Pion-Pion Interactions*†

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An experiment is being carried out at Lawrence Radiation Laboratory in the Alvarez 72-in. bubble chamber, using a π^+ and π^- beam designed by Frank Crawford at 1.03 Bev/c. The choice of this energy was motivated by the $\Sigma - K$ threshold, which lies in this energy region. It also coincides in energy with the third pion-nucleon resonance. It is hoped that these effects will not significantly affect the investigation of the final-state interactions carried out here.

The experiment was inspired by a prescription given by Chew and Low,[1] who considered the diagram shown in Fig. 1, where p^2 is the momentum transfer squared and ω^2 the total energy squared of the $\pi-\pi$ system in its center of mass.

The statement is that the matrix element for this diagram has a pole at $p^2=-\mu^2$ with residue $\propto f\times A(\pi,\pi)$, where f is the pion-nucleon coupling constant and $A(\pi,\pi)$ the pion-pion scattering amplitude. All other diagrams contributing to the process $\pi+N\rightarrow\pi+N+\pi$ contribute to a branch cut from $p^2=-9\mu^2\rightarrow\infty$.

The complex p^2 plane thus looks like the sketch in Fig. 2. In terms of cross sections we can write

$$\lim_{p^2\rightarrow -1}\frac{\partial^2\sigma}{\partial p^2\partial\omega^2}=\frac{p^2}{(p^2+1)^2}\frac{F(\omega^2)}{q_{1L}^2}\sigma_{\pi\pi}(\omega^2)\frac{f^2}{2\pi}, \tag{1}$$

where p^2 is in units of μ^2, the pion mass squared.

A successful application of this method has been carried out by a Yale Group,[2] who obtained the $\pi^+-p(3,3)$ resonance by looking at the process $p+p\rightarrow n+p+\pi^+$.

The possible reactions in hydrogen and the corresponding final-state $\pi-\pi$ I-spin amplitudes are

$$\begin{array}{lll}
\pi^-+p\rightarrow p+\pi^-+\pi^0, & \tfrac{1}{2}A_1+\tfrac{1}{2}A_2, & \text{(a)}\\
\qquad\rightarrow n+\pi^-+\pi^+, & \tfrac{1}{3}A_0+\tfrac{1}{2}A_1+\tfrac{1}{6}A_2, & \text{(b)}\\
\qquad\rightarrow n+\pi^0+\pi^0, & -\tfrac{1}{3}A_0\quad+\tfrac{1}{3}A_2, & \text{(c)}\\
\pi^++p\rightarrow p+\pi^++\pi^0, & +\tfrac{1}{2}A_1+\tfrac{1}{2}A_2, & \text{(d)}\\
\qquad\rightarrow n+\pi^++\pi^+, & A_2. & \text{(e)}
\end{array}$$

From the combined data of these experiments, it is possible in principle to separate the three I-spin amplitudes. The amplitude A_1 is important from a theoretical point of view in order to explain the observed nucleon isotopic vector form factor and the low-energy pion-nucleon phase shifts.[3] It is suggested by Bowcock *et al.*[4] that $|A_1|^2$ should show a resonance somewhere in the region $\omega^2\approx 22\mu^2$.

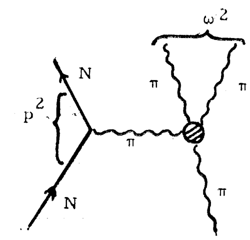

FIG. 1. Pole diagram for the process $\pi+N\rightarrow\pi+N+\pi$.

Reactions (a) and (d) are being investigated in the 72-in. chamber; 1275 events of the former and 450 events of the latter type have been found so far. We are thus looking at the amplitude $(\frac{1}{2}A_1+\frac{1}{2}A_2)$. The 72-in. chamber is a particularly convenient instrument for this investigation, in which we are interested mainly in events with $p^2\leqslant 9\mu^2$. This corresponds to recoil proton momenta below about 400 Mev/c (range$\leqslant$60 cm). Events in which the proton stops and goes forward of 70 deg (lab) are of necessity inelastic (two or more pions in the final state), and there is a one-to-one correspondence between p^2 and ω^2 and the measured range and laboratory angle of the proton. Only events in which the proton stops are accepted, and an IBM 704 correction program corrects for the fact that not all protons that would have had a range $\leqslant$60-cm stop in the chamber. The scanning table measurement does not distinguish between events with two pions and those with more than two pions in the final state. Those with more than two pions can occur for $\omega^2\geqslant 9\mu^2$ and form a background contamination to our events. Those with three pions in the final state do not have a pole at $p^2=-\mu^2$ but have a branch cut starting at $p^2=-4\mu^2$, while those with four pions in the final state have a pole at $p^2=-\mu^2$ and a branch cut starting from $p^2=-9\mu^2$. These four-pion events contribute to the

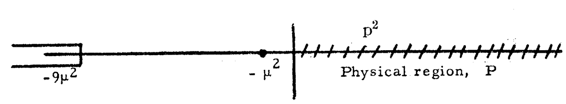

FIG. 2. The complex p^2 plane.

