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Accelerator Energy Calibrations

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I. INTRODUCTION

THE determination of the energy of a charged-particle beam from an accelerator is a necessary step in the measurement of nuclear reaction Q values or in the measurement of the energies of excited nuclear states from resonance reactions. An absolute voltage scale over an extended range was first established by Herb (21),[1] using a 1-m radius electrostatic analyzer. Several instruments are now in existence which allow absolute determinations of beam energies by either electric or magnetic deflection; these are listed in Table I.

TABLE I. Absolute beam energy analyzers.

Location	Type	Radius	Principal investigator
University of Wisconsin	Electric	1 m	R. G. Herb
Associated Electrical Industries (Aldermaston, England)	Electric	12 in.; 36 in.	S. E. Hunt
Rice University	Magnetic	0.35 m	G. C. Phillips
University of Zürich	Magnetic	0.5 m	H. H. Staub
Naval Research Laboratory	Electric	2 m	R. O. Bondelid

Energy measurements with all of these analyzers are "absolute" only in the sense that the energies are referred to standard meter bars, frequency standards (in the case of magnetic field measurements with proton moment devices), and voltage standards (in the case of electrostatic analyzers). The precisions attainable are, in general, better than 0.1% with all of the instruments of Table I. A number of measurements have been made with these analyzers of the energies of narrow, high-yield γ-ray resonances and of neutron thresholds. Such determinations may then be used to calibrate other nonabsolute analyzers for accurate Q-value and resonance energy measurements. It is the purpose here to examine the data for the five energies (four resonances and one threshold) which have been most widely measured with absolute analyzers in order to arrive at recommended values which can be used for calibration purposes. Also considered are eight other energies which have been investigated with both absolute and relative instruments. In these cases the relative measurements have been corrected for the adopted values of the standards used. Therefore, recommended values for five "primary" and eight "secondary" calibration points are given. In addition, several other energy points are discussed, none of which has been sufficiently investigated to warrant its inclusion in either of the other two categories, but which still have sufficient accuracy to be of considerable use.

II. METHOD OF OBTAINING WEIGHTED MEANS

In the process of arriving at recommended values based on various sets of data, it is necessary to adopt some weighting procedure. This would be a simple matter if it were true that the uncertainties in the measured values assigned by the various experimenters were all formulated on an identical basis. Unfortunately, this is not the case, and indeed the stated certainties which one finds in the literature cover the entire range from "limit of error" to "statistical uncertainty"; in fact, one often finds no mention of the manner in which the stated uncertainty was deduced. If an objective weighting procedure is used in the assessment of the data, then the weighting factor which is chosen for the jth measurement can be written in the form

$$\text{weighting factor} = w_j = (\Delta E_j)^{-n}, \qquad (1)$$

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[1] References in parentheses appear in numerical order in the bibliography.

where ΔE_j is the stated uncertainty. If all of the uncertainties were identically based, then statistical theory would apply, in which case $n=2$. Such a choice, however, would unduly penalize the careful worker who is overly cautious in error assignments and would weight too heavily the data of the experimenter who, though perhaps no more careful in his measurements, decides to list a statistical uncertainty. To say that a person who assigns a "limit of error" alone to his measurement *should* be penalized, although a valid criticism, is not a realistic attitude when it is necessary to arrive at a set of recommended values. Therefore, it seems necessary to choose a value for n which is less than 2. The choice $n=0$ can be attacked on the grounds that it gives no preference to the data which have been carefully treated; surely some more heavily weighted consideration should be given such data. As a result, more by default than by direct justification, $n=1$ has been chosen for the weighting factors for the individual measurements. This is the conclusion that was reached by representatives of the various groups active in the absolute energy measurement field at the conferences on calibration energies held in Washington, D. C., in April, 1959 (49) and in April, 1960. It should be emphasized that the choice to weight the measurements inversely as the stated uncertainties is entirely arbitrary, having no basis in any theory of measurements. However, the data under consideration do *not* form a statistical set, and therefore the choice of $n=1$ appears to be necessary in order to provide a realistic weighted mean as an adopted value.[2] It is hoped that in the future more uniform error assignments will be made (i.e., standard deviations) and that as more data become available a truly statistical weighting procedure can be adopted.

