310 Mev Pi$^+$—p Polarization and Cross-Section Experiments. Phase-Shift Analysis*

ERNEST H. ROGERS, OWEN CHAMBERLAIN, JAMES H. FOOTE, HERBERT M. STEINER, CLYDE WIEGAND, AND TOM YPSILANTIS

Lawrence Radiation Laboratory, University of California, Berkeley, California

INTRODUCTION

LOW-ENERGY pion-nucleon scattering experiments are traditionally analyzed in terms of phase shifts, which describe the interaction for each state of the pion-nucleon system. The motivation for this approach comes from the short-range character of the pion-nucleon force which implies that the first few angular momentum states dominate the interaction. For example, at 300 Mev an impact parameter of one-pion Compton wavelength corresponds to $l=2$. Therefore, one expects to be able to describe the interaction fairly accurately by only S- and P- or S,P- and D-wave phase shifts. Higher order terms are expected to become progressively smaller. In order to obtain phase shifts from experimental data, one has to set all phase shifts equal to zero for l greater than some value $l_{\max}$. The nonzero phase shifts are then required to fit the experimental data. The assumption is that by neglecting the small phase shifts one does not distort the values of the large phase shifts which are obtained from the analysis. As we shall see, this may be a dangerous assumption.

The earlier analyses of low-energy pion-proton scattering have generally been made in terms of only S- and P-wave phase shifts ($l_{\max}=1$). Only S and P waves were necessary to give a satisfactory fit to the data, and also the data were not sufficiently accurate to obtain meaningful results for $l_{\max}=2$. The data consisted entirely of differential and total cross-section measurements. Attempts to obtain phase shifts from cross-section data were hindered by ambiguities. These ambiguities are of several types, but all give rise to the same situation; that is, they give a prescription for taking a given set of phase shifts and producing another set, which yields the same or nearly the same differential cross section as the first. Therefore, there are several sets of phase shifts which fit the differential cross section data equally well. However, these several sets of phase shifts in general predict different values of the polarization of the recoil proton.[1] So, in principle, one can resolve these ambiguities by measuring the polarization. The various ambiguities are: (1) the Fermi-Yang, in which the sign of ($P_{3,3}-P_{3,1}$) is reversed[2]; (2) a similar D-wave ambiguity first pointed out by Clementel and Villi[3]; and (3) the Minami ambiguity in which one interchanges all phase shifts of the same J and different l. The Minami ambiguity has been resolved by comparing the energy dependence of the phase shifts at low energy with general theoretical predictions.[2] Our phase-shift notation is explained in Table III. Fermi's choice of P-wave phase shifts has been generally accepted, mostly because they are strongly favored from a theoretical standpoint. There is also some indication that the Yang-type solutions may not agree with the requirements of the dispersion relations for the spin-flip forward-scattering amplitude.[4] The knowledge about the phase shifts from earlier experiments was then:

$P_{3,3}$ is positive. It rises rapidly and passes through a resonance (90 deg) at $\sim$190 Mev.

$S_{3,1}$ is negative. Its magnitude, which increases with energy, was not well known above the resonance, where the inclusion of small D waves into the analysis can affect its value substantially.

$P_{3,1}$ is small. Its sign was not reliably determined.

$D_{3,3}$ and $D_{3,5}$—virtually nothing was known about D waves except that they were probably less than 15° up to 300 Mev, or they would have been needed to obtain an adequate fit to the data. No experiment showed that D-wave phase shifts must be different from zero.

The most striking feature of low-energy pion-nucleon scattering is the resonance in the $P_{3,3}$ state. This state dominates the cross section to such an extent that it has been difficult to determine the other phase shifts. However, the polarization of the recoil proton, which is caused by interference between the various phase shifts, is more sensitive to the values of the smaller phase shifts than is the cross section. Therefore, in an effort to resolve the various ambiguities experimentally, and to obtain accurate values for the smaller phase shifts, we have made relatively accurate measurements of the polarization of the recoil proton, and also of the differential cross section. These experiments were done at an incident pion energy of 310 Mev. This qualifies as low-energy scattering since the inelastic cross section ($\sim$0.5 mb) is very small compared to the elastic cross section (60 mb).

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] E. Fermi, Phys. Rev. **91**, 947 (1953).

[2] H. A. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston, Illinois, 1955), Vol. II, p. 72.