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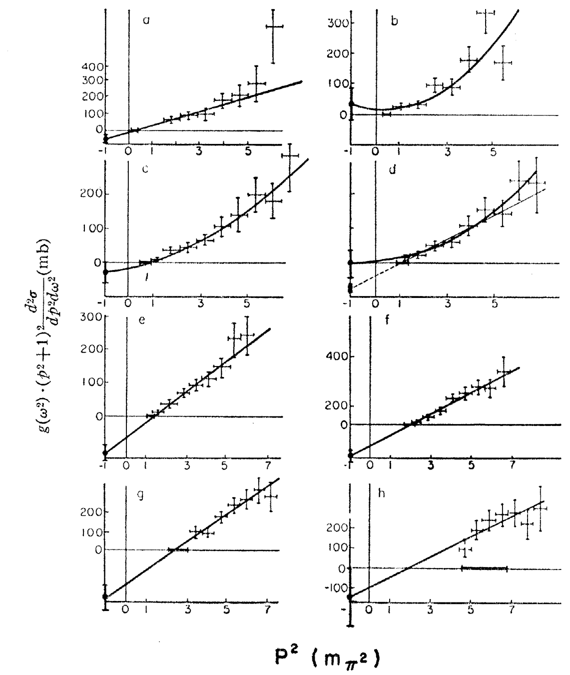

FIG. 3. Extrapolation curves at fixed ω^2. (a) $\omega^2=5$ to $8.2m_\pi^2$, (b) $\omega^2=8.2$ to 11, (c) $\omega^2=11$ to 13.7, (d) $\omega^2=13.7$ to 16.5, (e) $\omega^2=16.5$ to 19.2, (f) $\omega^2=19.2$ to 22, (g) $\omega^2=22$ to 24.7, and (h) $\omega^2=24.7$ to 27.5.

total $\pi-\pi$ cross section for $\omega^2 \gtrsim 16\mu^2$. Franckenstein measurements can eliminate events with more than two pions in the final state and are now being carried out. We are now scanning film at an incident momentum of 1.275 Bev/c so that we may study the higher ω^2 region and reduce the extrapolation distance at $\omega^2=20$.

To carry out the extrapolation, it is necessary for theory to provide some analytical form for the nonpole terms. A reasonable assumption for the behavior of the cross section is

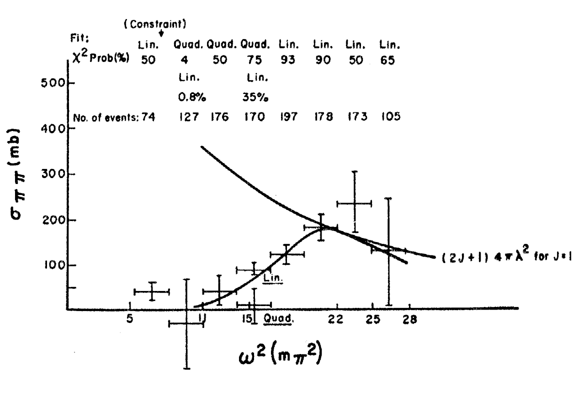

FIG. 4. The $\pi^- -\pi^0$ cross section as a function of the total dipion mass squared as determined by the Chew-Low method. Also shown are the maximum height of a p-state resonance and the shape of the Frazer-Fulco resonance [Phys. Rev. Letters **2**, 367 (1959), Eq. (10)], assuming the parameters $\nu_r=3.5$, $\Gamma=0.3$.

$$\frac{d^2\sigma}{dp^2d\omega^2}=\sum_{\text{spins}}\left|\frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{(p^2+1)}A+B_0+B_1(\boldsymbol{\sigma}\cdot\mathbf{p})+\cdots\right|^2, \quad (2)$$

where the terms $B_0+B_1(\boldsymbol{\sigma}\cdot\mathbf{p})+\cdots$ represent the effect of the branch cut at $p^2=-9\mu^2$ in the physical region. Equation (2) suggests a fitting procedure,

$$(p^2+1)^2(d^2\sigma/dp^2d\omega^2)$$
$$=A_0+A_1(p^2+1)+A_2(p^2+1)^2+\cdots, \quad (3)$$

and, using (1) and (3), we obtain

$$\sigma_{\pi\pi}(\omega^2)=-A_0(2\pi/f^2)[q_{1L}^2/F(\omega^2)]=-A_0g(\omega^2).$$