After the weighted mean averages are obtained, it is necessary to establish criteria for the assignment of uncertainties. Even though the sets of data under consideration do not form statistical samples, if the uncertainties are to be objectively chosen, there appears to be no alternative to using the statistical expressions. Therefore, the internal and external errors were computed in the following manner. The internal error is given by

$$(e_{\text{int}})^2=\sum_j w_j^2(\Delta E_j)^2/[\sum_j w_j]^2, \qquad (2)$$

where w_j and ΔE_j are the same as in Eq. (1). For the choice $w_j=(\Delta E_j)^{-1}$, then

$$(e_{\text{int}})^2=N[\sum_j(\Delta E_j)^{-1}]^{-2}, \qquad (3)$$

where N is the total number of contributing measurements. The external error is given by the standard expression

$$(e_{\text{ext}})^2=(N-1)^{-1}[\sum_j w_j\delta_j^2]/[\sum_j w_j], \qquad (4)$$

where $\delta_j=E_j-\bar{E}$ is the deviation of the jth measured value from the weighted mean. The values of the $\bar{E}$ are given only to the nearest 0.1 kev, and the larger of e_{int} or e_{ext}, increased to the next larger 0.1 kev, is given as the uncertainty in the mean value.

In the case of γ-ray resonances it is necessary also to give a mean value for the natural width Γ. Unfortunately, there are usually fewer accurate measurements of Γ than of E. Therefore, it has been necessary to list values for the Γ which are somewhat arbitrary. In general, a "mean value" of sorts was taken, with a heavy weight given to those measurements which listed a small uncertainty.

III. PRIMARY CALIBRATION POINTS

The basic data for the five primary calibration points are exhibited in Tables II–VI[3]; a summary is also given in Table XI. These points cover the energy range from 0.16 to 1.88 Mev. It is unfortunate that the number of accurate measurements at energies above the $Li^7(p,n)$ threshold is too few to warrant placing any of these values in the "primary category."

All absolute measurements of these five energies known to the author have been considered in arriving at the weighted mean values. The only measurement which has been discounted is the value of 1881.3±0.7 kev for the $Li^7(p,n)Be^7$ threshold energy which was obtained by Sturm and Johnson (45). These authors compared the threshold energy with the energy of the α particles from RaC′ (7.6802 Mev). If this threshold energy is corrected for the best present value for the RaC′ energy (7.6895 Mev) (50), then the value becomes 1883.6 kev, a result that appears unreasonably high. The probable reason for the apparent discrepancy in this older measurement is the use of faulty α-source technique (33). Browne (10) and others have since shown the great importance of using clean fresh sources when precision experiments are undertaken. It therefore seems reasonable to base the weighted mean value for the $Li^7(p,n)Be^7$ threshold energy only on the remaining seven absolute measurements.

TABLE II. $B^{11}(p,\gamma)C^{12}$ resonance energy data.

E_R (kev)	Γ (kev)	Method	Reference
162 ±1	5.3±1	Absolute voltmeter	48
162.8±0.2	4.5±1.5	Absolute electric	38
163.8±0.3	7.3±0.5	Absolute electric	27
	Weighted mean: 163.07 kev		
	Internal error: 0.19		
	External error: 0.42		

Adopted value: 163.1±0.5 kev; $\Gamma=6.3\pm1.0$ kev

[2] The weighting procedure based on $n=1$ yields a value of 1880.7 kev for the $Li^7(p,n)Be^7$ threshold energy (see Table VI), while $n=2$ would yield 1880.5 kev. In general, a slight lowering of the weighted mean values will result if $n=2$ is used since Staub's measurements have very small stated uncertainties and these results always are lower than the mean values (see Table VII).

[3] The data in these tables are listed in chronological order by date of publication or report.

TABLE III. $F^{19}(p,\alpha\gamma)O^{16}$ resonance energy data.

E_R (kev)	Γ (kev)	Method	Reference
340.4±0.4	2.0±1.0	Absolute electric	38
	2.9±0.2		26
	3.3±0.2		11
340.5±0.3	2.4±0.3	Absolute electric	6
340.7±0.3	2.6	Absolute electric	30
340.6±0.5		Absolute magnetic	47
	Weighted mean: 340.54 kev		
	Internal error: 0.18		
	External error: 0.07		

Adopted value: 340.5±0.2 kev; Γ=2.7±0.4 kev

TABLE V. $Al^{27}(p,\gamma)Si^{28}$ resonance energy data.

E_R (kev)	Γ (kev)	Method	Reference
993.3±1.0	0.1	Absolute electric	21
	0.06±0.03		11
992.4±0.5	0.1 ±0.05	Absolute electric	6
992.9±1.0		Absolute magnetic	3
993.5±0.8	<0.4	Absolute electric	30
991.1±0.2		Absolute magnetic	47
	Weighted mean: 992.04 kev		
	Internal error: 0.22		
	External error: 0.49		

