A Unified Dynamical Approach to High- and Low-Energy Strong Interactions*

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I. INTRODUCTION

In the past, theoretical approaches to strong interactions have differentiated sharply between the high- and low-energy regions. At high energies, where the wavelength is short compared to the interaction distance and many partial waves interact, semiclassical methods have generally been employed—optical models to describe total and elastic cross sections and statistical models for inelastic processes. At low energies, where only a few partial waves are significant, the analyticity of the S matrix has played a dominant role, and many semiphenomenological dynamic calculations have been made on this basis—with a limited but significant success in correlating experimental facts. Some empirical connection between the two energy regions has been achieved through forward dispersion relations, and very recently analyticity arguments have been applied to give a crude description of high-energy peripheral collisions—as described by Drell.[1] However, the dynamics of high and low energies have not been related heretofore in a direct way. This report deals with the recent efforts of a number of Berkeley theorists in this direction. This work is entirely post-Rochester and still tentative; we are anxious, however, to stimulate discussion of some of the points encountered. The people involved include John Charap, Steve Frautschi, Marcel Froissart, Virendra Singh, and B. M. Udgaonkar. We have benefited very much already from vigorous criticism of our ideas by M. Gell-Mann and S. Mandelstam.

Frautschi and I got started in this direction because of frustration with attempts to use the Mandelstam representation to make self-consistent low-energy dynamical theories that include P resonances. In particular, we had found in the $\pi\pi$ problem that, although large low-energy S phase shifts could be accommodated without involving the high-energy region, strong elastic P scattering at low energies seemed to require conditions on inelastic cross sections at high energies.[2] There is as yet no direct evidence for a P-wave $\pi\pi$ resonance, but Frautschi and Walecka[3] and, independently, Frazer and Fulco[4] confirmed the same difficulty in the πN problem with respect to the famous $(\frac{3}{2},\frac{3}{2})$ resonance. Mandelstam had noticed the possibility of trouble in his first paper, but had hoped that the coupling between high and low energies would be weak.[5] However, by the time of the Rochester Conference last September, it was indicated by a number of detailed calculations that nature is not going to permit a realistic dynamic theory of strong interactions which is self-consistent within the low-energy domain.

Frautschi and I are proposing[6] the simplest extension of the original Mandelstam program[5] that we feel can conceivably accommodate low-energy P resonances. We have arrived at a set of equations that may or may not be self-consistent and complete; but quite apart from the validity of our particular approximation, we have been led to a way of thinking that suggests to us a startling circumstance. If total cross sections are to approach constants systematically at very high energies, then in the low-energy elastic region the forces should be strong enough to produce large S *and* P phase shifts, but not sufficiently strong to make resonances or bound states for $J \geqslant 2$. We are encouraged, in fact, to propose a universal definition of strong interactions: That they are always as strong as possible—consistent with the requirements of unitarity and analyticity. It has been plausible for some time that such a definition would correctly lead to constant high-energy total cross sections corresponding to a radius of the order of a pion Compton wavelength. One object of this presentation is to suggest that the same definition may lead to *low-energy* forces of the order of magnitude observed.

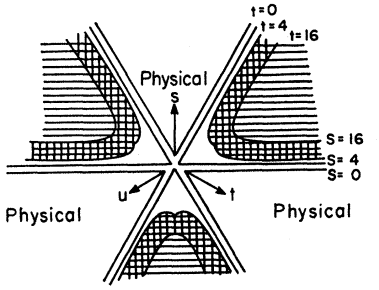

FIG. 1. Mandelstam diagram for $\pi\pi$ scattering.

* Work done under the auspices of the U. S. Atomic Energy Commission.

[1] S. D. Drell, Revs. Modern Phys. **33**, 458 (1961), preceding paper.

[2] G. F. Chew and S. Mandelstam, Nuovo cimento (to be published); see also G. F. Chew, Proc. Ann. Rochester Conf. High Energy Phys. **10**, 273 (1960).

