of four alpha clusters, and could be the basis for the success of Dennison's treatment[3] of a model with this symmetry. The fact that Elliott and Flowers[4] obtained some of the same results from the shell model suggests that there is a close correspondence between these two models just as there is between the shell model and the deformed-shell model discussed earlier.

NUCLEAR MODELS IN LITHIUM

Let us return to the lithium isotopes and other very light nuclei. Here, we may ask whether we have an extreme manifestation of surface clustering, so that the cluster model is valid, or whether the shell model applies and there are too few particles to favor a full development of the surface clustering. By "full development" is meant a clustering more pronounced than is inherent in the first-order symmetries, one which from the shell-model point of view is enhanced by a substantial amount of configuration mixing. It may also be described in terms of a molecular model, following Hafstad and Teller's discussion back in 1938, and others even earlier.

This is emphasized because there is already a large amount of close association or clustering of the nucleons in the first-order treatment of the shell model, as we have seen in the discussion of He^6. This is particularly pronounced in the ground state of Be^8, discussed in *LS* coupling by Wildermuth and Kanellopoulos,[5] who referred to this ground-configuration clustering as substantiation for a cluster model. Here, there are four nucleons in the p shell and the symmetry of their space wave function makes them tend to cluster fleetingly on one side of the shell. This means that they are predominantly on one side of the center of gravity whereas the s nucleons are clustered on the other side. While this seems like a description of a separation into two alpha particles, the limitation imposed by the shell model (with oscillator wave functions) is that there is a prescribed (and quite large) amount of overlap between the two clusters. The average distance between the two clusters and the sizes of the clusters are dictated by the same size parameter of the oscillator well. Perhaps the most serious limitation is that, in minimizing the energy of the system, the necessary compromise with the presence of the p nucleons forces the s shell to be considerably larger (about 50% larger in radius) than the free alpha particle. If the shell model is to give a more stable ground state than does a molecular-type cluster model, the gain in binding energy through overlap of the s shell and p shell must be great enough to compensate the loss of internal binding energy accompanying this expansion of the s shell.

It has long been questioned whether the shell model or a cluster model (it used to be called an alpha model or alpha-particle model) applies to the lithium isotopes. Such properties as magnetic moment and strength of spin-orbit coupling have been estimated with both

[2] D. H. Wilkinson, *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961*, edited by J. B. Birks (Academic Press Inc., New York, 1961), p. 339.
[3] D. M. Dennison, Phys. Rev. **96**, 378 (1954).
[4] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A242**, 57 (1957).
[5] Th. Kanellopoulos and K. Wildermuth, Nuclear Phys. **9**, 349 (1960).

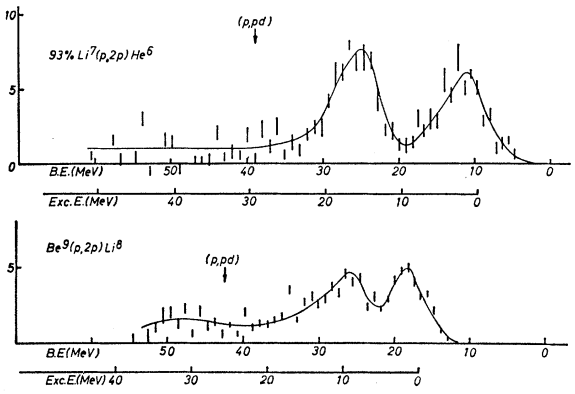

FIG. 5. Energy groups for protons ejected from the s and p shells in the $(p,2p)$ reaction.

models during the late 1930's and 1940's. (Since these nuclei display LS coupling, the famous 1949 discovery of jj coupling in heavier nuclei does not invalidate these old and incomplete calculations.) Fairly serious attempts to calculate binding energies of these isotopes on the basis of various force assumptions have been made at that time and also more recently.[6]