Adopted value: 992.0±0.5 kev; Γ=0.08±0.04 kev

The results of Jones *et al.* (31) deserve some additional comments since the values quoted in Table VI can be revised if some further accurate γ-ray measurements become available. In this experiment the energy of the $Li^7(p,n)Be^7$ threshold was compared (using inelastic scattering techniques) with the energy of the first excited state of Mg^{24}. This latter state is populated in the β decay of Na^{24} and the energy of the γ ray can be deduced from the measurement (22) of the ratio of the momenta of the photoelectrons from this γ ray and from the 1.33-Mev γ ray in the Co^{60} decay. The energy of this latter γ ray has in turn been compared (35) with that of the 1.42-Mev γ ray from RaC′, which is known from absolute measurements (35). When recoil corrections are made, the energy of the Mg^{24} excited state is 1368.68±0.45 kev. If, however, the energy of the Co^{60} γ ray is obtained by comparison with the 0.41-Mev transition in the decay of Au^{198} (40), then the Mg^{24} energy is 1369.95±0.40 kev. Therefore, if accurate measurements of the γ-ray energy in the Na^{24} decay become available, then perhaps the discrepancy in the energy values listed previously can be resolved and a more precise $Li^7(p,n)$ threshold energy can be computed.

The uncertainty in the weighted mean for the $Li^7(p,n)$-Be^7 threshold energy is relatively smaller than that for any of the other primary calibration points. This fact probably stems from ease with which it is possible to observe this threshold and from the uncomplicated nature of target preparation. For these reasons and because of the fact that the $Li^7(p,n)Be^7$ threshold occurs at a convenient energy, readily reached by most of the research accelerators in existence at present, it is recommended that this threshold serve as the truly *primary* calibration point on the energy scale with a value of 1880.7±0.4 kev. The actual recommended value is subject to slight changes as more measurements become available.

TABLE IV. $F^{19}(p,\alpha\gamma)O^{16}$ resonance energy data.

E_R (kev)	Γ (kev)	Method	Reference
	5.2		18
873.5±0.9		Absolute electric	21
872.5±1.8		Absolute magnetic	16
	5.4±0.3		28
	4.5±0.2		11
872.4±0.4	4.5±0.3	Absolute electric	6
872.63±0.75		Absolute magnetic	3
873.5±0.7	4.2	Absolute electric	30
871.5±0.4		Absolute magnetic	47
	Weighted mean: 872.50 kev		
	Internal error: 0.26		
	External error: 0.33		

Adopted value: 872.5±0.4 kev; Γ=4.5±0.3 kev

TABLE VI. $Li^7(p,n)Be^7$ threshold energy data.

E_{th} (kev)	Method	Reference
1882.2±1.9	Absolute electric	21
1881.2±1.9	Absolute velocity	44
1879.7±1.1	$Co^{60}\gamma$, 1.3325	31
1881.4±1.1	$Au^{198}\gamma$, 0.411770	31
1881.2±0.9	Absolute electric	6
1880.1±1.0	Absolute magnetic	3
1880.3±0.5	Absolute magnetic	47
	Weighted mean: 1880.69 kev	
	Internal error: 0.38	
	External error: 0.29	

Adopted value: 1880.7±0.4 kev

It is interesting to compare with the weighted mean values the results of the five groups which have contributed the largest amount of data for the primary calibration list. This comparison is made in Table VII, where the deviations in parts per 10^4 are given for the $F^{19}(p,\alpha\gamma)$ resonance at 872.5 kev, the $Al^{27}(p,\gamma)$ resonance at 992.0 kev, and the $Li^7(p,n)$ threshold at 1880.7 kev. Table VII shows that the energy scales of Herb and Hunt appear to be too high by about 0.1% and that Staub's scale appears consistently low but by a smaller amount. The measurements of Bondelid and Phillips do not appear to contain systematic deviations. This

TABLE VII. Deviations of the individual measurements from the weighted mean values (in parts/10^4).

Reaction	Herb	Hunt	Bondelid	Staub	Phillips
$F^{19}(p,\alpha\gamma)$ (872.5)	+11.5	+11.5	−1.1	−11.5	+1.5
$Al^{27}(p,\gamma)$	+13.1	+15.1	+4.0	−9.1	+9.1
$Li^7(p,n)$	+7.9	···	+2.7	−2.1	−3.2

TABLE VIII. Secondary calibration points.