[3] S. Frautschi and D. Walecka, Phys. Rev. **120**, 1486 (1960).

[4] W. Frazer and J. Fulco, Phys. Rev. **119**, 1420 (1960); see also W. Frazer, Proc. Ann. Rochester Conf. High Energy Phys. **10**, 282 (1960).

[5] S. Mandelstam, Phys. Rev. **112**, 1344 (1958).

[6] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **5**, 580 (1960); Lawrence Radiation Lab. Rept. UCRL 9510 (1960).

II. MANDELSTAM DIAGRAM AND STRIP APPROXIMATION

It is assumed that the audience is familiar with the Mandelstam variables and the Mandelstam representation.[5] Let us consider for simplicity the case of equal masses (e.g., $\pi\pi$ scattering) in which the three physical regions are as shown in Fig. 1. The three double-spectral functions are nonzero in the shaded domains, and it is the geometry of these regions that is central to the approach to be described.

The basic idea behind all successful approximations in strong-interaction theory (in which there are no small coupling constants) is to take accurate account of "nearby" singularities and to approximate or neglect "faraway" singularities. Unitarity restricts the strength of all singularities, so generally speaking the inverse dependence on distance given by the Cauchy formula for an analytic function in terms of its poles and branch cuts makes the "nearby" singularities more important. Now the notion of the "distance" of a real physical point from a singularity is straightforward for a function of a single complex variable, particularly when all singularities lie on the real axis, but we are dealing here with a function of two variables. As long as the only large low-energy phase shifts are in the S state, it turns out that the double-spectral functions are unimportant in comparison to the single-spectral functions, and so the problem can be made one-dimensional. However, in the interesting case, we must decide what is "near" and what is "far" in the two-dimensional Mandelstam diagram. (For substantial parts of the following argument I am indebted to Froissart.)

Consider the term

$$\int ds'dt'[\rho_{st}(s',t')/(s'-s)(t'-t)]. \tag{1}$$

The strength of the "source" is ρ_{st}, but the relevant "distance" is the product of the two one-dimensional displacements from "source" point to physical point. Thus, the double-spectral regions of constant "distance" from a low-energy physical point are bounded by hyperbolas running more or less parallel to the boundaries of the double-spectral region. The "nearest" region is the heavily shaded strip just inside the boundary. It is this region that we must accurately represent in a consistent theory of low energies.

One should realize that it is really only to the principal value part of formula (1) that the foregoing argument applies. That is to say, for example, in the s physical region the $t'-t$ denominator cannot vanish, but we must interpret $(s'-s)^{-1}$ as

$$P(s'-s)^{-1}+i\pi\delta(s'-s). \tag{2}$$

Thus, the imaginary part comes entirely from the single value $s'=s$. At low energies, the real part of the amplitude is generally of the same order of magnitude or larger than the imaginary part. However, at high energies, there is reason to believe that the imaginary part dominates the forward diffraction peak. Such a dominance is suggested by all classical models and has substantial experimental support. We believe that it will emerge naturally from the Mandelstam approach when the latter is thoroughly understood, but for the moment our faith is of classical origin. On combining formulas (2) and (1), we see that the imaginary part of the amplitude in the s physical region, arising from ρ_{st}, is

$$\int dt'[\rho_{st}(s,t')/t'-t]. \tag{3}$$

Thus, the "distance" from source point to physical point is only one-dimensional, and so far as small angles in the physical region are concerned, the "nearest" part of the double-spectral region is the strip on which we have already focused attention in connection with low energies. Thus, we have been led to the conclusion: If one can calculate the double-spectral function in a strip adjacent to the boundary, of width $\Delta t \gtrsim 16m_\pi^2$ (it must be substantially wider than the gap between the double-spectral boundary and the physical region), then one should achieve a theory appropriate to *all* energies (s) for *small* momentum transfers (t). "Small" means

