the Bethe-Salpeter equation is one way of putting unitarity into the calculation. Of course unitarity is in here in a somewhat complicated way because if you go above the threshold for producing mesons, etc., the potential which you get out of this automatically becomes complex and has some of the inelastic channels taken into account indirectly.

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Recent Progress in the Dispersion Theory of Pion-Nucleon Interactions

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DURING the last two years a considerable amount of work has been dedicated to the application of dispersion theory to different problems of pion physics, such as pion-pion, pion-nucleon, and nucleon-nucleon scattering, and photopion production, and very interesting results have been obtained.

Here the attempt is made to explain the basic physical ideas which are at the origin of this development and to discuss the main points where our theoretical understanding of the experimental situation has improved. Only the case of pion-nucleon scattering is considered in detail, because the success of any attempt of treating the other problems, such as nuclear forces and photopion production, depends on our understanding of this fundamental problem.

The first successful approach to pion-nucleon scattering was the one based on the existence of a strong resonant interaction in the $J=\frac{3}{2}$, $T=\frac{3}{2}$ state. This approach was first based on the static model of Chew and Low, then on the relativistic dispersion treatment of Chew, Goldberger, Low, and Nambu (CGLN). In this model the pion-nucleon interaction is of short range, taking place essentially in the P-wave state. Since the details of such a short-range interaction are not known, the position of the (33) resonance cannot be determined by the theory but can be fitted to the experimental data.

One may ask why it is necessary to make use of dispersion theory and not simply to try the experimental data by means of a Breit-Wigner formula. One of the reasons is that dispersion theory allows a clear theoretical comparison between the phenomenological constants appearing in different phenomena involving the same basic interaction. For example, it has been possible to verify that the same renormalized coupling constant is obtained by comparison with experimental data on P-wave π-N scattering, S- and P-wave photoproduction, and high l nucleon-nucleon scattering.

One of the important problems treated recently is the study of possible corrections of the resonant model and, in particular, investigation as to whether, in addition to the short-range interaction of the pion with the core, there is a long-range pion-nucleon potential due to the interaction of the incoming pion with the pions of the nucleon cloud. This potential is the analog of the nucleon-nucleon potential due to two-pion exchange and has the same origin in the meson cloud effect which is responsible for the electromagnetic structure of the nucleon. The success of the resonant model in explaining the main features of low-energy pion-nucleon scattering and photoproduction might indicate that such a long-range term is negligible. However, there are many reasons that suggest the existence of the long-range pion-cloud interaction.

(1) The high-energy pion-nucleon cross sections are rather large and can be interpreted by means of an optical model with a nucleon radius of the order of the pion Compton wavelength. A similar radius appears in the optical model for high-energy nucleon-nucleon scattering and in the Hofstadter form factors of the nucleon.

(2) The existence of the $d\frac{3}{2}$ and $f\frac{5}{2}$ resonances shows the importance of the scattering with high l at energies of the order 600–900 Mev. This fact is difficult to understand on the basis of a short-range pion-nucleon interaction.

(3) At low energy the prediction of the CGLN theory for waves different from the (33) resonant one are in disagreement with experiment.

(4) The CGLN theory applied to the electromagnetic form factors of the nucleon gives results which are very difficult to reconcile with the experimental findings. Frazer and Fulco have shown that a satisfactory explanation of the data can be obtained by assuming the existence of a strong pion-pion interaction in the $T=J=1$ state.

Let us now discuss how one can evaluate the effect of the pion-pion interaction on pion-nucleon scattering. Here also the use of the dispersion method has definite advantages because it allows one to use the parameters which specify the strength and range of the pion-nucleon potential in connection with other problems, such as nucleon electromagnetic structure and nuclear forces.

Let us see what new singularities in the S

matrix are caused by this long-range pion-nucleon potential. It is clear that, whereas the existance of resonances gives rise to important singularities in the energy variable, a scattering by a long-range potential is connected with singularities in the momentum transfer t. This can be understood by recalling that the scattering amplitude given by a Yukawa potential of range μ^{-2} has a pole for $t=-\mu^2$ which approaches the physical region where μ becomes smaller, i.e., the range gets larger. This shows that the effect of pion-pion interaction cannot be adequately taken into account in the framework of the conventional dispersion relations at fixed momentum transfer, but that one needs a theoretical scheme in which the analytic properties in t enter in an essential manner. This scheme is provided by the two-dimensional Mandelstam representation which allows one to locate the singularities of the scattering matrix both in the energy and in the momentum transfer variable.