Reaction	E_R or E_{th} (kev)	Γ (kev)	Method	Adjusted E_R or E_{th} (kev)	Reference	Adopted values[a] (kev)
$Li^7(p,\gamma)Be^8$	440 ±2	11 ±2	Absolute voltmeter		48	
	441.4±0.5	12	$F^{19}(p,\alpha\gamma)$, 873.5	440.9±0.5	18	E_R= 441.2±0.3
	442.4±1.5		$F^{19}(p,\alpha\gamma)$, 873.5	441.9±1.5	25	Γ= 12.2±0.5
	441.5±0.5	12.2±0.5	Absolute electric		26	
	441.3±0.6	12 ±1	Absolute magnetic		47	
$Be^9(p,\gamma)B^{10}$	1087.0±2.0	4	$F^{19}(p,\alpha\gamma)$, 873.5	1085.8±2.0	18	
	1085 ±2	4	$F^{19}(p,\alpha\gamma)$, 874	1083.1±2.0	24	E_R=1083.9±0.6
	1083.7±0.7	3.8±0.5	Absolute electric		29	Γ= 3.8±0.5
	1084 ±2	3	$F^{19}(p,\alpha\gamma)$, 873	1083.4±2.0	39	
$F^{19}(p,\alpha\gamma)O^{16}$		5.6±0.5			28	E_R=1346.6±1.1
	1347.7±1.0	4.2	Absolute electric		30	Γ= 5 ±1
	1345.5±1.0		Absolute magnetic		47	
$F^{19}(p,\alpha\gamma)O^{16}$		11 ±1			28	E_R=1373.5±0.6
	1373.7±1.2	12	Absolute electric		30	Γ= 11 ±1
	1373.4±0.7		Absolute magnetic		47	
$C^{13}(p,\gamma)N^{14}$	1746.9±0.8	≤0.4	$Li^7(p,n)$, 1881.1	1746.5±0.8	37	E_R=1746.5±0.5
		0.077±0.012			20	Γ= 0.077
	1746.2±0.6		$Li^7(p,n)$, 1881.1	1745.8±0.6	53	±0.012
	1747.6±0.9	0.075±0.050	Absolute electric		6	
$Ni^{58}(p,\gamma)Co^{59}$	1843.7±0.9	0.1 ±0.05	Absolute electric		6	E_R=1843.2±0.5
	1842.9±0.45		Absolute magnetic		47a	Γ= 0.1±0.05
$Be^9(p,\alpha\gamma)Li^6$	2565 ±5	39±2	$F^{19}(p,\alpha\gamma)$, 873.5	2562 ±5	13	E_R=2564.6±1.8
	2567 ±2	41±3	Absolute electric		29	Γ= 39 ±2
	2562 ±4	38±3	$Be^9(p,n)$, 2059		36	
$C^{13}(p,n)N^{13}$	3236 ±3	···	$Li^7(p,n)$, 1882	3234 ±3	42	
	3237.2±1.6	···	Absolute electric		6	E_{th}=3235.7±1.2
	3235 ±2	···	Absolute magnetic		3	

[a] Based on weighted mean values plus the larger of the internal and external probable error. The widths Γ have been chosen arbitrarily; see text.

latter point is reassuring since Bondelid has used an electrostatic analyzer while Phillips' group has employed a magnetic instrument. It therefore appears that there is no difference within the accuracy of the present results between electric and magnetic measurements, a point that has been questioned previously (46).

IV. $Li^7(p,n)Be^7$ THRESHOLD AS AN ENERGY STANDARD

Unlike resonance energy determinations, values for neutron threshold energies can be obtained not only from direct measurements but also from Q-value analyses. For example, the most recent mass-adjustment calculation has yielded as output data Q values for all reactions in the light- and medium-weight element region (15). The output Q value for the $Li^7(p,n)Be^7$ reaction from this compilation[4] corresponds to a (relativistically corrected) threshold energy of 1879.56±1.26 kev. *It must be noted, however, that one is here comparing a Q value, which is based on a least-squares adjustment involving other mesaurements of Q values [in this case primarily those for the* $B^{10}(p,\alpha)Be^7$ *and the* $B^{10}(n,\alpha)Li^7$ *reactions], with a measurement of an energy which is intended as a laboratory energy standard.* That is, the adjustment procedure which yields a consistent set of masses and Q values does *not* yield threshold values which should be used for calibration purposes. Therefore, the list of threshold values which was used to arrive at a recommended $Li^7(p,n)$ threshold energy did not include any results based on calculated Q values.

A similar point has been raised by Brown *et al.* (10) in a precision comparison of the $Li^7(p,n)$ threshold energy and the energy of Po α particles. This group was interested in comparing these two energies in the manner in which they are used as *laboratory energy standards*, quite apart from whether the measured ratio corresponds to the *actual* energy ratio. The value obtained was

$$E(\text{Po }\alpha)/E(\text{Li}^7)=2.8221\pm0.0015. \quad (5)$$

Thus, if the $Li^7(p,n)$ threshold energy (*laboratory standard*) is 1880.7±0.4 kev, then the Po α energy (also, *labatory standard*) is 5.3075±0.003 Mev. Procedures for the use of Po α particles as energy standards are discussed in detail by Browne *et al.* (10).

V. SECONDARY CALIBRATION POINTS

In addition to the five primary calibration points for which considerable data exist, several other γ-ray resonances and one neutron threshold are suitable for calibration purposes. In general, the sets of data for these other points are composed of both absolute and

[4] This least-squares analysis did not include some of the more recent threshold measurements and did include the Sturm and Johnson (45) value.

relative measurements. It is possible to correct the relative measurements for the standards used if these standards are among the primary calibration points. In some cases, however, an average of several calibrations was used [see, for example, (34)] and the correction of such data is rather uncertain; these data along with measurements based on "uncorrectable" calibrations have therefore been omitted. Table VIII lists the seven γ-ray resonances and one neutron threshold chosen as secondary points. The averaging procedure for the resonance energies and for the natural widths was the same as for the primary calibration points. A summary of the results is also given in Table XII.

