REVIEWS OF

MODERN PHYSICS

VOLUME 33, NUMBER 1 JANUARY, 1961

# Completeness of Quantum Mechanics and Charge-Conjugation Correlations of Theta Particles*

D. R. INGLIS

*Argonne National Laboratory, Argonne, Illinois*

## INTRODUCTION

THE conclusion of Einstein, Podolsky, and Rosen[1] (EPR) that a "wave function does not provide a complete description of physical reality" has been discussed for many years. The limitation that the existence of a finite quantum of action $h$ imposes on simultaneous measurement of certain pairs of dynamical variables is successfully taken into account in quantum theory and is the very basis of its triumphant difference from classical mechanics. It is clear that the measurement of the position of a single particle influences the particle and the knowledge of its momentum. The view of EPR is that the possibility of evaluating a dynamical variable exactly under some circumstances establishes it as an element of reality which entitles it to have a counterpart (they imply a numerical counterpart) in the theory under all circumstances. They provide a striking example by considering two systems correlated by past interaction in such a way that measurement on the first gives information about the second, apparently without disturbing the latter.

Bohr[2] convincingly pointed out that in the measurement on the first system there is "an influence on the very conditions which define the possible types of prediction regarding the future behavior of the system" and that these conditions constitute a part of "physical reality." To avoid the difficulties of making clean measurements of continuous variables, Bohm[3] has suggested the conceptually simple consideration of the spin coordinates of two previously interacting particles. Furry[4] has even questioned whether a fundamental conservation law such as the conservation of angular momentum applies to a situation in which it has not been tested; e.g., to the total spin of two particles whose wave packets do not overlap. Such drastic renunciation seems unnecessary, but Bohm has discussed the possibility of some hidden interaction by which a measurement on the second particle affects the first, and finds reason for doubt in the fact that the measurement on the first can be outside of the light cone from the second. The fact that theta particles exist as two kinds (two kinds characterized by their interactions, transforming into two kinds characterized by their lifetimes) can be described in terms of a formalism analogous to the quantum-mechanical treatment of spin. Lee and Yang[5] have suggested an experiment involving thetas that is related to the EPR question. The possibility of investigating in more detail the properties of such a formalism adds current interest to this question. It is our purpose here to present the problem in terms of an analogy based on a descriptive treatment of Bohm's two-spin model.

The properties of the spin of a particle may be associated with the behavior of the particle in a magnetic field. The standard way to measure a component of the spin is by means of the Stern-Gerlach experiment. The particle moving along the $y$ axis first enters a fairly homogeneous field, such as the fringing field of the magnet, and then proceeds to a region where there is a strong gradient of the field between the poles, $\partial H_z/\partial z$, and is deflected "up" or "down" by an amount depending upon the component of its magnetic moment. In a homogeneous magnetic field $\mathbf{H}$, a torque is applied

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).

[2] N. Bohr, Phys. Rev. 48, 696 (1935).

[3] D. Bohm, *Quantum Theory* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1951), Chap. XXII; D. Bohm and Y. Aharonov, Phys. Rev. 108, 1070 (1957).

[4] W. H. Furry, Phys. Rev. 49, 393, 476 (1936).

[5] T. D. Lee and C. N. Yang (unpublished); Professor Lee (personal communication by letter and at a meeting of the "ZGS Users Group" at Argonne, May 28, 1960).