Figure 3 shows the least-squares fit obtained to $(p^2+1)^2d^2\sigma/dp^2d\omega^2$ for eight equal intervals of ω^2 varying from 5.5 to $27.8\mu^2$. Only the π^- data are included in these plots. The end of the physical region is marked on each graph as an extended heavy line on the p^2 axis. Only in the first plot does the fitted curve go through the p^2 axis before the end of the physical region. We

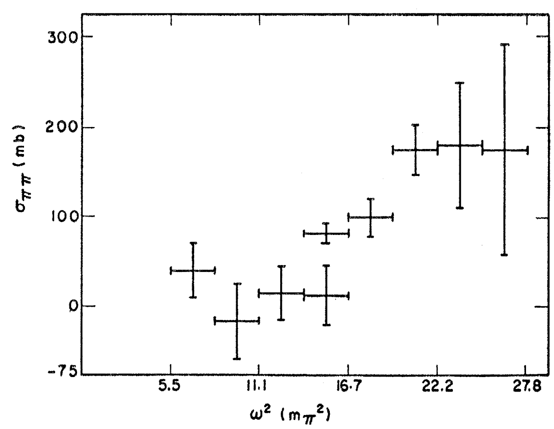

FIG. 5. The $\pi^\pm-\pi^0$ cross section as a function of the total dipion mass squared as determined by the Chew-Low method for the combined data (1725 events).

constrained this curve, therefore, to go through the end of the physical region. For the second and third plots, a quadratic fit was found necessary; for the fourth plot, however, it was not immediately obvious whether a quadratic was better than a linear fit. We show both. For the four last plots, a linear fit was definitely adequate even though there were more events for the fifth and sixth plots than for the second and third plots where quadratic fits were required. The eighth plot is shown although it is rather insignificant because of lack of data and distance of the extrapolation. Figure 4 shows the value of the $\pi^- -\pi^0$ cross section as a function of ω^2, the values being obtained from the fitted curves at $p^2=-\mu^2$. If we accept the extrapolation procedure used, then we see an increase in $\sigma_{\pi\pi}(\omega^2)$ beginning at $\omega^2\approx 15$ to $18\mu^2$, rising to about 200 mb at $\omega^2\approx 20$ to $22\mu^2$. However, it is just in this region of ω^2 that our extrapolation distance begins to get larger, making the extrapolation procedure less conclusive.

Also, if more data in this region show that a quadratic term is definitely required, then the results may be modified.

One conclusion from our data is that a Frazer-Fulco resonance at $\omega^2 \approx 10$ to $12\mu^2$ is very hard to understand. Our data are very close to the pole in this ω^2 region, so that extrapolation does not present the same problems as at higher energies. A large cross section is possible in this region only if there is a strong cubic term with positive sign in expansion (3). We see no evidence for this at present.

On the other hand, Bowcock *et al.*[4] found on a later analysis of the nucleon electromagnetic structure and the low-energy pion-nucleon phase shifts that the Frazer-Fulco resonance should be shifted to about $\omega^2 = 22$. This is consistent with our present results. *If* we assume that our data peak at $\omega^2 = 20$ to 22 (our incident energy is insufficient to examine the high-energy side of the peak), then the height is in accord with $(2J+1)4\pi\lambda\!\!\!^{-2}$ for a p-state resonance. Our half-width

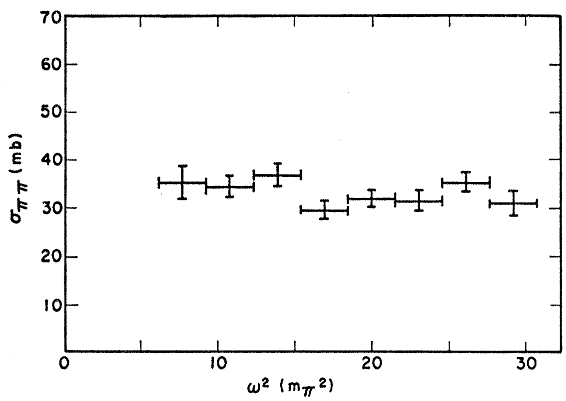

FIG. 6. Physical region ($p^2 \leqslant 9\mu^2$) plot of the $\pi^- - \pi^0$ cross section as a function of ω^2.