VI. OTHER CALIBRATION POINTS

Because of the target preparation problems for some of the primary and secondary calibration points and because of inconvenient energy spacing in some regions, it is clearly advantageous to have more calibration points, especially at higher bombarding energies. Table IX lists 12 additional points which can also be used for calibration purposes. Although the lists of the primary and secondary points included values only for proton-induced reactions, Table IX also gives results for two (α,n) resonances and one (d,n) threshold. Since the magnetic rigidity of deuterons and singly ionized He^4 ions is considerably greater than that for protons, the points given for these bombarding particles actually correspond to quite high "equivalent proton energies" for magnetic analyzing systems.

Even though several measurements each have been reported for most of the resonances and thresholds listed in Table IX, only the single most recent (and in all cases, most precise) value has been given. The calibration used to obtain some of the results is given as "several reactions." It was the policy to exclude such measurements from the previous tables, since they are "uncorrectable." However, there appears to be a need for calibration points in the energy region covered by these values, and it was therefore considered appropriate to include these points in this tabulation. In general, the uncertainties indicated for these points include the uncertainty which results from the calibration procedure used.

A unique method of using the $O^{15}(d,n)F^{17}$ threshold to extend the range of calibration has been used by the Chalk River group (19). In the process of calibrating the magnetic analyzer for their Tandem accelerator, these experimenters accelerated O^{16} ions (ionized to 4+ and 5+) and bombarded a deuterium target. The threshold for the $H^2(O^{16},n)F^{17}$ reaction occurs at approximately 14.75 Mev and the uncertainty in the threshold energy based on the value given in Table IX is only 0.033%, or 5 kev. By using 4+ and 5+ ionized O^{16} ions, it was possible to obtain calibration points at 14.47 and 9.33 Mev equivalent proton energy, respectively.

VII. RECOMMENDED PROCEDURES FOR MEASURING γ-RAY RESONANCES AND NEUTRON THRESHOLDS

A. Energy Measurements with Molecular Beams

Since accurate measurements of resonance and neutron threshold energies are not available over the entire energy range which is attainable with present-day accelerators, it has been customary in some laboratories to calibrate beam energy analyzers by observing resonances or thresholds with molecular we well as atomic beams [see, for example, (34)]. Thus, it should be possible to obtain three calibration points by observing a given (p,γ) resonance or (p,n) threshold with the H^+, HH^+, and HHH^+ beams. By properly taking into account the aggregate particle masses and the effective energy spreads in the beams due to the internal motion of the protons in the molecular ions (2, 23, 41), it would then be possible to obtain three calibration points with zero relative error. Recently, however, detailed measurements of resonance processes with molecular beams have been carried out by Bondelid and Kennedy (5), Anderson *et al.* (2), and Dahl *et al.* (14) which indicate that asymmetrically shaped thick-target resonance curves are obtained with molecular beams, even though atomic beam experiments on the same targets and

TABLE IX. Other calibration points.

Reaction	Resonance or threshold energy (kev)	Γ (kev)	Method	Reference
$T(p,n)He^3$	1019.7±0.5	Thresh	Absolute electric	7
$Ni^{58}(p,\gamma)Co^{59}$	1424.1±0.7	0.050±0.050	Absolute electric	6
$O^{16}(d,n)F^{17}$	1829.2±0.6	Thresh	Absolute electric	8
$O^{18}(p,n)F^{18}$	2573.4±0.8	Thresh	Absolute electric	9
$C^{13}(\alpha,n)O^{16}$	2800 ±3	~4	$Li^7(p,n)$, 1881.1	54
$Si^{28}(p,p'\gamma)Si^{28}$	3105 ±6	12	Several reactions	52
$Si^{28}(p,p'\gamma)Si^{28}$	3340 ±7	12	Several reactions	52
$Na^{23}(\alpha,n)Al^{26}$	3492 ±3	<1	$Li^7(p,n)$, 1881.1	54
$N^{15}(p,n)O^{15}$	3780.8±1.1	Thresh	$Li^7(p,n)$, 1881.1	32
$F^{19}(p,n)Ne^{19}$	4233 ±3	Thresh	Absolute magnetic	3
$Si^{28}(p,p'\gamma)Si^{28}$	4240 ±8	16	Several reactions	52
$Si^{28}(p,p'\gamma)Si^{28}$	4887 ±10	12	Several reactions	52

under otherwise identical conditions yield symmetrical curves. Furthermore, the resonance energy positions obtained with molecular beams are slightly lower for thick-target measurements than for thin-target measurements. This latter effect amounts to some 0.05 to 0.08% (5, 14). The reason for this effect appears to be associated with the Coulomb energy of the two protons which result from the stripping of the electron from the ion (14).