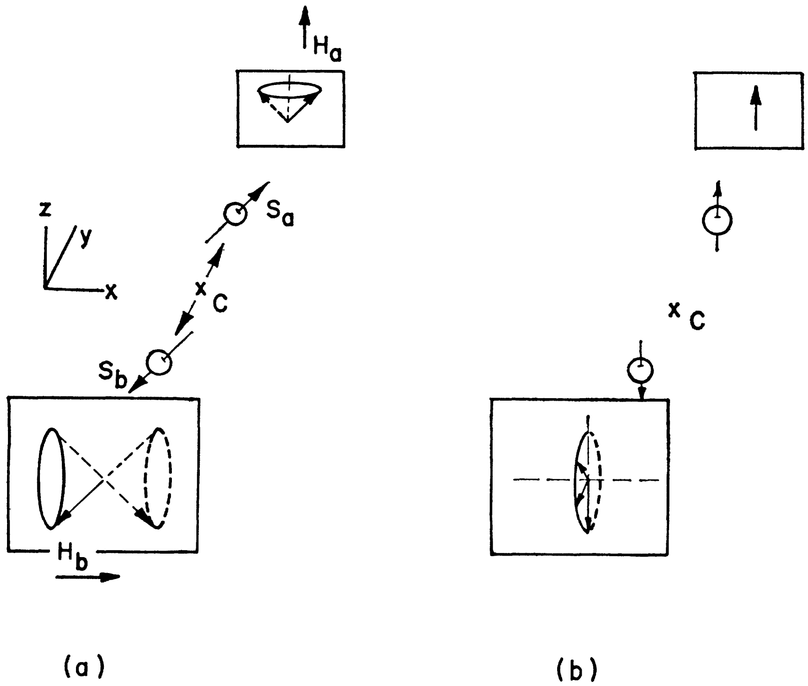


FIG. 1. Two particles with total spin zero flying apart to enter magnetic fields. The rectangles indicate the plane boundaries between free space and homogeneous-field regions.

to the particle but the torque is at right angles to **H**. This means that the classical angular momentum vector precesses in a cone having the field direction as its axis. If **H** is in the $z$ direction, the $z$ component of angular momentum remains constant while the $x$ and $y$ components vary rapidly. In quantum mechanics this rapid variation is associated with the nondiagonal nature of their matrix elements in the representation in which $s_z$ is diagonal. The component of **s** in the $xy$ plane is a vector whose length is definite but whose orientation in the plane is completely indeterminate, so the vector $s$ may be thought of as uniformly distributed over the cone of the classical precession. In quantum mechanics this $xy$ component of **s** is never zero and the length of the spin vector is greater than the maximum value of $s_z$ in keeping with $s^2 \rightarrow s(s+1)$. This uncertainty of direction of **s** is a familiar example[6] of the uncertainty principle. A later measurement by means of a magnetic field in the $x$ direction may alter the situation so that $s_x$ becomes definite but the orientation in a cone about the $x$ axis is indefinite.

## CLASSICAL AND QUANTUM DESCRIPTION OF TWO-SPIN SYSTEM

Consider a compound system $C$ having total angular momentum $S=0$ that breaks apart into two particles, $a$ and $b$, one having spin $\mathbf{s}_a$ and the other $\mathbf{s}_b$. While this remains a thought experiment, they are preferably neutral particles such as two similar $^2S$ atoms from the breakup of an excited molecular $S$ state, since the Stern-Gerlach observation of spin projection is carried out on neutral atoms. In Fig. 1(a), we see $\mathbf{s}_a$ flying away and entering a vertical magnetic field $\mathbf{H}_a$ which exists behind the plane of the rectangle in the distance. Its precession in the field is indicated by a cone. The measurement of $s_{az}$ in this field makes it possible to predict the result of a measurement of $s_{bz}$ in the foreground: If there we apply a field $\mathbf{H}_b$ also in the vertical direction, $\mathbf{s}_b$ precesses in a cone similar to that of $\mathbf{s}_a$ but inverted, with its base downward. If instead $\mathbf{H}_b$ is horizontal in the $x$ direction, $\mathbf{s}_b$ would precess around some cone with a horizontal axis, but we would have no prediction of $s_{bz}$. Classically, the measurement of $s_{az}$ left it uncertain whether $\mathbf{s}_a$ was a vector upward to the right, as indicated by a solid line, or upward to the left, as indicated by a broken line, for example. Correspondingly, $\mathbf{s}_b$ might precess on the cone to the left or to the right in the foreground of Fig. 2(a), or anywhere between.

In classical mechanics, which would apply for very large values of $s$, it is possible with this experiment to eliminate this uncertainty as suggested in Fig. 1(b). Out of many repetitions of the experiment, one might select a case in which $s_{az}$ is equal to $s$, so that the spin vector is in the $z$ direction. One would then know that $s_{bx}$ is equal to 0 and the field in the $x$ direction in the foreground would produce a circular precession in a plane rather than a cone, as indicated in the sketch. This is of course not possible with finite $s$ in quantum mechanics.

The fact that $S=0$ as the two particles fly apart

[6] Compare E. U. Condon, Science **69**, 573 (1929).