(obtained from the low-energy side) is approximately $5m_\pi{}^2$. Our data do not rule out a nonresonant rise in the cross section composed of $s, p, d, f, \cdots$ states, which just happens to satisfy $12\pi\lambda\!\!\!^{-2}$ at $\omega^2 = 22$.

In Fig. 5, we show the result of including our 450 $\pi^+ - p$ events in the extrapolation. We use the fact that the $\pi^+ - p$ and $\pi^- - p$ data both have the same residue at the pole although having very different contributions coming from the branch cut. The general characteristics of the $\pi - \pi$ cross section is unmodified and the errors are reduced slightly.

If the $\pi - \pi$ cross section does show an increase in the region $\omega^2 \approx 17$ to $22\mu^2$, then it is expected to be reflected in the behavior of our reactions in the nearby physical region, P. If we assume that only the pole term is significant for $p^2 \leqslant 9\mu^2$, then we can write, using Eq. (1),

$$\sigma_{\pi\pi}{}^P(\omega^2) = \frac{2\pi}{f^2} \frac{q_{1L}{}^2}{F(\omega^2)} \left\langle \frac{(p^2+1)^2}{p^2} \frac{d^2\sigma}{dp^2 d\omega^2} \right\rangle, \qquad (4)$$

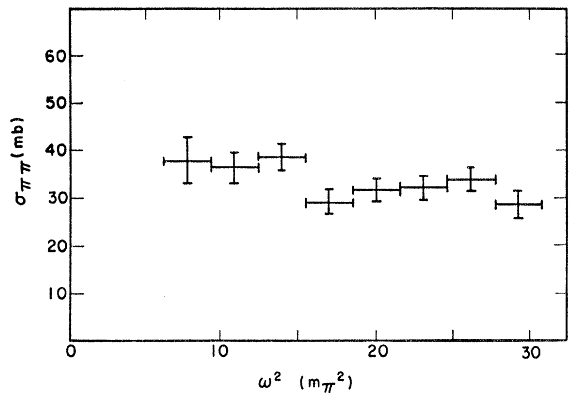

FIG. 7. Physical region ($p^2 \leqslant 9\mu^2$) plot of the combined $\pi^\pm - \pi^0$ cross section as a function of ω^2.

where the angular brackets indicate an average over our range of $p^2 \leqslant 9\mu^2$. Figure 6 shows $\sigma_{\pi\pi}{}^P(\omega^2)$ plotted for the π^- data and Fig. 7 shows $\sigma_{\pi\pi}{}^P(\omega^2)$ plotted for the combined $\pi^+\pi^-$ data. In both figures very little effect is noticeable. Certainly there is no rise in the $\sigma_{\pi\pi}{}^P(\omega^2)$ to the order of magnitude 200 mb that our extrapolation suggests is the value of $\sigma_{\pi\pi}(\omega^2)$ in the region of $\omega^2 \approx 20$ to $22\mu^2$. An alternative but illuminating way of looking at the physical-region data is shown in Fig. 8. Here we plot the number of events with $p^2 \leqslant 9\mu^2$ and also $F(\omega^2)dp^2d\omega^2$ arbitrarily normalized to have the same maximum value. If there were only the pole term, then any departures of the data from the theoretical curve would be evidence for a variation in the $\pi - \pi$ cross section. We see no such evidence in the physical region.

If we believe that $\sigma_{\pi\pi}(\omega^2)$ does increase to about 200 mb at $\omega^2 = 20$ to $22\mu^2$, then the reason for the nonappearance of such gross effects as considered in Figs. 6–8 is the appearance of higher order terms in the expansion (2). For example, final-state pion-nucleon interactions, as represented in the diagram shown in Fig. 9, could cause trouble. This is one of the many contributions from the branch cut. If the final pion and nucleon relative momenta are correct, then

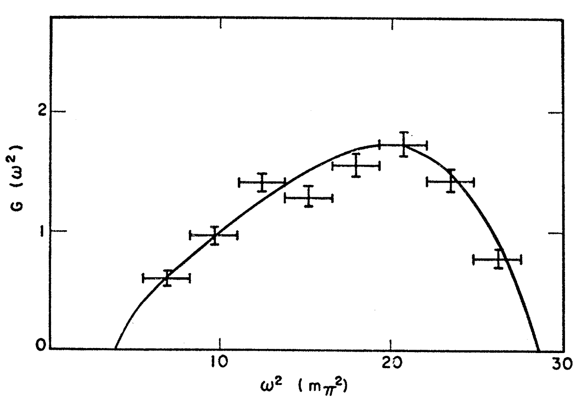

FIG. 8. Number of events with $p^2 \leqslant 9\mu^2$ and $F(\omega^2)dp^2d\omega^2$ arbitrarily normalized as functions of ω^2.

we may expect a large contribution in the physical region from a final-state pion-nucleon (3,3) resonance interaction. We are looking into such effects.