[3] E. Clementel and C. Villi, Nuovo cimento **5**, 1343 (1957).

[4] W. C. Davidon and M. L. Goldberger, Phys. Rev. **104**, 1119 (1956).

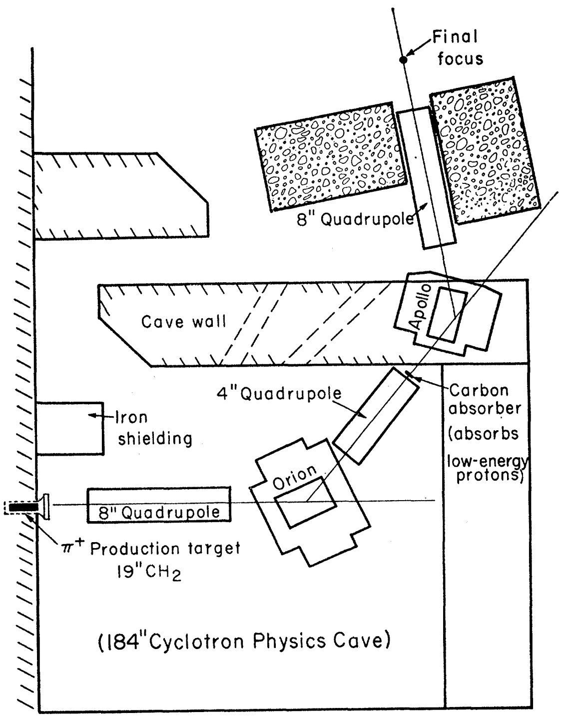

FIG. 1. Plan view of the pion spectrograph.

EXPERIMENTS

A. Beam

In order to carry out these experiments, it was necessary to develop a very intense pion beam.

Figure 1 shows a plan view of the pion spectrograph. The polyethylene target was bombarded by 740-Mev protons from the Berkeley 184-in. cyclotron. Pions produced in the forward directions were gathered by an 8-in.-diam quadrupole magnet and focused at the 4-in. quadrupole. We had a mono-momentum beam at this point because of the dispersion in the first bending magnet. The 2-in. carbon absorber stopped the proton component of the beam, while slowing the pions only slightly. The beam was deflected once more and focused onto the liquid-hydrogen target at the final focus. At this point the beam was 2 in. in diameter and contained 2×10^6 pions/sec with a contamination of 4% muons and $\frac{1}{4}$% positrons.

TABLE I. Experimental measurements of the polarization P of the recoil proton. The sign of the polarization is positive when a preponderence of the protons have their spin pointing in the direction $\mathbf{P}_i\times\mathbf{P}_f$, where $\mathbf{P}_i$ and $\mathbf{P}_f$ are initial and final pion momentum vectors.

$\theta_{c.m.}$ (deg)	P
114.2	$+0.044\pm0.062$
124.5	-0.164 ± 0.057
133.8	-0.155 ± 0.044
145.2	-0.162 ± 0.037

B. Measurement of Polarization

Figure 2 shows the counter arrangement used to measure the polarization of the recoil proton. An interesting event occurred when an incident pion scattered from a proton in the hydrogen target so that the proton recoiled through counters A and B. In this case, the pion must have passed through counter C, and was counted in coincidence with the recoil proton. The polarization of these protons was measured by scattering them from a carbon target and observing the left-right asymmetry, $e=(N_R-N_L)/(N_L+N_R)=P\bar{P}_c$. P is the polarization from hydrogen being measured, and $\bar{P}_c$ is the average polarization from the carbon analyzing target. A separate experiment was done to measure $\bar{P}_c$. N_L and N_R are the number of protons which scatter left and right, respectively, from the carbon target. These scattered protons were detected by a two-counter telescope III, D_0 or IV, D_e. A severe limitation to this method of measuring polarization is the fact that $\bar{P}_c\rightarrow0$ for proton energies $\lesssim90$ Mev. This limited the angles at which we could measure P to $\theta_{c.m.}\gtrsim110°$. The counting rate was ~1 count/min in the analyzing telescope with 10^6 pions/sec incident. The data are listed in Table I and plotted in Fig. 5.