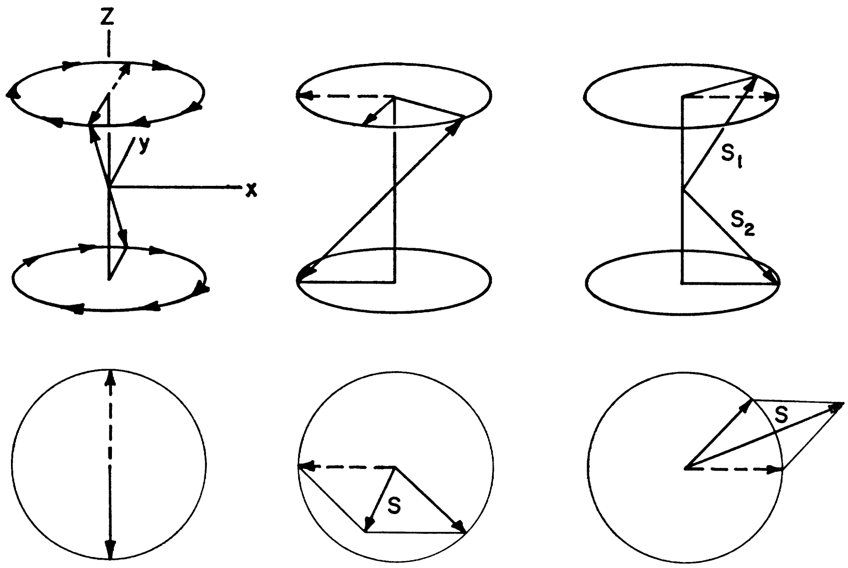


FIG. 2. Change of relative phase of the components of two spin vectors in the $xy$ plane when one precesses twice as fast as the other about the $z$ axis.

gives three equations:

$$s_{ax}+s_{bx}=0 \quad (1a)$$

$$s_{ay}+s_{by}=0 \quad (1b)$$

$$s_{az}+s_{bz}=0. \quad (1c)$$

In quantum mechanics we can select a representation such that the quantities in one of these equations are represented by a diagonal matrix, those in the other two by nondiagonal matrices. This means, for example, that we select spin wave functions that would be appropriate if we should apply **H** in a vertical direction, and make a measurement to find out whether $s_{az}$ is $+\frac{1}{2}$ or $-\frac{1}{2}$. If we do this, Eq. (1c) tells us without further measurement that $s_{bz}$ is either $-\frac{1}{2}$ or $+\frac{1}{2}$, the opposite of $s_{az}$. We could just as well have decided to put **H** in the $x$ direction, measure $s_{ax}$, and infer $s_{bx}$ from Eq. (1a). But we could not do both. If we decide to measure $s_{az}$, for example, we apply to particle $a$ a field in the $z$ direction. Since the magnetic moment $\boldsymbol{\mu}_a=g\mathbf{s}_a$ is not parallel to the $z$ direction, we thereby apply a torque $\boldsymbol{\mu}_a\times\mathbf{H}$ to the combined system and it is no longer true that the total angular momentum remains zero. However, the torque is normal to the $z$ axis (classically giving rise to a precession about that axis), so only Eqs. (1a) and (1b) are rendered invalid; Eq. (1c) remains valid and permits us to transfer the result of the measurement to the second particle. We could have left Eq. (1a) valid instead. Thus the determination of either $s_{bz}$ or $s_{bx}$ by a measurement on $\mathbf{s}_a$ destroys the possibility of determining the other in that way. It invalidates a conservation law which is a part of physical reality concerning the second particle. This is a graphic illustration of the words quoted from Bohr in the Introduction.

EPR claim that the observation on $\mathbf{s}_a$ in no way affects $\mathbf{s}_b$. It does, however, affect the information which can be extracted by means of the operators $s_{bx}$ and $s_{bz}$. The wave function which stores the information about the past interaction of the two particles is a wave function for the system, namely,

$$\psi(a,b)=[u_+(a)u_-(b)-u_-(a)u_+(b)]/\sqrt{2}, \quad (2)$$

where $u_\pm$ corresponds to $s_z=\pm\frac{1}{2}$. This wave function with its rotational properties implies the conservation laws (1). The operators $s_{bx}$ and $s_{bz}$ operating on it give information that is correlated with information given by a similar use of the operators $s_{ax}$ and $s_{az}$. A measurement that $s_{az}=+\frac{1}{2}$, for example, by invalidating Eqs. (1) also invalidates Eq. (2) and selects its first term as the wave function of the system. This is separable and it is possible then, but not before, to say that the wave function of particle $b$ is $u_-(b)$. This tells us how it will react to the second measuring apparatus. We have prepared the system with particle $b$ in a pure state appropriate to a field in the $z$ direction.

The time-independent wave function (2) implies that we have factored out a factor depending on some local time, for example, the time near the measurement of particle $b$. If the measurement on $a$ is made at some remote place and perhaps outside the light cone, the local wave function of particle $b$ becomes $u_-(b)$ only after arrival of a signal giving the result of the first measurement, that is, when the first measurement is in the past.