$$-\Delta t \lesssim t \leqslant 0. \tag{4}$$

Recall that

$$t=-2q_s^2(1-\cos\theta_s) \quad \text{and} \quad s=4(q_s^2+m_\pi^2).$$

For energies such that $s \lesssim \Delta t$, all physical angles are included, but at high energies, only the first diffraction peak is covered by such a theory. However, this first peak comprises most of the observed collisions, and so rather few experimental phenomena are excluded. The theory also could handle backward scattering in an angular interval corresponding to

$$-\Delta u \lesssim u \leqslant 0, \tag{5}$$

where

$$u=-2q_s^2(1+\cos\theta_s).$$

Whether backward peaks systematically occur at high energies is a most interesting question.

Some check on our conclusion that the strip regions should be dominant is given by the very existence of the forward-diffraction peak, which in all measured cases has a width corresponding to $\Delta t \sim 10-20m_\pi^2$. If in formula (3) there were important contributions from $t' \gg \Delta t$, it would be difficult to understand why the physical amplitude is so small for $t \ll -\Delta t$. Further assurance is given by the circumstance that it is only in the low-energy (resonance) region that total cross sections ever become really large. A glance at expression (3) shows that in the s channel this fact implies the existence of maxima in $\rho_{st}(s,t)$ for low s, while the

corresponding argument for the t channel implies maxima for small t. Again, the strip regions are emphasized.

III. ASYMPTOTIC BEHAVIOR IN STRIP DIRECTIONS

It is indicated experimentally and very plausible from classical arguments that (to within logarithmic factors) total cross sections approach constant high-energy limits and that the widths (in t) of the corresponding forward-diffraction peaks also approach constants. Now the imaginary part of the amplitude at $t=0$ (in the s channel) is proportional to s times the total cross section. Thus, we conclude from expression (3) that[7]

$$\lim_{\substack{s\to\infty \\ -\Delta t\lesssim t<0}} \int dt' \frac{\rho_{st}(s,t')}{t'-t} \to sf(t). \tag{6}$$

We find it hard to believe that if the amplitude goes linearly with s for small negative t, it does not also go roughly linearly with s for small positive t, even beyond the branch point at $t=4m_\pi^2$, the physical threshold for the t channel. Mandelstam has repeatedly pointed out to us that such a conclusion is not required if oscillations are important in the double-spectral function, and that in nonrelativistic scattering such oscillations necessarily occur. There is no analog of the limit (6) in potential scattering, however.

A linear increase in the modulus of the amplitude for $t>4m_\pi^2$ would have potent physical implications for the low-energy behavior in the t channel, where s is to be interpreted as the (negative) square of momentum transfer. It has been shown rigorously by Regge for potential scattering that the asymptotic behavior of the amplitude for large momentum transfer is $\sim s^{\alpha(t)}$, where $\mathrm{Re}\,\alpha$ is positive for an attractive potential and increases with potential strength.[8] Regge also showed that partial waves for $l\leqslant[\mathrm{Re}\,\alpha]_{\max}$ may have bound states or resonances, while those for $l>[\mathrm{Re}\,\alpha]_{\max}$ necessarily have small phase shifts. Now if one examines the Mandelstam representation with regard to the relative amount of scattering in states of different angular momenta at low energy in the t channel, one finds that a natural way to achieve large and fluctuating phase shifts for $l\leqslant l_{\max}$, with a smooth variation for $l>l_{\max}$ according to the usual range criterion, is to have an asymptotic behavior $\sim s^\alpha$, where $[\mathrm{Re}\,\alpha]_{\max}\approx l_{\max}$. Thus, we believe Regge's criterion to be of general validity, and we conclude that if $[\mathrm{Re}\,\alpha_{\max}]$ is approximately one for strong interactions, as suggested previously, they should be capable of producing large S and P phase shifts in the low-energy elastic region but only weak scattering in states for $J\geqslant 2$.