In view of the uncertainties concerning the behavior of molecular ions in matter, molecular-beam calibration points should probably not be used, except if *very thin targets* are employed, when accuracies approaching 0.1 per cent are desired.

B. Target Conditions

In the measurement of resonance or threshold energies it is important to insure that the target surface is clean. The contamination of targets by carbon buildup from the cracking of organic vapors in the vacuum system is a frequent cause of sizeable apparent energy shifts. Cold trapping in the immediate vicinity of the target is an effective means of preventing carbon deposits; these techniques have been discussed, for example, by Butler and Gossett (12) and by Richards (43). In the case of α-particle bombardment, carbon buildup is extremely rapid and even a highly trapped target may show appreciable carbon deposits after short bombardments (54). The only effective means of dealing with this problem at present seems to be frequent changes of the target spot or of the entire target.

In the measurement of γ-ray or neutron resonances the thickness of the target used in the experiment enters directly into the analysis of the data [see, for example, (17) and (43)]. Targets for resonance energy determinations may be divided into three thickness categories.

(a) Thin targets. If the thickness of the target is much less than the natural width of the resonance observed, then the position of the peak counting rate corresponds to the resonance energy. Some of the resonances listed in the preceding tables, however, have extremely small natural widths and the preparation of targets which have an even smaller energy loss for the bombarding particle beam is quite difficult, if possible at all. Therefore, as a general technique for precision experiments, the thin target method is not particularly useful, although in some cases, it is adequate.

(b) Targets with thickness comparable to the natural resonance width. If targets of this type are used, then it is necessary to measure the thickness to an accuracy (usually) of a fraction of a kev since the position of the peak counting rate will differ from the resonance energy by one-half of the target thickness. Hunt (30) has determined target thicknesses by observing the difference in the position of the peak counting rate between a resonance curve measured with the particle beam striking the target at normal incidence and a curve measured with the beam striking the target at an angle of 60°. Since the effective target thickness is increased by a factor of 2 in the 60° rotation, the difference in the positions of the peak counting rates is equal to one-half of the target thickness. Results of these measurements were always checked with thicknesses calculated from a knowledge of the amount of material evaporated on the target backing (30).

(c) Thick or semithick targets. Perhaps the easiest and most straightforward technique in the measurement of resonance energies is to use a target whose thickness is somewhat greater than the natural width of the resonance studied. If a measurement of the "thick-target step" is made, then the mid-point of the rise corresponds to the resonance energy (17, 43). Furthermore, the natural width of the resonance can be obtained from an analysis of the shape of the thick-target step, even for extremely narrow resonances [see, for example, (5) and (6)]. Because of the ease with which targets may be prepared and because of the straightforward nature of data analysis, the use of semithick targets is recommended for resonance energy measurements.

C. Neutron Threshold Measurements

The energy dependence of the *total* neutron emission cross section for s waves in the region immediately above threshold is

$$\sigma \sim (\Delta E)^{\frac{1}{2}}, \tag{6}$$

where ΔE is the difference between the bombarding energy and the threshold energy. Therefore, if a target of finite thickness is used, the total neutron *yield* is proportional to the integral of Eq. (6):

$$Y \sim (\Delta E)^{\frac{3}{2}} \text{ (thick target; } s \text{ waves).} \tag{7}$$

This result indicates that in order to extrapolate linearly a yield curve to zero yield for the purpose of determining the threshold energy, a plot of $Y^{\frac{2}{3}}$ vs ΔE must be made. The necessity of using a $Y^{\frac{2}{3}}$ plot was first realized by Stephens, Spruch, and Schiff [quoted by (4)] in an investigation of the $C^{14}(p,n)N^{14}$ reaction.

Equation (7) gives the expression for the *total* neutron yield from a thick target in the energy region immediately above threshold. Since the neutrons are confined to a narrow cone about the forward (beam) direction in this energy region, it is possible to measure the total yield with a single detector. A paraffin- or polyethylene-moderated $B^{10}F_3$ proportional counter is commonly used as a detector. Such a counter must be placed so that it subtends a half-angle at the target which is greater than the half-angle of neutron emission at the maximum bombarding energy used for the extrapolation to threshold. For example, if a 5-kev interval is used for extrapolation of the yield from the $Li^7(p,n)Be^7$ reaction, then the detector must be located so that the sensitive area subtends a half-angle of at least 21° at the target.

Some care must be exercised in this procedure, however, since in order to measure the total yield by the above method, the detector must have a uniform sensitivity within the solid angle filled by the emerging neutrons; data on this type of efficiency function are lacking for most detectors.