The system consisting of two particles, each experiencing a measurement, is to be compared with the system of one particle undergoing two successive Stern-Gerlach measurements. Before it enters the first measuring apparatus, the particle has a wave function

$$\psi(b)=c_+u_+(b)+c_-u_-(b) \quad (3)$$

with arbitrary coefficients $c_i$. The first measurement selects the term $u_-(b)$, for example, and the particle is thus prepared in this pure state, just as it was by measurement on the other particle in the two-particle system. The uncertainty principle applies to the particle in this pure state in which $s_{bz}$ is determined and $s_{bx}$ is just as likely to be $+\frac{1}{2}$ as $-\frac{1}{2}$. This is a more stringent uncertainty than that represented by the arbitrariness of the $c_i$ in Eq. (3). The former is based on knowledge, the latter merely reflects ignorance of the past history of the system.

When the particle is made part of a two-particle system, ignorance of the past history can be replaced by partial knowledge of the past history in the form of correlations with the other particle [such as is represented by the minus sign in Eq. (2)]. So far as the quantum-mechanical formalism is concerned, we have seen that this partial knowledge merely provides us with an alternative means for, in effect, determining the $c_i$ from experiment.

## QUESTION OF COMPLETENESS

So much for the formalism itself. EPR raise the question whether the theory embodying this formalism is complete in some physical sense. Their query focuses attention on the meaning of separating a system into two parts. They seem to be in agreement with the quantum-mechanical purists' point of view on the role of measurement when they say[1] "The elements of physical reality cannot be determined by *a priori* philosophical considerations but must be found by an appeal to the results of experiments and measurements." Strictly on those grounds, their criticism seems invalid, as has been shown by Bohr. In particular, the *a priori* philosophical concept that a part of a composite system (particle $b$) has some physical reality as an independent system is tacitly introduced when they state their criterion for reality: "If, without in any way disturbing a system (meaning particle $b$), we can predict with certainty the value of a physical quantity ($s_{bx}$ or $s_{bz}$), then there exists an element of physical reality corresponding to this physical quantity." By the criterion of an appeal to any experiment which has been proposed, the physical quantity associated with particle $b$ of which use is being made is its spin correlation with particle $a$. This correlation is disturbed by the measurement on which the prediction is based, as we have seen.

The lingering doubt which has sometimes been dubbed the "EPR paradox" exists outside the ground rules laid down by EPR. Many physicists are inclined on the basis of "physical intuition" or a desire for simple concepts to accept as evident the philosophical concept that the physical properties of particle $b$ are somehow inherent in particle $b$, even when it is part (particularly a remote part) of a composite system and even if we have not yet thought of an experimental way to give meaning to this concept.

In full awareness that we may be going beyond the realm of the experimentally meaningful as specified by EPR, let us consider the consequence of the *assumption* that the predetermined behavior of a system of two particles may be separated into individually predetermined behavior of the separate particles after they are out of range of interaction with each other. The two particles flying apart with their spins correlated in keeping with Eqs. (1) and $\psi(a,b)$ of Eq. (2) are thereby predestined to have equal and opposite values of $s_x$ or of $s_z$, whichever we choose to measure. According to the assumption of separate predetermination, it must be predetermined how each will behave when it enters either a vertical or a horizontal field. This implies more information than can be made explicit in an equation of the form of (3), since any specification of the $c$'s defines a pure single-particle state suitable for only one orientation of $\mathbf{H}$. In a popular vernacular, we may be inclined to assume that "the particle knows" how it will behave, even though principles of quantum mechanics do not permit *us* to know how it will behave. To encompass this we would need a theory more comprehensive than quantum mechanics, perhaps one containing "hidden variables." EPR are inclined to the belief that "such a theory exists" and indeed the quest cannot be considered closed; although quantum mechanics is so broadly successful and convincing that the quest does not seem hopeful. The dual predetermination cannot be verified experimentally in any straightforward manner we know of, for the measurement of one component of $s_b$ establishes a pure state and erases the predetermination of the other. As already noted, we know how to verify experimentally no more than the correlation with the other particle implied by Eq. (2) (and even this has apparently not been done for separated particles).

In looking for something more than the correlated behavior implied by the two-body wave function, we are seeking something in the two-particle system that from the quantum-mechanical point of view is similar to insisting on giving conceptual meaning to the coordinates of a single particle when we know that its position is governed by a wave. Or, again in a two-particle system, we may make comparison with the familiar and well-accepted situation governed by a wave function antisymmetric in the space coordinates of two particles that have no interaction (or only a very short-range interaction). There is a correlation between the two particles: *If* particle $a$ is at $x$, particle $b$ is unlikely to be near $x$ (or near an integral number of wavelengths from $x$). This is a wave property—a consequence of the symmetry of the two-particle wave function—and the spin correlation that we have discussed, with a view to transcending it, is similarly a wave property.