If we had the pole term alone, then our expansion would reduce to

$$(p^2+1)^2(d^2\sigma/dp^2d\omega^2)=A_0-A_0(p^2+1), \quad (5)$$

and our data would be fitted by a curve that goes through the origin. Figure 3 shows that this behavior certainly does not apply to our experimental data. That is, we are seeing evidence for a strong nonpole contribution to the physical-region behavior.

If we expand Eq. (2), we obtain

$$(p^2+1)^2(d^2\sigma/dp^2d\omega^2)=|B_0|^2+(|A|^2+2|B_0|^2 +2\,\mathrm{Re}(A^+B_1)+\cdots)p^2+O(p^4); \quad (6)$$

we can thus look at the term $|B_0|^2$, which in terms of our expansion in powers of (p^2+1) is given by

$$|B_0|^2=A_0+A_1+\cdots. \quad (7)$$

B_0 is the first correction to the pole term coming from the cut. Figure 10 shows the variation of $|B_0(\omega)|^2$ with ω^2. As expected, $|B_0(\omega)|^2$ shows a negative increase where our pole term showed a positive increase. We can thus understand the absence of an effect in the physical region as being caused by a cancellation between the pole term and $|B_0(\omega)|^2$.

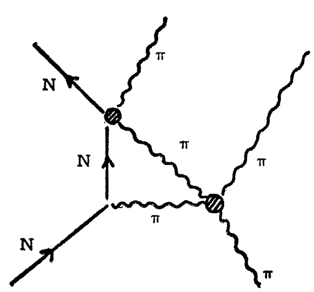

FIG. 9. A nonpole diagram for the process $\pi+N\rightarrow\pi+N+\pi$.

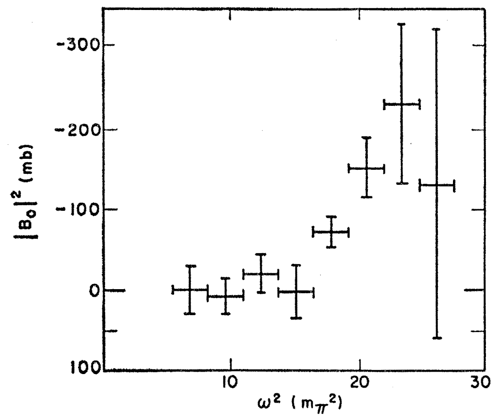

FIG. 10. $|B_0|^2$, the first correlation to the pole term, as a function of ω^2.

Effort is now being put into separating out the s- and p-wave dependence of the pion-pion cross section, particularly in the region $\omega^2=15$ to $25\mu^2$. In this way it is possible in principle to separate the contributions from the $I=1$ and $I=2$ states to our observed cross section. This separation requires Frankenstein measurement and fitting of our events, which is being carried out.

ACKNOWLEDGMENTS

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DISCUSSION

T. Fulton, *Johns Hopkins University, Baltimore, Maryland:* In terms of the expression which you have written, the cross section goes to zero at some point before it reaches the value of infinity at $p^2=-\mu^2$. How confident are you that this extrapolation through zero would give you reasonable answers?

P. G. Burke: The only thing I know that one should require of the data is that the extrapolation should be made positive in the physical region of the data, and I think in all cases this criterion is satisfied. In carrying out the extrapolation, this should be a positive constraint, and in all cases this followed automatically from our data except in the very first interval from $w^2=5$ to 7.5 in which the data, without imposing a constraint, would go on very close to but a little bit to one side of this p^2.

T. Fulton: In the region of your extrapolation, the variation of this cross section is extremely rapid aside from this positive condition, so that you are extrapolating in a region where extremely large variations take place. This is what actually disturbs me somewhat.

P. G. Burke: It is certainly true that $d^2\sigma/dp^2d\omega^2$ has a rapid variation in the physical region. In order, therefore, to say anything about the value of the extrapolated cross section, we must assume some analytic form for this variation. We, therefore, plot $(p^2+1)^2d^2\sigma/dp^2d\omega^2$, which theory tells us is a straight line through the origin when only a pole term is present. We find that such a plot has a rather smooth behavior in the physical region and in fact a straight line fit is very good over a large region of ω^2 for our data. I must emphasize, however, that the result depends very critically on the procedure which we used to take out the rapid p^2 variation before fitting. This procedure can only come from a theoretical knowledge of the analytic form of the matrix elements involved.