[7] All statements here are uncertain with respect to logarithmic factors. Only the powers are intended to be taken seriously.

[8] T. Regge, Nuovo cimento **14**, 951 (1959); **18**, 947 (1960).

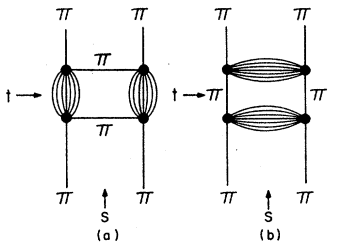

FIG. 2. Cutkosky diagram for the strips.

Such an argument in the $\pi\pi$ problem, where all three channels refer to the same process, immediately eliminates the so-called "S-dominant" solutions[9] as being physically uninteresting and focuses attention on solutions where P as well as S phase shifts are large at low energies. Moreover, we expect that no resonances occur for D or higher waves. For problems such as πN, where spin is present and where the three channels do not all correspond to the same process, the situation is more complicated, but the same basic principles apply; that is, the region of high-energy and low-momentum transfer in one channel is closely related to low-energy and high-momentum transfer in another. Unitarity puts on the asymptotic behavior in the former an absolute maximum, which by analytic continuation is carried over to the latter.

The most exciting aspect of such considerations is the prospect that coupling constants, heretofore regarded as independent, will turn out to be determined by the principle of "maximum strength" for strong interactions. For example, the magnitude of the pion-nucleon constant, $f^2=0.08$, corresponds to strong nucleon-nucleon low-energy scattering in states of $J=0$ and 1 and to strong pion-nucleon scattering in states of $J=\frac{1}{2}$ and $\frac{3}{2}$. Semiphenomenological analyses already made indicate that if f^2 were much smaller or much larger than its actual value, this situation would not be possible. The pion-pion constant λ is not yet known, and it will be an exciting race to see if theory can predict its value before measurement is made. Whether a prediction is possible depends on the formulation of a reasonable program for calculating the double spectral function in a strip of sufficient width.

IV. DYNAMICAL EQUATIONS FOR STRIPS

Cutkosky has proposed a general recipe for calculating Mandelstam's double-spectral functions in which one must consider all four-vertex graphs and insert at the corners the appropriate (complete) scattering amplitudes.[10] In the $\pi\pi$ problem, the outermost-strip contributions arise from graphs of the type shown in Fig. 2. In one channel, only two pions occur in intermediate states, but in the other an arbitrary number of particles is allowed. Figure 2(a) controls the strip for $4m_\pi^2<t<16m_\pi^2$ as s approaches infinity, while Fig. 2(b)

[9] G. F. Chew, S. Mandelstam, and H. P. Noyes, Phys. Rev. **119**, 478 (1960).

[10] R. Cutkosky, Phys. Rev. Letters **4**, 624 (1960); J. Math. Phys. **1**, 429 (1960).

controls the strip for $4m_\pi^2<s<16m_\pi^2$ as t approaches infinity. Cutkosky's formula for Figs. 2(a) and 2(b) is identical with that given in Mandelstam's first paper[5] and involves only the absorptive parts of elastic $\pi\pi$ amplitudes. Thus, one can hope to calculate the strip regions without becoming involved with the full fury of a many-body problem.

Whether the strip bounded by $16m_\pi^2$ is of sufficient width to constitute a reasonable first approximation is another matter. Clearly, Fig. 2 corresponds to considering inelastic processes reached by the exchange of a single pion; this phenomenon has been discussed by Drell, who has expressed doubt that this mechanism can account quantitatively for a majority of inelastic processes.[1] However, the estimate from which this doubt arose was based on an extension of physical cross sections off the mass shell, an extension that is not necessary in the Mandelstam-Cutkosky approach. To put the situation differently, the one-pion exchange approximation according to Mandelstam-Cutkosky is not really the same as that surveyed by Drell. We continue to hope, therefore, that Fig. 2 does constitute a reasonable first approximation.