Since the question raised by EPR is one of groping for concepts, let us further describe some relevant classical concepts whose validity is limited by quantum mechanics. Let a particle having a classical spin vector with a definite orientation be about to enter the fringing field of a Stern-Gerlach apparatus. For any arbitrary choice of field direction, we know in advance the angle of the cone about which the spin vector will start to precess. Let the field be $H_z$ and the spin not parallel to the $z$ axis. The measurement, if made with ordinary care, would not include a completely accurate mapping of the deflecting and fringing fields, though this would in principle be possible. The spin would undergo an indefinite number of Larmor precessions during the passage, and emerge with $s_z$ unaltered and measured, but with $s_x$ bearing no simple relation to its previous value. We knew $s_x$ before, but the measurement has destroyed this knowledge. The fact that we could in principle do better classically is clearly dependent on an exact knowledge of the trajectory in an inhomogeneous field. In quantum theory we cannot know the trajectory exactly and this discussion provides a model for the loss of knowledge of a phase of $s_x$ because of the measurement of $s_z$. In the two-particle system represented by Fig. 1(a), with $\mathbf{H}_a=\mathbf{H}_b$ and appropriate inhomogeneous fields beyond and with complete symmetry on the two sides, any phase relationship that might exist between $s_{ax}$ and $s_{bx}$ as they enter the apparatus is lost when they leave it. (Classically, with the possibility of identical orbits, the phase relationship could in principle survive the measurement.) One expects such an initial phase relationship to exist because of the validity of Eqs. (1) until the particles enter the fields. It is perhaps because such a conceptual model is thus far satisfactory that one is tempted to push the concepts a little further, and to ask whether there can be not only a phase *relationship* between $s_{ax}$ and $s_{bx}$, but also some meaning to the individual phases beyond what is allowed by the quantum description and of a nature to help predetermine the behavior of an individual spin in a field of arbitrary orientation.

## DESCRIPTION OF TRANSVERSE SPIN COMPONENT

While it may not help answer that question, it is instructive to remind ourselves how far we may go with the quantum description of the behavior of $s_x$ if, and only if, we relinquish knowledge of $s_z$. The spin wave functions $v_\pm$ with reference to the $x$ direction are given in terms of $u_\pm$, which refer to the $z$ direction, by the following specification of the $c$'s of Eq. (3):

$$\begin{aligned} v_+&=(u_++u_-)/\sqrt{2}, \quad u_+=(v_++v_-)/\sqrt{2},\\ v_-&=(u_+-u_-)/\sqrt{2}, \quad u_-=(v_+-v_-)/\sqrt{2}. \end{aligned} \tag{4}$$

Thus, for example, when $s_z=\frac{1}{2}$ and thus $u_-=0$, $v_+$, and $v_-$ are equal and $s_x=\frac{1}{2}$ and $-\frac{1}{2}$ are equally likely.

If the particle is in a vertical field $H_z$ we have the wave equation

$$i\hbar(\partial/\partial t)(c_+u_++c_-u_-)=2\mu s_z(c_+u_++c_-u_-), \tag{5}$$

where the $c$'s are functions of time. On equating appropriate coefficients, we have

$$\begin{aligned} \dot{c}_+&=-i\omega c_+, \quad c_+(t)=c_+(t_0)e^{-i\omega(t-t_0)}\\ \dot{c}_-&=i\omega c_-, \quad c_-(t)=c_-(t_0)e^{i\omega(t-t_0)}. \end{aligned} \tag{6}$$

Here $\omega=\mu H/\hbar=eH/2mc$ is the angular frequency of the Larmor precession. In particular, if we start with $s_x=-\frac{1}{2}$, that is, with $\psi(t_0)=v_-$, the subsequent evolution of the wave function is given by

$$\begin{aligned} \psi(t)&=(u_+e^{-i\omega(t-t_0)}-u_-e^{i\omega(t-t_0)})/\sqrt{2}\\ &=[v_-\cos\omega(t-t_0)-v_+i\sin\omega(t-t_0)]. \end{aligned} \tag{7}$$

Here we see that, if we give up knowledge of $s_z$ and let the two states $u_+$ and $u_-$ interfere, we can write a wave function to display the $x$ component of the Larmor precession explicitly, but in terms of oscillating probability amplitudes rather than classical vector component amplitudes.