C. Measurement of Differential Cross Section

The hydrogen target and counter arrangement used to measure the differential cross section are shown in

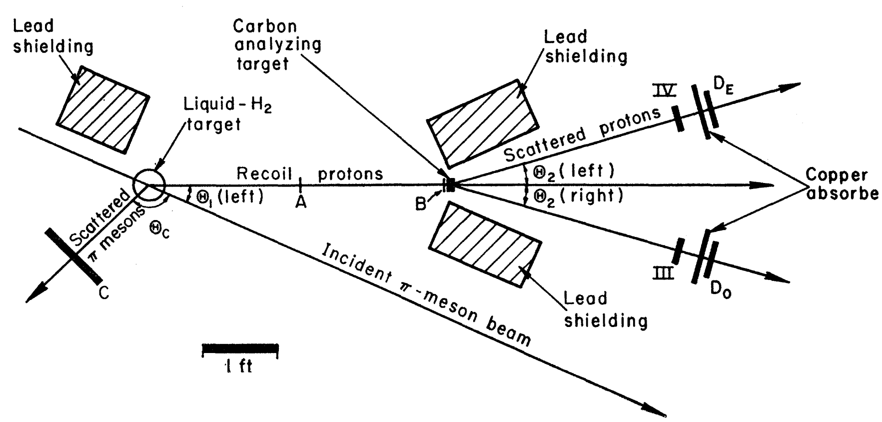

FIG. 2. Scale drawing (plan view) of the targets and counter arrangement used to measure the polarization of the recoil proton.

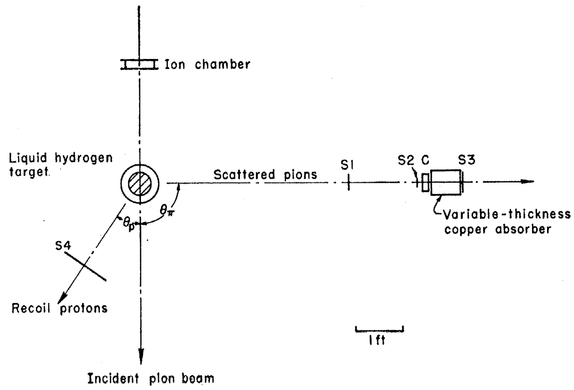

FIG. 3. Scale drawing (plan view) of the hydrogen target and counter arrangement used to measure the differential cross section.

Fig. 3. An ion chamber monitored the pion flux. Scattered pions were counted by a threefold coincidence among scintillation counters $S1$ and $S2$ and Čerenkov counter C. The Čerenkov counter rejected recoil protons. For scattering angles ~ 90 deg it was possible to check the three counter telescope ($S1$, $S2$, C) by counting the recoil proton in coincidence with the pion. Counter $S3$ and the copper absorber were used to make range curves of the scattered pions at a few angles. Several very small corrections were made to the data. The main ones corrected for the contamination in the pion beam, double scattering in the hydrogen target, and efficiency of the telescope. A summary of the differential cross section data is given in Table II. The data are plotted in Fig. 4.

TABLE II. Experimental measurements of the differential cross section (c.m.s.). The errors given are standard deviations and are independent. Not included is an rms error of $\pm 2.5\%$ in the absolute differential cross-section scale.

$\theta_{\mathrm{c.m.}}$ (deg)	$(d\sigma/d\Omega)_{\mathrm{c.m.}}$ (mb/sr)
14.0	18.37 ± 0.59
19.6	15.76 ± 0.45
25.2	13.57 ± 0.30
30.6	12.76 ± 0.25
34.6	12.06 ± 0.27
36.2	11.44 ± 0.27
44.0	9.65 ± 0.15
51.8	8.43 ± 0.26
56.8	7.40 ± 0.27
60.0	6.46 ± 0.22
69.6	4.64 ± 0.10
75.3	3.55 ± 0.09
81.6	2.72 ± 0.08
97.8	1.63 ± 0.07
105.0	1.48 ± 0.06
108.1	1.59 ± 0.07
120.9	2.04 ± 0.08
135.2	2.88 ± 0.14
140.6	3.30 ± 0.12
144.7	3.69 ± 0.15
152.2	4.03 ± 0.21
156.4	4.43 ± 0.17
165.0	4.79 ± 0.12