Another precaution to be considered in the measurement of neutron threshold energies is the energy spread or resolution of the bombarding beam. Newson *et al.* (41) have shown that if a bombarding beam with energy spread Δ is used, then the *apparent* threshold energy will be found approximately $\Delta/2$ below the *true* threshold energy. Their calculation, however, probably overestimates the effect, since when high resolutions are used ($E/\Delta \gtrsim 2000$) Browne *et al.* (10) and others have have found no measurable difference in the extrapolated thresholds as the resolution is altered. However, high resolutions should be used whenever precision energy measurements are undertaken.

The recommended procedures for the measuring of neutron threshold energies are therefore: (a) bombarding beams with the highest possible resolution should be used; (b) the neutron detector should subtend a sufficiently large half-angle at the target to intercept all of the neutrons from the reaction in the energy range investigated; and (c) the extrapolation procedure for determining the threshold energy should utilize a plot of $(\text{yield})^{\frac{3}{2}}$ vs bombarding energy.

VIII. OTHER METHODS OF ANALYZER CALIBRATION

One rather powerful technique which has not yet been applied to the measurement of the energy of accelerator beams has been used by White *et al.* (51) in a precision determination of the energy Po α particles. This method does not attempt to measure absolutely the magnetic field or radius of the analyzer, but compares directly the energy of the α particles with the voltage necessary to accelerate Lu^{175} singly charged ions to the same $B\rho$ value. Lutetium was chosen as the comparison ion since it is essentially monisotopic (Lu^{175} abundance = 97.4%), easy to ionize, and its high mass number permits the use of relatively low voltages to produce large $B\rho$ values. In order to make the comparison with Po α particles (5.30 Mev) it is necessary to accelerate the Lu^{175} ions only to about 30 kev, and voltages of this order can be measured with high precision. Since the mass of Lu^{175} is known to within about 0.002%, it is possible to make energy measurements which are essentially limited only by the accuracy with which it is possible to locate the line in the spectrometer due to the particle of unknown energy.

This Lu^{175}-ion comparison technique could be easily adapted for use in an accelerator by constructing an ion source which can be inserted into the drift space between the accelerator proper and the analyzing system. The major problem would be to ensure that the Lu^{175} beam and the accelerator beam actually follow the same path, but a properly designed system of slits and apertures should make this possible.

Another technique which utilizes a spectrometer to calibrate a beam analyzer (neither of which is an absolute instrument) is an extension of the scattering technique used, for example, by Jones *et al.* (31). This method would be based on one fixed energy point [the $Li^7(p,n)Be^7$ threshold energy] and on the excitation energies of the first excited states of Mg^{24} (1.369 Mev) and Li^7 (0.477 Mev) which are known to precisions of 0.05 and 0.06%, respectively (31, 1). The calibration procedure would be as follows: With the lithium target in place, the $Li^7(p,n)$ threshold would be located and the analyzing magnet would be stabilized at a field setting corresponding as closely as possible to the threshold position. Next, the magnesium target (preferable enriched Mg^{24}) would be placed in position at the object point of the spectrometer and the spectrometer located at, say 90° with respect to the beam direction. The spectrometer field would then be varied until the elastic scattering edge corresponding to Mg^{24} is located and the spectrometer field stabilized at that point. The analyzing magnet and beam energy would then be increased until the inelastic scattering edge is located in the spectrometer. The beam energy would then be 3.31 Mev and would be known to an accuracy of 0.05% relative to the $Li^7(p,n)$ threshold. If, after establishing the position of the threshold, the scattering measurements were made with deuterons, then an analyzer calibration point at an equivalent proton energy of 4.87 Mev could be obtained, again with an accuracy of 0.05%. The scattering of singly charged He^4 ions would yield a point at 8.46 Mev. Such measurements could be cascaded to obtain calibration points to energies as high as desired. Such cascading would decrease the accuracy, but this would not be a serious limitation until three or four steps were involved. Furthermore, scattering from Li^7 would give points at intermediate energies. Points lower in energy than the $Li^7(p,n)$ threshold energy could be obtained by reversing the order of the elastic and inelastic scattering measurements; i.e., the inelastic scattering edge would first be located and then the beam energy lowered until the elastic scattering edge is located. Table X lists several of the calibration points that could be obtained by this method in the range of equivalent proton energies from 0.5 to 15 Mev; obviously, the number of combinations using Li^7 and Mg^{24} as targets and scattering protons, deuterons, He^3, and He^4 ions is extremely large. The notation used in Table X is as follows: the Mg^{24} scattering first discussed in the foregoing is denoted by $+(Mg^{24}+p)+(Mg^{24}+p)$; the reverse process, e.g., the inelastic scattering of protons from Li^7 followed by elastic scattering to obtain a low energy point, is denoted by $-(Li^7+p)$; etc.