We have considered two particles flying through free space until they enter magnetic fields. For the sake of the later discussion of theta particles, let us here consider the similar situation of a combined system with $S=0$ in a homogeneous magnetic field disintegrating into two particles (atoms) $a$ and $b$ flying apart in the field until they emerge from it into free space at times $t_a$ and $t_b$, respectively, with $t_b>t_a$. They subsequently enter further fields for the sake of spin measurements.

We treat the same problem in two ways. The first treatment considers the combined two-particle system until $t_a$, and the one-particle system of particle $b$ thereafter. Select a case in which the measurement on $a$ shows that $s_{ax}=\frac{1}{2}$. Since there was no net torque on the two-particle system until $t_a$ and Eqs. (1) and (2) were valid until then, we have $\psi(b,t_a)=v_-$ and from Eq. (7) we see that the wave function for particle $b$ when it emerges from the field contains a term $v_+i\sin\omega(t_b-t_a)$. Thus the probability that both particles are observed to have $s_x=\frac{1}{2}$ is

$$P_{v_+,v_+}(t_a,t_b)=\tfrac{1}{2}\sin^2\omega(t_b-t_a), \tag{8}$$

where the factor $\frac{1}{2}$ comes from the probability of observing $s_{ax}=+\frac{1}{2}$ in the first place. This result is nicely visualizable in terms of the Larmor precession.

In that treatment it does not matter whether the measurement of $s_{ax}$ is made at $t_a$ or later. In the second treatment we assume that both measurements are made after $t_b$, and consider the system as a two-particle system until $t_b$. In the interval from $t_a$ to $t_b$ the wave equation is

$$i\hbar\partial\psi/\partial t=2\mu(s_{az}H_a+s_{bz}H_b)\psi, \tag{9}$$

with $H_a=0$ and with

$$\begin{aligned} &\psi=f_C\psi_C+f_D\psi_D\\ &\psi_C=u_+(a)u_-(b)\equiv u_+u_-, \quad \psi_D=u_-(a)u_+(b)\equiv u_-u_+. \end{aligned} \tag{10}$$

In this case $f_C$ and $f_D$ obey Eq. (6) just as do $c_+$ and $c_-$, and the time-dependent wave function that reduces to Eq. (2) so as to satisfy $S=0$ at $t_a$ is

$$\psi(a,b,t_a,t)=(u_+u_-e^{i\omega(t-t_a)}-u_-u_+e^{-i\omega(t-t_a)})/\sqrt{2}$$
$$=[(v_-v_+-v_+v_-)\cos\omega(t-t_a)$$
$$+(v_-v_--v_+v_+)i\sin\omega(t-t_a)]/\sqrt{2}. \quad (11)$$

The coefficient of $v_+v_+$ again gives the result stated in Eq. (8). Here we see the triplet components $v_-v_-$ and $v_+v_+$, symmetrical in exchange of $a$ and $b$, appearing and disappearing at the rate at which the Larmor precession classically brings the $xy$ components of the spin vectors into parallelism and antiparallelism by the rotation of one of them. The singlet term, $(v_-v_+-v_+v_-)$, which is antisymmetric in $a$ and $b$, is of course the only term at $t=t_a$, in keeping with the initial condition, and remains just out of phase with the triplet.

As a slight generalization which is not needed in the following section but which ties in more closely with the discussion in Bohm's book[3] of the role of inequalities of the fields (due to nonidentical paths in the inhomogeneous fields), we consider the case in which particles $a$ and $b$ are in different homogeneous fields $H_a$ and $H_b$ in the $z$ direction, either *ab initio* or after a flight in free space until $t=0$. Equation (9) is written to include this case, and the coefficients $f$ of Eq. (10) satisfy

$$\dot{f}_C=i(\omega_b-\omega_a)f_C, \quad \dot{f}_D=i(\omega_a-\omega_b)f_D. \quad (12)$$

Thus, starting with a singlet at $t=0$, we obtain in place of Eq. (11)

$$\psi(a,b,t)=(u_+u_-e^{i(\omega_b-\omega_a)t}-u_-u_+e^{-i(\omega_b-\omega_a)t})/\sqrt{2}$$
$$=[(v_-v_+-v_+v_-)\cos(\omega_b-\omega_a)t$$
$$+(v_-v_--v_+v_+)\sin(\omega_b-\omega_a)t]/\sqrt{2}. \quad (13)$$

Thus the triplet term appears and disappears with a frequency given by the difference of the two Larmor frequencies. This is easily understandable classically in terms of one vector catching up with the other in phase, as is portrayed in Fig. 2. This figure pictures the particle with the fast precession having its spin up ($s_z$ positive). This has meaning only classically, but the precession would be the same and nothing would be altered about the phases if the picture were drawn as a mirror image with this spin down instead. If we wish to use such a picture as an aid to intuition in quantum mechanics, we should think of the picture and its image in some way superposed to correspond to terms in $u_+u_-$ and $u_-u_+$ in Eq. (13). Each of these terms contains the differences $(\omega_b-\omega_a)$ emphasized by the picture, yet the superposition of both terms is required to display the alternation in singlet and triplet amplitudes (no matter how far apart the particles may be).