PHASE-SHIFT ANALYSIS

A. Search Program

A phase-shift analysis of these data was made with the aid of an IBM 704 computer. The computer made a least-squares fit to the experimental data using the grid search method. It computed the usual quantity, $\chi^2=\sum_i[(x_i^c-x_i^E)/E_i]^2$. Where x_i^E is the experimental value of x_i and E_i is the experimental error, x_i^c is the value of x_i calculated from a given set of phase shifts. The summation is over all experimental polarization and differential cross-section data. On starting from a given set of phase shifts, the computer varied each phase shift by small increments until χ^2 was minimized. It cycled through all phase shifts several times until it reached a minimum χ^2, where a small change in any phase shift caused χ^2 to increase. This minimum was not necessarily the absolute minimum, but only a depression in the hyperspace in which χ^2 is plotted as a function of the phase shifts. There may be several such minima. Which one the computer finds depends upon the starting set of phase shifts. In making our search we started at several hundred different random sets of phase shifts in order to find all these minima.

In order to relate experimentally observable quantities with phase shifts, the no spin-flip scattering amplitude $f(\theta)$ and the spin-flip amplitude $g(\theta)$ are expanded in terms of partial waves[5]:

$$f(\theta)=\lambda\!\!\!^{-}\sum_{l=0}^{\infty}\{(l+1)\exp(i\delta_l^+)\sin\delta_l^+ +l\exp(i\delta_l^-)\sin\delta_l^-\}P_l(\cos\theta)$$

$$g(\theta)=\lambda\!\!\!^{-}\sum_{l=1}^{\infty}\{\exp(i\delta_l^+)\sin\delta_l^+-\exp(i\delta_l^-)\sin\delta_l^-\}P_l^1(\cos\theta).$$

$\delta_l^\pm$ is the phase shift for the state $J=l\pm\frac{1}{2}$, where l denotes orbital angular momentum. The phase shifts are real quantities in the absence of inelastic scattering. $\lambda\!\!\!^{-}$ is the center-of-mass wavelength (over 2π) of the pion. P_l and P_l^1 are associated Legendre polynomials of degree l and order 0 and 1, respectively. The differential cross section $d\sigma/d\Omega$ and the polarization of the recoil proton P are expressed in terms of $f(\theta)$ and $g(\theta)$:

$$d\sigma/d\Omega=|f(\theta)|^2+|g(\theta)|^2$$

$$P=2\,\mathrm{Im}[g^*(\theta)f(\theta)]/[|f(\theta)|^2+|g(\theta)|^2].$$

The polarization is taken in the direction of $\mathbf{P}_i\times\mathbf{P}_f$, where $\mathbf{P}_i$ and $\mathbf{P}_f$ are pion momentum vectors before and after scattering. The effects of Coulomb scattering were included in the analysis. The method used was essentially that of Stapp, Ypsilantis, and Metropolis.[6]

B. *SP* and *SPD* Fit

The first thing that was apparent from this analysis was that we could not adequately fit our data using

[5] J. Askin, Nuovo cimento Suppl. **14**, 221 (1959).
[6] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105**, 302 (1957).

TABLE III. Phase shift solutions to 310 Mev π^+-p scattering data. The rms uncertainty in the mean energy of the pion beam was $\sim\pm 3$ Mev.

	Phase shifts (deg)[a]								Expected
Solution	$S_{3,1}$ (α_3)	$P_{3,1}$ (α_{31})	$P_{3,3}$ (α_{33})	$D_{3,3}$	$D_{3,5}$	$F_{3,5}$	$F_{3,7}$	χ^2	χ^2
SP-Fermi	−22.3	−8.1	136.1	b	b	b	b	92	24
SPD-Fermi I	−18.5±0.6	−4.7±0.6	134.8±0.6	1.9±0.4	−4.0±0.4	b	b	15.8	22
SPDF-Fermi I	−17.2±2.6	−2.9±4.0	135.0±0.6	3.1±2.6	−4.9±2.1	0.5±0.6	−0.6±1.4	14.1	20
SPDF-Fermi II	−35.5±0.7	−16.1±0.7	151.4±0.8	−11.4±0.5	13.1±0.5	−1.1±0.5	−1.8±0.3	18.3	20
SPDF-Minami-Yang I	123.1	−22.4	3.1	158.6	0.2	−2.8	−0.1	17.6	20
SPDF-Yang II	−32.0	142.2	160.4	17.8	−6.4	−1.7	−1.3	26.6	20