A new and exciting experimental result obtained at Orsay, France, has shed light on this question. In fact, that is the reason for reviewing the question of clustering at this time. The power of careful observations of the $(p,2p)$ reactions (with incident proton energy in the neighborhood of 150–450 Mev) was first demonstrated at Uppsala, Sweden, about four years ago, by Tyrén, Hillman, and Maris.[7] Their observations of numbers of events as dependent on the energy required to knock a proton out of Li^7 or Be^9 are shown in Fig. 5. The right-hand peak in each case corresponds to knocking a proton out of the p shell and the left-hand peak to knocking out a more tightly bound proton from the s shell. This is a beautiful confirmation of the separation of the protons into shells, and one sees that the p protons are more tightly bound in Be^9 than in Li^7, a manifestation of the familiar four structure in nuclear binding energies.

In addition to recognizing them thus by their energies, it is also possible to identify the s and p nucleons experimentally by their angular-momentum distributions. One sets the two counters at about 45° from the direction of incidence and gates them to observe the highest-energy range occurring in coincidence. At this position, which is appropriate for observing the scattering from a free proton (hydrogen) at a slightly higher energy, one observes the knocking out of protons which had no momentum in the nucleus. By shifting to slightly smaller or larger angles, while keeping the two angles equal, one can observe instead the scattering from particles moving either in the direction of the beam or in the opposite sense, respectively.

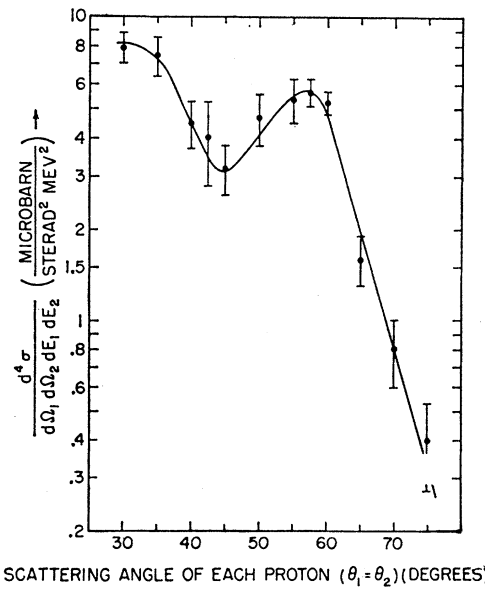

FIG. 6. Angular distribution in the $(p,2p)$ reaction on C^{12}, for the p-shell group. See reference 8.

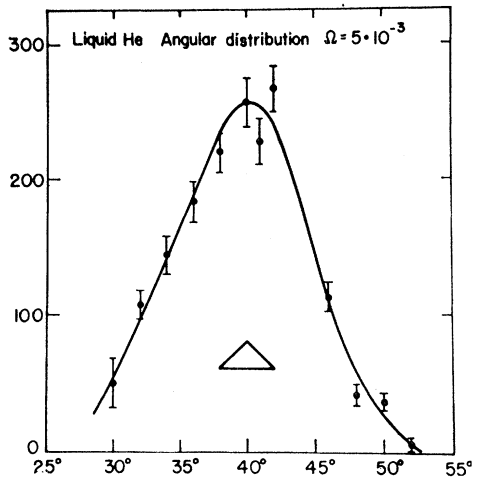

FIG. 7. Angular distribution in the $(p,2p)$ reaction on He^4.

With oscillator functions, the wave functions in momentum space are very similar to those in space. The momentum distribution for an s state has a maximum at zero momentum and for a p state it goes to zero at zero momentum. Classically, a particle having a finite angular momentum keeps going round and round and never stops, while a particle with zero angular momentum swings back and forth through the center and stops at the turning points at the ends of the swing. Thus, with the experimental arrangement just described, one obtains a maximum near 45° for scattering from s protons and a minimum for p protons. This was first

[6] H. Margenau and K. G. Carrol, Phys. Rev. **54**, 705 (1938); D. R. Inglis, Phys. Rev. **51**, 531 (1937); Y. C. Tang, K. Wildermuth, and L. D. Pearlstein, *ibid.* **123**, 548 (1961); N. Austern and P. H. Wackman; L. Foldy (private communication).