Frautschi and I have made a preliminary examination of the iterative solution of the problem defined by keeping only the diagrams of Fig. 2. We conclude that the iteration should converge for the S-dominant type of solution and probably not differ appreciably from the result found earlier[9] by a method in which the double spectral function was treated much more crudely. Wilson has independently arrived at exactly the same equations and is actually attempting the numerical iteration.[11] However, a straightforward iteration may not converge for the interesting case in which the high-energy total cross section approaches a constant. Whether the equations, without modification to represent some effect from the neglected Cutkosky diagrams, have *any* consistent solutions of the true strong-interaction type remains to be seen. It is correspondingly uncertain as to whether the $\pi\pi$ constant λ will be required to have a unique value.

[11] K. Wilson, Harvard University Physics Department preprint, 1960 (unpublished).

V. CONCLUSION

This, then, is the situation at present. The $\pi\pi$, NN, and πN problems are being attacked at Berkeley by the new approach. At the same time, Charap is studying nonrelativistic potential scattering by the analogous technique in the hope of discovering general properties of the double spectral function (in particular, the oscillations) that will guide us in the relativistic theory. We are full of hope, but at the same time aware from past experience that the strong-interaction problem has a knack of continuously developing new twists so as to defy systematic analysis. It will be surprising to us, in any event, if nothing useful develops from the notion that unitarity puts an upper limit on interaction strengths, at low as well as at high energies, and that nature seems to approach this limit closely.

Gell-Mann often reminds us that the analyticity properties of S-matrix elements on which all our thinking is based, may in fact not be correct at the high energies to which they are now being pushed. There is no answer to this remark except that at the moment we see nothing to be gained by abandoning Mandelstam and Cutkosky. If they go down, there is nothing in sight to save us from the abyss.

Post-conference note: V. N. Gribov has presented arguments closely related to the subject discussed here [Proc. Ann. Rochester Conf. High Energy Phys. **10**, 340 (1960)].

DISCUSSION

S. Mandelstam, *University of Birmingham, Birmingham, England:* Essentially the spirit of what Dr. Chew has proposed is the same as the spirit of what we have been doing up till now. We generally neglect the terms in which *one* of these denominators was large, as Dr. Fubini in particular explained, whereas the aim now is only to neglect terms in which *both* of these energy denominators is large, and in that case you would take more into account so you expect to get more accurate results. You also have very much more work to do. In particular, one difficulty will have to be faced, which I think probably can be gotten over but which has not been gotten over yet. The difficulty lies in the asymptotic behavior of these spectral functions which Dr. Chew is talking about. The point is that he did not mention some fearful oscillations that could occur, that *do* in fact occur in potential theory. This has been shown by Regge,[a] who finds that the scattering amplitude behaves like t^α as momentum transfer t approaches infinity. The power α depends on the energy s and may be interpreted as the highest bound state at s. It is complex above the threshold in s, thus producing the oscillations I referred to. The oscillations lead to cancellations in the integrals over absorptive parts, such that α may exceed one for positive s in certain cases, and yet be less than one for negative s. In potential theory the region s negative, θ_∞, is unphysical, but in field theory there is crossing and it is a physical region. Although Regge's proof applies only to potential theory, I do not see any reason why the power of α should not also increase as s becomes positive in field theory, thus reconciling the power $\alpha=1$ which Dr. Chew has proposed at small negative s with higher powers (and therefore higher angular momentum bound states in the s channel) at small positive s. That is why I do not find the argument against high angular momentum resonances as convincing as Dr. Chew does, though I think we all agree that it would make life easier.

I should like to emphasize that my comments are directed to this one aspect only, and not to the approach in general, which I should very much like to see carried out.

[a] T. Regge, Nuovo cimento **14**, 951 (1959); **18**, 947 (1960).