[a] Phase shift notation: The capital letter denotes l: $S(l=0)$, $P(l=1)$, $D(l=2)$, etc.; the first subscript $=2I+1$ (always 3 for π^+p system); the second subscript $=2J+1$.
[b] Held at zero throughout the analysis.

only S and P waves ($l_{\max}=1$), but that the data could be fit with S-, P-, and D-wave pause shifts ($l_{\max}=2$). We also found that there is only one set of S-, P-, and D-wave phase shifts which fit the data. Other solutions were found, but none had a χ^2 low enough to have more than a 2 or 3% chance of being a valid solution. Table III lists the phase shifts and the χ^2 for the *SPD* solution (*SPD*-Fermi I), and the best *SP* fit (*SP*-Fermi). The large χ^2 of the *SP*-Fermi set indicates that it is a very poor fit to the data. Each solution is designated by a name which shows the position that this solution takes with regard to the various ambiguities, e.g., Fermi or Yang. I and II refer to the D-wave ambiguity.[3] I indicates the type of solution in which $D_{3,3}-D_{3,5}>0$.

Figure 4 shows the cross-section data. The dotted line represents the *SP*-Fermi solution which does not fit the data adequately in the backward direction. The solid line represents the *SPD*-Fermi I fit which does fit the data. The dashed line shows the same *SPD*-Fermi I phase shifts but with all signs reversed. The solid and dashed lines differ at small angles because of the Coulomb-nuclear interference. One sees how the forward points indicate constructive Coulomb interference, thus determining the sign of the phase shifts.[7] Figure 5 shows the experimental polarization data and the calculated values from *SP*-Fermi and *SPD*-Fermi I. This clearly shows how poor the best *SP* fit is.

Besides resolving the ambiguities, the polarization data reduced the errors on the small phase shifts to $\frac{1}{2}$ or $\frac{1}{3}$ the values obtained when only our cross-section data was used in the analysis. These errors (from our cross-section data only) were, in turn, only $\frac{1}{2}$ to $\frac{1}{3}$ as large as errors in previous analyses. At this point the original goals had been reached. The S-, P-, and D-wave phase shifts were uniquely determined with very small errors purely on the basis of these experiments. We were overwhelmed by our success. These errors were, in fact, so small that an investigation of the effects of F waves was called for, since it was feared that their inclusion might well cause changes of greater than 0.4 deg in the D-wave phase shifts.

[7] J. Orear, Phys. Rev. **96**, 1417 (1954).

C. *SPDF* Fit

Two very surprising things happened when a search was made for *SPDF* solutions. As expected, the Fermi I solution turned up with small F waves, $\sim\frac{1}{2}$ deg, and with the S-, P-, and D-wave phase shifts essentially unchanged from the *SPD* solution, but the errors on the phase shifts had increased by a factor of 5. These

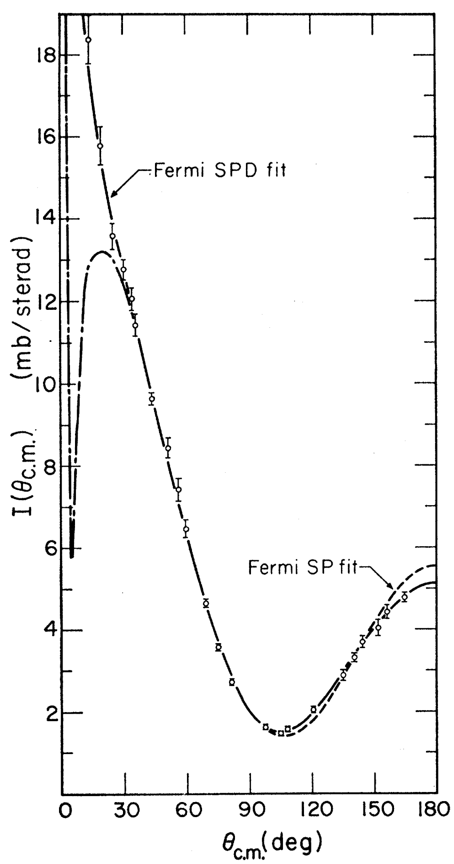

FIG. 4. The experimental differential cross section data. The solid curve is obtained from the *SPD*-Fermi I set of phase shifts, and the dotted curve represents the *SP*-Fermi set (see Table III). The dashed curve, which deviates from the solid curve at small angles, corresponds to the *SPD*-Fermi I set but with the signs of all the phase shifts reversed.