## CORRELATIONS BETWEEN TWO THETA PARTICLES FROM $P+\bar{P}\rightarrow\theta^0+\bar{\theta}^0$

Lee and Yang[5] have pointed out the interesting expectation that a theta and an antitheta resulting from proton-antiproton annihilation are correlated in their time evolution and that the correlation depends on the fact that the charge parity ($C=-1$) of the original pair of fermions (arising in field theory from the anticommutation of their creation operators) is carried over to the pair of bosons by the charge independence of the strong interactions involved in the annihilation. They find the striking result if one of the thetas is observed to be an antitheta ($\bar{\theta}^0$ which we write simply $\bar{\theta}$), the other cannot also be a $\bar{\theta}$ at the same time. This is a further example of the specification of the state of one particle by means of a measurement on a distant particle, and the discussion of spins in the preceding section provides a model that may be helpful in contemplating the behavior of the thetas.

The wave function for the thetas at $t=0$ is

$$\psi_{t=0}=(\theta_1\theta_2-\theta_2\theta_1)/\sqrt{2}=(\theta\bar{\theta}-\bar{\theta}\theta)/\sqrt{2}, \quad (2')$$

antisymmetric under charge conjugation in keeping with $C=-1$. These expressions are analogous to Eq. (2) and to Eq. (11) at $t=0$, the two representations for the theta being related by[7]

$$\theta_1=(\theta+\bar{\theta})/\sqrt{2}, \quad \theta=(\theta_1+\theta_2)/\sqrt{2}$$
$$\theta_2=(\theta-\bar{\theta})/\sqrt{2}, \quad \bar{\theta}=(\theta_1-\theta_2)/\sqrt{2}. \quad (4')$$

The exact analogy with Eq. (4) defines the analogous quantities involved. In particular, the $\theta_1$ and $\theta_2$ are analogous to $u_+$ and $u_-$ in that both sets carry time dependence in $\psi$, the $u$'s an imaginary exponential $e^{\pm i\omega t}$ through the coupling to the magnetic field and the $\theta_1$ and $\theta_2$ a real exponential arising from a coupling causing a decay process. On an appropriate time scale, $\theta_2$ may be considered constant (it decays relatively slowly) while the decay of $\theta_1$ is specified by

$$\theta_1(t)=\theta_1e^{-\lambda t}. \quad (6')$$

(We ignore another time-dependent factor $e^{-imt}$, by which $\theta_1$ differs from $\theta_2$, arising from a small mass difference $m$.) The $\theta$ evolves in such a way as to become partly a $\bar{\theta}$ because of the difference of decay rates of the $\theta_1$ and $\theta_2$ in a manner analogous to the way $v_+$ evolves into $v_-$ because of the difference in the imaginary exponentials associated with the Larmor precession.

The problem is to find the probability $P_{\bar{\theta},\bar{\theta}}(t_a,t_b)$ that both particles will be observed to be antithetas, particle $a$ at time $t_a$ after the $P\bar{P}$ annihilation and $b$ at time $t_b$. We find an analog in this discussion of two atoms with $S=0$ leaving a homogeneous magnetic field at $t_a$ and $t_b$, respectively, and here again there are two ways to treat the problem. Equation (2′) indicates that either particle $a$ or $b$ is an antitheta, but not both, and as long as there is no observation this remains true, with $C=-1$, in the subsequent evolution of the two-particle wave function

$$[(\theta_1e^{-\lambda t})\theta_2-\theta_2(\theta_1e^{-\lambda t})]/\sqrt{2}=(\theta\bar{\theta}-\bar{\theta}\theta)e^{-\lambda t}/\sqrt{2}. \quad (14)$$

[7] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955); A. Pais and O. Piccioni, *ibid.* **100**, 1487 (1955).