Many recent attempts have been made in order to construct theories of low-energy pion-nucleon interaction based on the two-dimensional representation. The main idea common to all these attempts is to try to construct forms of the scattering matrix whose singularities in both variables are located in agreement with the Mandelstam prescription and which satisfy unitarity. These approaches are necessarily approximate since no simple way has yet been found to deal with the higher singularities caused by many particle states. The discontinuities of these singularities are related to the physical production amplitudes; no dispersion treatment of these amplitudes has yet been possible. Thus, the effect of these higher singularities on low-energy scattering is usually either neglected or approximated by constants.

Many approaches have been tried by the different groups working in low-energy pion physics. They are all based on similar physical ideas, but the mathematical techniques used are rather different.

The method discussed here is the one with which I am more familiar; it is based on the approximate separation of the scattering amplitude into a resonant scattering and a potential scattering part.

Starting from the Mandelstam representation, it has been shown that the pion-nucleon scattering amplitude can be divided into two parts—a resonant part depending strongly on only the energy and a potential part depending strongly on the momentum transfer. The first term is equivalent to the (33) resonance contribution evaluated by Chew, Goldberger, Low, and Nambu; the term depending on the momentum transfer describes the effect of the interaction of the incoming pion with the meson cloud of the nucleon. The parameters appearing in this new contribution depend on the form of the pion-pion scattering matrix at low energy.

Unfortunately, since pion targets are not easily available, the present experimental information on $\pi\pi$ scattering is still rather rough and indirect. On the other hand, it is possible to obtain from the Mandelstam representation a self-consistent equation for the pion-pion scattering amplitude. This equation has given very important information on the general structure of the scattering amplitude and on the interconnection between the different $\pi\pi$ phase-shifts but, owing to its nonlinear character, it has not allowed one to select unambiguously among the possible different models of pion-pion scattering.

A possible way of dealing with this situation is to try to check the validity of the different models by comparing experiment with their consequences in the different phenomena involving $\pi\pi$ interactions. The existing experimental data on the electromagnetic form factors of the nucleon strongly suggest the existence of a π-π resonant interaction in the $J=1$, $T=1$ state.

In the original paper of Frazer and Fulco, a mass $\sim 3.2 m\pi$ was proposed; however, a more careful analysis of the experimental form factors suggests a somewhat higher value around $4.7 m\pi$ with a total width of about one pion mass. This higher value of the resonance energy seems to be confirmed by experiments on final-state $\pi\pi$ interaction in $\pi+N\rightarrow\pi+\pi+N$.

Returning to pion-nucleon scattering, the first attempt for evaluating the π-π effect was based on the (1,1) resonant model using the parameters suggested by the analysis of the electromagnetic form factors of the nucleon. Because of the isotopic spin 1 of the $\pi\pi$ resonance, this term affects only the isotopic spin-flip scattering amplitudes (i.e., the difference between $T=\frac{3}{2}$ and $T=\frac{1}{2}$).

The $\pi\pi$ contribution computed in this manner is of the right sign and order of magnitude to give agreement with the existing data of low-energy pion-nucleon scattering in the isotopic spin-flip combination. Work is now in progress to evaluate the effect on πN scattering of the S-wave $\pi\pi$ interaction. The inclusion of the effect of the $T=J=1$ $\pi\pi$ resonance on photopion production, in this case, also improves the agreement between theory and experiment both for the π^+/π^- ratio and for the π^0 angular distribution.

Work toward understanding the nuclear force problem using the same theoretical ideas is in a very advanced state. Numerical results are not yet available, but the first indications look rather encouraging.

The main points of an attempt to understand the main features of low-energy pion physics on the basis of dispersion theory have been outlined here. Although the results already obtained look rather promising, this theoretical development is still in a very preliminary stage. Much work, both theoretical and experimental, is still needed before we shall have a completely satisfactory picture of low energy pion-nucleon interactions.

DISCUSSION

J. J. Sakurai, *University of Chicago, Chicago, Illinois:* I would like to make two remarks on pion-nucleon scattering.

The numerical results obtained by you, or rather by Bowcock, Cottingham, and Lurie at CERN, are expected on the basis of lowest order perturbation theory, when the nucleon and the pion are allowed to exchange a particle with $T=1$, $J=1$. These days, even if one does Born approximation, one calls it either polology or the Cini-Fubini approximation to double dispersion relations.