[7] H. Tyrén, P. Hillman, and Th. A. J. Maris, Nuclear Phys. **7**, 1 (1958).

demonstrated experimentally by Gottshalk and Strauch at Harvard[8] (Fig. 6) and has also been shown by Tyrén at Chicago[9] (Figs. 7 and 8). One sees a beautifully clean distinction between the s and p protons, in keeping with the identification of the groups from their energy and in keeping with the expectations of the shell model.

This brings us to the recent observations on Li^6 and Li^7 at Orsay.[10] In the case of Li^7, we see in Fig. 9 that the low-binding-energy group has a dip in the middle, characteristic of a p proton, just as in the earlier observations on C^{12}. The surprise comes in the observations on Li^6 shown in Fig. 10. Here again, there are two energy groups, but both groups have angular distributions characteristic of s protons.

This would be expected by a cluster model but not by a shell model. Another experimental result which singles out Li^6 as qualitatively different from the other p-shell nuclei is found in the Stanford observation of nuclear densities by electron scattering.[11] These data indicate that Li^6 is the only one of these nuclei with distinctly

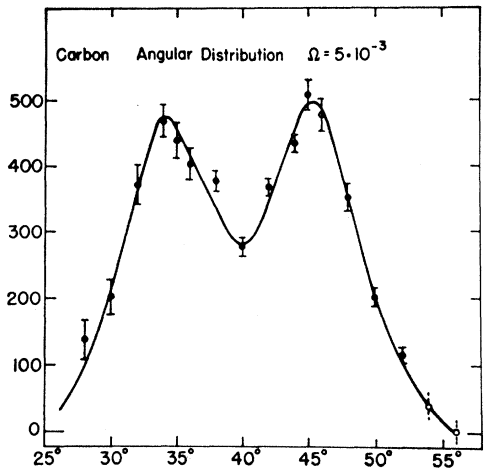

FIG. 8. Angular distribution in the $(p,2p)$ reaction on C^{12}, for the p-shell group. See reference 9.

less than normal density. This also suggests, but less decisively than do the $(p,2p)$ results, that Li^6 is a unique example of the validity of a fully developed cluster model.

The simplest description of a cluster model of Li^6 is a deuteron clinging feebly to an alpha particle. Then, one of the groups would correspond to ejecting a proton from the s state of the deuteron, the other from the s state of the alpha particle. The actual binding energies suggest that the deuteron clings to the alpha by a

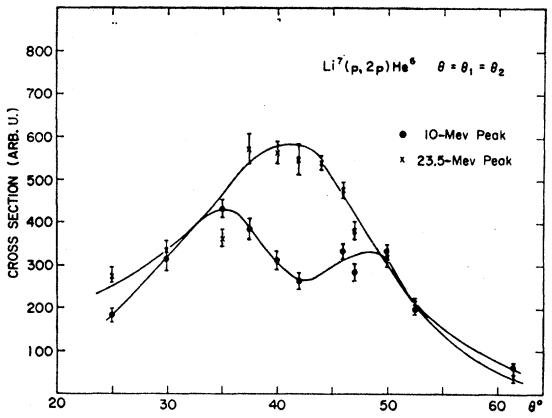

FIG. 9. Angular distribution in the $(p,2p)$ reaction on Li^7.[10] As in Figs. 6 and 8, the dip is characteristic of the p shell.

cohesion energy almost as large as the internal binding energy of the deuteron, so the picture is probably not as simple as this, but perhaps almost. The energies of cohesion and internal binding energies of the clusters in the cluster models of Li^6 and of Li^7 are shown in Table I. We note that the cohesive energy is smaller, relative to the binding energy of the cluster, in Li^7 than it is in Li^6. From the point of view of the question, "Is the cohesive energy strong enough to break up the internal structure of the cluster?" one might then naively conclude that the cluster model would be more apt to be valid in Li^7 than in Li^6.