The probability that we have $\bar{\theta}(a)$ and $\theta(b)$ at $t_a$ is $\frac{1}{2}e^{-2\lambda t_a}$, the square of the coefficient of the last term. If we now start afresh with the one-body problem with the initial condition $\psi(b,t_a)=\theta(b)$, we have from Eqs. (4′) and (6′)

$$\psi(b,t)=(\theta_1 e^{-\lambda(t-t_a)}+\theta_2)/\sqrt{2}=[\theta(e^{-\lambda(t-t_a)}+1)+\bar{\theta}(e^{-\lambda(t-t_a)}-1)]/2. \quad (7')$$

The last term gives us a factor in the combined probability for observing both particles to be antithetas:

$$P_{\bar{\theta},\bar{\theta}}(t_a,t_b)=\tfrac{1}{8}e^{-2\lambda t_a}(e^{-\lambda(t_b-t_a)}-1)^2. \quad (8')$$

In the second treatment we use only a two-body wave function. In the preceding analog we could turn off an interaction by letting particle $a$ emerge from the field at $t_a$ and leave a two-body wave equation valid later. Here instead one may formally introduce a separate time coordinate for each particle (on a par with the space coordinates) and thus write[5]

$$\begin{aligned}\psi&=[\theta_1(a)\theta_2(b)e^{-\lambda t_a}-\theta_2(a)\theta_1(b)e^{-\lambda t_b}]/\sqrt{2}\\&=[(\theta\theta-\bar{\theta}\bar{\theta})(e^{-\lambda t_a}-e^{-\lambda t_b})\\&\qquad-(\theta\bar{\theta}-\bar{\theta}\theta)(e^{-\lambda t_a}+e^{-\lambda t_b})]/2\sqrt{2}. \quad (11')\end{aligned}$$

The square of the coefficient of $\bar{\theta}\bar{\theta}$ again gives Lee and Yang's result, Eq. (8′).

No matter which way we derive it, the most striking aspect of the result is the simple conclusion that we cannot observe both $a$ and $b$ to be $\bar{\theta}$ at the same time, and this comes directly from the validity of Eq. (14) up to the time of the observation. The mechanism for this is that the decay attenuates both the terms $\theta_1\theta_2$ and $\theta_2\theta_1$ at the same rate, since they each contain a factor $\theta_2$, and leaves their ratio unchanged.[8] This is analogous to the way the Larmor precession in a homogeneous field keeps the spin vectors 180° out of phase [which follows from the constant ratio of the two terms in the first line of Eq. (13) with $\omega_a=\omega_b$].

[8] This attenuation of a real exponential is analogous to a varying phase in that it converts $\theta$ into $\theta+\bar{\theta}$, though not periodically, and there can also be an actual phase variation between the two terms if we take into account the small mass difference $m$ between the $\theta_1$ and $\theta_2$, replacing $\lambda$ by $\lambda-im$.

## RELATION OF THE THETA EXPERIMENT TO THE EPR QUESTION

The experimental verification of this simple conclusion would be very interesting as a further test of the validity of the Pais-Gell-Mann scheme, Eq. (4′), as a basis for the quantum-mechanical description of the behavior of theta particles.[9] However, this would not alone constitute a test of the EPR question in all its profundity. There is only one variable, the charge-conjugation operator, involved in the proposed observation, whereas a choice between observing either one of two noncommuting variables is the heart of the EPR question.

In the two-spin analog, the EPR question is properly put in terms of the two atoms flying apart in free space, so that neither orientation of the measurement is preferred and a free choice of measuring either $s_{ax}$ or $s_{az}$ could provide corresponding information about particle $b$. One concludes that either (a) the assumption of separate predetermination for the two particles separated in space is not tenable, or (b) the theory (quantum mechanics based on conservation of angular momentum) predicting the spin correlation does not apply to the separated particles, for which it has not been tested, or (c) the result of the theory is correct but the theory is incomplete. For those who resist (a), conclusion (b) would provide a way out of the difficulty if one could observe that $s_{az}$ and $s_{bz}$ were not appropriately correlated. Such observation would be immediately applicable to $s_{ax}$ and $s_{bx}$.

The observation that $\theta$ and $\bar{\theta}$ are not appropriately correlated would likewise provide a way out. However, if this correlation should be experimentally established, as expected, a further experiment to establish a similar correlation between $\theta_1$ and $\theta_2$ would be called for before one could experimentally eliminate conclusion (b) as applied to thetas.

## ACKNOWLEDGMENTS

The author wishes to thank Drs. G. R. Ringo, W. Davidon, and K. Tanaka for helpful discussions, and Professor T. D. Lee for his stimulating remarks.

[9] There would be a different correlation if we had $C=1$, rather than $-1$, or if we had some mixture of the two corresponding to $P\bar{P}$ annihilation not predominantly in the $S$ state (footnote 5).