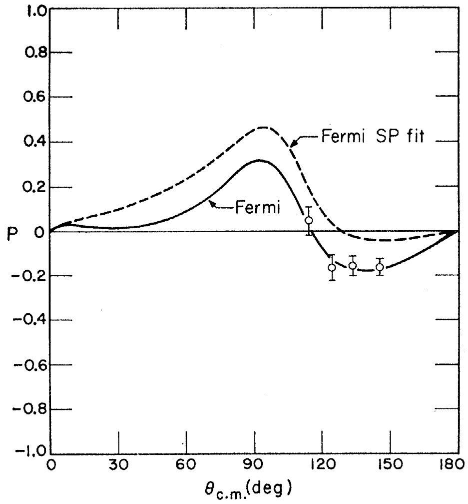

FIG. 5. The experimental recoil proton polarization data. The solid curve is obtained from the *SPD*-Fermi I set of phase shifts, and the dashed curve is from the *SP*-Fermi set (see Table III).

large errors are very disturbing! However, it should be pointed out that these phase shifts are strongly correlated. Therefore, more information is contained in this solution than the errors indicate. Secondly, those old ambiguities, which the polarization data had resolved in the *SPD* analysis, reappeared. *F*-wave phase shifts of only 1 and 2 deg allowed Fermi II to become a good fit to the data. Other type solutions also became good fits to the data (Yang II and the Minami-Yang I). The solutions to the *SPDF* analysis are also given in Table III. We certainly no longer have a unique set of phase shifts if the data is analyzed in *SPDF* waves. The data was not fit significantly better when *F* waves were allowed to be nonzero, but this is to be expected since the *SPD* fit was already very good. Even though we reject the two Yang-type *SPDF* solutions for the usual reason (Yang type solutions do not seem to agree with dispersion relations),[4] we are left with two Fermi-type *SPDF* solutions. We have two Fermi solutions because we are unable to resolve the *D*-wave ambiguity if *F*-wave phase shifts are allowed to be nonzero.

CONCLUSION

Our investigations indicate that is difficult to obtain a meaningful set of phase shifts using this method of analysis. It is very depressing to see that by allowing small *F*-wave phase shifts (1 or 2 deg), we have introduced a new solution (Fermi II), which differs by 13 to 18 deg in *S*-, *P*-, and *D*-wave phase shifts from the original Fermi I solution. This is precisely the kind of thing which we had assumed would not happen.

We have not found a valid reason for discarding either of the two Fermi-type *SPDF* solutions. The *D*-wave phase shifts of the Fermi I solution seem to show some agreement with the values predicted by Chew, Low, Goldberger, and Nambu from dispersion relations[8] ($D_{3,3}=+0.3$, $D_{3,3}=-2.5$ deg at 310 Mev). However, these predictions do not include the effect of the pion-pion interaction. We had hoped to determine the *D*-wave phase shifts accurately enough to obtain some information about the pion-pion interaction by comparing the experimental phase shifts with the predicted values of Chew *et al.*, but we have not reached this point yet.

It seems that we are unable to determine accurately even the larger phase shifts at this time even though ours is the most extensive and most accurate π^+-p scattering data available. At this time there does not appear to be any theoretical way of simplifying the analysis. However, this kind of help may appear in the near future. (See comments by Chew in the discussion of this paper.)

There is some reason to hope that these difficulties can be cleared up purely on the basis of the experimental data. The fact that we have four *SPDF* solutions instead of one is probably due to the very limited angular region of our polarization data. Figure 6 shows the behavior of *P* vs θ predicted by the various *SPDF* solutions. As expected, they differ violently at angles where no experimental data ties them down. One or two measurements in this region may well result in a unique *SPDF* solution, depending on where these additional points fall. If enough points were measured so that *P* is well determined as a function scattering angle, it is even possible that a meaningful *SPDFG* fit could be obtained. The hope would be that after including polarization data taken over a wide angular region, one could still obtain an adequate fit to the data using only a few phase shifts ($l_{max}=2$ or 3), and that the results of the analysis would remain essentially unchanged when l_{max} is increased by one. Although we were unable to measure polarization *P* for $\theta_{c.m.}<114$