This type of calibration procedure should be particularly useful for Tandem or other high-energy accelerators, since, for example, by two successive scat-

TABLE X. Partial list of calibration points possible with the scattering technique. The energies given are for an observation angle of 90° throughout. The notation is described in the text.

Equivalent proton energy (Mev)	Method	Accuracy possible
0.54	$-(Li^7+d)$	0.06%
1.32	$-(Li^7+p)$	0.06%
1.8807	$Li^7(p,n)$	Standard
2.44	$+(Li^7+p)$	0.06%
2.75	$+(Mg^{24}+p)-(Li^7+p)$	0.08%
3.22	$+(Li^7+d)$	0.06%
3.31	$+(Mg^{24}+p)$	0.05%
4.31	$+(Mg^{24}+d)-(Li^7+p)$	0.08%
4.74	$+(Mg^{24}+p)+(Mg^{24}+p)$	0.07%
4.87	$+(Mg^{24}+d)$	0.05%
6.30	$+(Mg^{24}+p)+(Mg^{24}+d)$	0.07%
7.86	$+(Mg^{24}+d)+(Mg^{24}+d)$	0.07%
8.46	$+(Mg^{24}+He^{4+})$	0.05%
9.88	$+(Mg^{24}+p)+(Mg^{24}+He^{4+})$	0.07%
11.44	$+(Mg^{24}+d)+(Mg^{24}+He^{4+})$	0.07%
15.03	$+(Mg^{24}+He^{4+})+(Mg^{24}+He^{4+})$	0.07%

terings of singly charged He^4 ions from Mg^{24}, it should be possible to obtain a calibration point at an equivalent proton energy of 15 Mev with an uncertainty of only 0.07% (or 10 kev) due to the uncertainty in the energy of the first excited state of Mg^{24}; other uncertainties in the measurements may limit the actual final result to about 0.1%. By using scattering angles of greater than 90° (the value chosen for the list given in Table X), it would be possible to obtain points at even higher energies. The precisions possible are increased if more accurate measurements of the energies of the first excited states of Li^7 and Mg^{24} become available.

IX. SUMMARY

Since many measurements of nuclear reaction Q values require a precise knowledge of the energy of the bombarding particle beam initiating the reaction, it is desirable to establish several energy calibration points for use in laboratories not possessing absolute beam energy analysis systems. An arbitrary (but, it is hoped, realistic) procedure has been adopted to obtain weighted mean values for some of the more common calibration points. The recommended values for the primary and secondary energy points are summarized in Tables XI and XII. In addition, several other points recently measured with precision instruments are listed in Table IX; the number of independent determinations of these energies was insufficient to warrant their inclusion in either the primary or secondary category and only the most recent result has been listed.

Recommended procedures for the measurement of γ-ray resonance and neutron threshold energies have been given and, again, it is hoped that these procedures will be followed by groups performing precision energy measurements. Two "new" methods for calibrating beam energy analyzers have been discussed which should allow accurate measurements to be performed even at relatively high bombarding energies.

TABLE XI. Summary of primary calibration points. (All contributing measurements made on an absolute basis.)

Reaction	Adopted resonance or threshold energy (kev)[a]	Adopted Γ (kev)[b]
$B^{11}(p,\gamma)C^{12}$	163.1±0.4	6.3 ±1.0
$F^{19},(p,\alpha\gamma)O^{16}$	340.5±0.2	2.7 ±0.4
$F^{19},(p,\alpha\gamma)O^{16}$	872.5±0.4	4.7 ±0.3
$Al^{27}(p,\gamma)Si^{28}$	992.0±0.5	0.08±0.04
$Li^7(p,n)Be^7$	1880.7±0.4	Thresh

[a] Weighted mean value plus the larger of the internal and external probable error.
[b] Arbitrarily chosen.

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TABLE XII. Summary of secondary calibration points. (Contributing measurements both absolute and relative; the latter have been corrected for the adopted values of the standards used.)

Reaction	Adopted resonance or threshold energy (kev)[a]	Adopted Γ (kev)[b]
$Li^7(p,\gamma)Be^8$	441.2±0.3	12.2 ±0.5
$Be^9(p,\gamma)B^{10}$	1083.9±0.6	3.8 ±0.5
$F^{19}(p,\alpha\gamma)O^{16}$	1346.6±1.1	5 ±1
$F^{19}(p,\alpha\gamma)O^{16}$	1373.5±0.6	11 ±1
$C^{13}(p,\gamma)N^{14}$	1746.5±0.5	0.077±0.012
$Ni^{58}(p,\gamma)Co^{59}$	1843.2±0.5	0.1 ±0.05
$Be^9(p,\alpha\gamma)Li^6$	2564.6±1.8	39 ±2
$C^{13}(p,n)N^{13}$	3235.7±1.2	Thresh

[a] Weighted mean value plus the larger of the internal and external probable error.
[b] Arbitrarily chosen.

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