Instead, the true situation seems to be that the twelve interactions between the three nucleons of the p shell and the four nucleons of the s shell in Li^7 are sufficiently

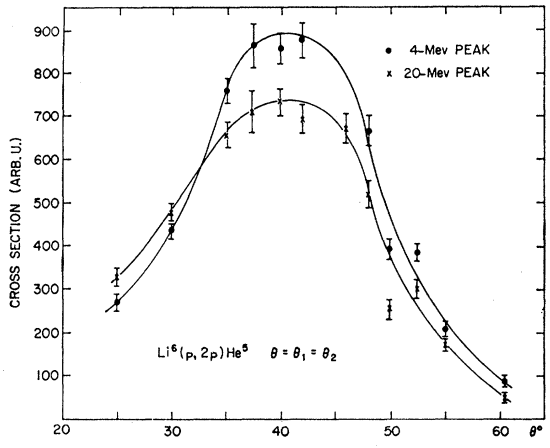

FIG. 10. Angular distribution in the $(p,2p)$ reaction on Li^6 (data from reference 10). In this case, the low-binding-energy group does not show the pronounced dip characteristic of a p state. There remains room for more precise observation and for careful interpretation to determine how closely the shape of the curve is characteristic of a simple s state. It is not clear to what extent it is excluded, for example, that there might be a modest admixture of d state. This could be caused in the cluster model by the action of a tensor interaction which may also be largely responsible for the energy separation of the ^3D excited states, involving rotation of the entire deuteron, with its spin, about the alpha.

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TABLE I. The energies of cohesion and internal binding energies of the clusters in the cluster models of Li^6 and Li^7.

	Li^6	Li^7
Binding energy of α (Mev)	28.30	28.30
Binding energy of other cluster	2.23	8.48
"Cohesive energy"	1.47	2.47
Total binding energy of nucleus	32.00	39.25

important when the two shells overlap that they can compensate for the energetic cost of distorting the two groups of nucleons, to make them fit into the same potential well, by expanding the alpha and introducing nodes within the "triton." The triton has three attractive interactions between its nucleons to benefit from the compactness imposed by the compromise with the alpha and to help pay for the introduction of three nodes in the nucleon wave functions, whereas the deuteron has only one attractive interaction to help pay for two nodes in the shell model of Li^6. The required change of kinetic energy is less serious in Li^7 than in Li^6 also because the free triton comes closer to matching the alpha particle in size than does the quite diffuse deuteron.

Perhaps another significant aspect of the situation is that in Li^7, where one p nucleon interacts with two others, we are beginning to get some of the pluralistic complexity characterized in heavier nuclei by the concept of quasi-particles, as an improved treatment of the shell model. This is absent (within the p shell) when there is only one other p nucleon as in Li^6.

In other words, the formation of a shell seems to be a cooperative phenomenon. Just as ferromagnetism has its Curie point, superconductivity its critical temperature, and the deformation of moderately heavy nuclei its critical neutron number 88, so the formation of the p shell has its critical nucleon number three.

In closing, a much over-simplified analog may be introduced which suggests that one does not have to go so far as to think of a separate deuteron in the cluster model of Li^6 in order to explain the experimental result. Consider first a simple pendulum constructed with a stiff rod, not a string. Consider it swinging perhaps even beyond the horizontal position, but not going "over the top." It still spends a considerable fraction of its time in the neighborhood of zero velocity. Give it more energy (or keep the same energy of oscillation but shorten the rod) so that it goes over the top. Now, it never stops and there is no zero-momentum component. To make the analog look more like Li^6, let us eliminate gravity and connect two such pendulums by a spring which makes them attract one another. The rods of the two pendulums are longer in such an analog of Li^6 than in the one for Li^7, so there is not enough energy to carry them on around through the position 180° apart. They vibrate relative to one another and have a zero-momentum component.

The ground state of Li^6 thus seems to contain something vaguely like a separate deuteron, perhaps very much distorted by its interactions with the alpha but so different from a shell-model state that a great deal of configuration mixing would be required to describe it in the shell-model terms. Thus, between Li^7 and Li^6 we see the demise of the shell model at the light-nucleus end.