The second remark is slightly more serious. Now if we try to find out how much of the isospin-dependent S-wave scattering length is due to the exchange of a resonating $T=1$, $J=1$ state, it turns out that about 90% of this scattering length can be explained; other singularities are quite unimportant. Now some time ago I made this remark, and everybody thought it was crazy because people said, especially dispersion experts said, that it is silly to try to compute a scattering length from first principle. But it turns out this way, in spite of the fact that you fit only the energy dependence of the S-wave scattering and do not make any attempt to fit the scattering length.

Now it is not my purpose to claim priority on this. Instead I would like to ask a rather serious question. Why do you think that the other singularities are unimportant? And this mystery is coupled to another mystery. If you work out the isospin-independent S-wave scattering length, it is extremely small. In fact, if you take the recent Hamilton value, it is essentially zero. As Professor Pais remarked the other day, it is not anybody's business to give a complete theory of accidents, and I wonder if anybody in this room would like to comment on this whole situation of S-wave scattering. Not only the isospin-dependent S wave at threshold, not only the energy dependence of the isospin-dependent amplitude, but the entire S-wave scattering can be explained simply by postulating that $T=1$, $J=1$ resonance.

S. Fubini: I thank Dr. Sakurai for his remark because this allows me to get five minutes more of time; because this was a remark which I really wanted to make but forgot. So first I would like to answer the first point about the relation of the representation that Cini and I obtained with quasi-particle bound states to the full Mandelstam representation. This is, in my opinion, one of the points with which we are most happy, because we feel that it is by no means trivial that our simple one-dimensional representation, for which you can write Feyman graphs if you like, has a very strict relation to the Mandelstam representation. Take the nucleon electromagnetic structure, for example, and consider the photon going to the nucleon pair by way of a $T=1$, $J=1$ quasi-particle. You can obtain the representation for the nucleon form factor and its relation with pion-nucleon scattering and nuclear forces in a simple natural manner without going through any complicated discussion of the analytic properties of the partial wave amplitudes. So at least one merit which we feel exists in our representation is to show that if you want to make a phenomenology based on dispersion relations, you can introduce quasi-bound states just by using a width function, as has to be done with unstable particles. Now one question is, can this be done independently of whether the quasi-particle is elementary or not? We feel that it is exceedingly difficult on the basis of our knowledge to know whether the quasi-particle is elementary or not, because this would involve a knowledge of extremely short-range things which we do not have. But I feel it is not a trivial fact, for example, that all the results of Chew, Goldberger, Low, and Nambu just come from the Feynman graph,

$$N+\pi \rightarrow 3-3 \text{ quasi-particle} \rightarrow N+\pi,$$

and from the pole terms.

G. F. Chew, *University of California, Berkeley, California:* I wanted to make a remark regarding Sakurai's question concerning the S-wave amplitudes. I think there is one pretty well recognized mechanism by which one could get the results Sakurai mentioned, and that is to have all the difficult singularities equivalent to short-range repulsion rather than attraction. In this case it is well known that a long-range attraction coming from the 2π $T=1$ exchange would dominate, and the short-range repulsion, even though it is exceedingly strong, just would not show up in low-energy scattering. I personally believe this is the situation, but the reason that theorists have been cautious is that there is only one of these short-range singularities that is easy to put your finger on, and that is the one that comes from single nucleon exchange—the pole term; this does correspond to a repulsion, but it is not quite short-range enough. All the calculations I have ever heard of end up with a substantial contribution from this term. Presumably the answer is that other short-range singularities combine with this one to knock it out somehow. But I do not think anybody has done a convincing calculation.

S. Mandelstam, *University of Birmingham, Birmingham, England:* I would just like to make one comment and ask one question.

The comment is again concerning how heavy the resonance has to be. Dr. Fubini remarked that if you take a resonance with the CERN value (that's about 4.5 pion masses or a little more), it does just what you want it to do to the 1-1 pion-nucleon state. In other words, the interaction for the 1-1 state was too strongly repulsive in Chew-Low theory; and now the pion-pion contribution takes the repulsion away. There is an essentially equivalent calculation done on the pion-nucleon problem by Frautschi and Walecka here and at Stanford with the parameters of the Frazer-Fulco resonance. They found that with the lower mass assigned to the resonance by Frazer and Fulco, it was too much of a good thing and certainly did give attraction, but so much attraction that you got a resonance. And this attraction that you got seems to be coming from long range, so it is something about which you would be pretty certain, and that is why at the time we were very worried about it. But now the electromagnetic structure data are consistent with this higher mass resonance, so it seems to be all right if we shift the mass of the resonance up.

The other question I want to ask is: Did I misunderstand Dr. Fubini, or did he say that they had made some progress in the s-state pion-nucleon problem?

S. Fubini: We are working now on that. But the results are not completed—the calculations are going on.