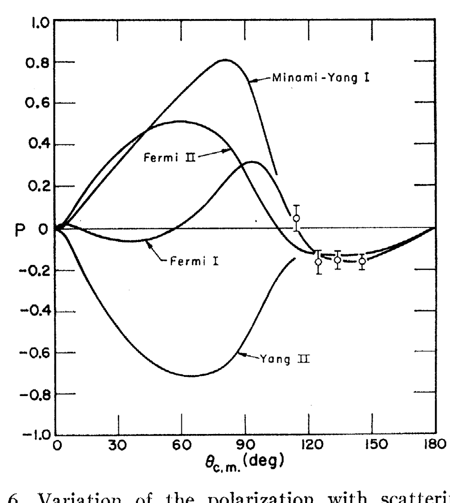

FIG. 6. Variation of the polarization with scattering angle predicted by the four *SPDF* solutions given in Table III. The experimental data are also shown.

[8] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

deg, it does not seem impossible that these data could be obtained in the near future. For instance, a helium analyzing target may be used in place of our carbon target to analyze the polarization of low energy protons. Counting rate would be the problem here, since this helium analyzer would be an order of magnitude less efficient than our carbon analyzer. Another method would involve starting with a partially polarized hydrogen target. Then a measurement of the azimuthal asymmetry in the differential cross section for scattering from this target would yield $P\cdot P_T$, where P_T is the polarization of the target. There is no inherent limitation to the angles at which P could be measured using this method.

Finally, one can measure another parameter

$$R=2\,\mathrm{Re}[g^*(\theta)f(\theta)]/[|g(\theta)|^2+|f(\theta)|^2],$$

which is independent of P and $d\sigma/d\Omega$, by measuring the polarization of recoil protons from a polarized hydrogen target. The polarization in the direction $\mathbf{P}_T\times(\mathbf{P}_i\times\mathbf{P}_f)$ is $R|\mathbf{P}_T|$. If R is calculated for the *SPDF*-Fermi I set of phase shifts and the *SPDF*-Fermi II set at 125 deg c.m., where both sets predict the same P and $d\sigma/d\Omega$, Fermi I yields -0.954, and Fermi II gives -0.230. Therefore, experimental information concerning this parameter should prove to be very useful in an analysis of this kind.

ACKNOWLEDGMENTS

We wish to thank Professor Emilio Segrè for his interest and encouragement throughout these experiments and their analysis. In addition, the help and cooperation of James T. Vale and the cyclotron crew was greatly appreciated. Thanks are also due the computer center, which is supervised by Kent K. Curtis.

DISCUSSION

G. F. Chew, *University of California, Berkeley, California:* One should point out the relevance to this kind of analysis of the notion of peripheral collisions. The point is that collisions in the pion-nucleon system, as well as in many other strong interaction combinations, which have the largest impact parameters and which therefore would be the ones to dominate the higher angular momentum states, lead to inelastic events rather than elastic. The single-pion exchange mechanism between the pion and the nucleon can produce pions, but it cannot elastically scatter pions. This may very well have the effect that the development of higher angular momentum scattering is closely associated with inelastic processes and that it is perhaps more appropriate to make the high-l phase shifts complex from the very beginning. The dividing line seems to come at $J\sim\frac{3}{2}$ for the pion-nucleon system. Theoretically, one would conjecture that between 200 and 400 Mev the old-fashioned type of analysis, in which you simply assign real phase shifts to the different states, would be appropriate for $J<\frac{3}{2}$, but for $J>\frac{3}{2}$ you should explicitly take account of inelastic processes. More parameters will be introduced; however, the theory, if you believe these peripheral collision ideas, will tell you something about the correlation between the different higher angular momentum states, so that it might be possible to parameterize the higher phase shifts in a way analogous to the techniques that have been used in nucleon-nucleon scattering. The total picture would then be something like this: for $J=\frac{3}{2}$ or less one uses the old-fashioned method; for J greater than $\frac{3}{2}$ one uses some sort of an integrated formula which includes all the phase shifts together and also includes inelastic processes at the same time. This hopefully might get us out of the kind of difficulties described by Rogers.

E. H. Rogers: We eagerly await this "integrated formula."

Our analysis was in terms of real phase shifts. Our only concern with the imaginary part of the phase shifts (inelastic scattering) has been to show that our values for the real part of the phase shifts are not affected by more than $\frac{1}{2}$ deg by neglecting the imaginary part (assuming only that the total inelastic cross section is less than 1 mb.)