Some Magneto-Fluid Dynamic Effects in a Finitely Conducting Medium

V. N. Zhigulev

N. E. Zhukovskii Central Aerodynamic Institute, Moscow, U.S.S.R.

Magneto-fluid mechanics of infinitely conducting fluids (or what is the same, the theory for $\mathrm{Re}_m=\infty$) has been considerably developed by now; here many interesting cases of motion have been found.

Since, however, all real media possess finite conductivity there occur the following situations. In one case motion can be regarded only on the basis of magnetohydromechanics of finitely conducting fluid; as is the case with the study of the aerodynamics of a gas discharge, some kinds of motion here having a limiting form at $\mathrm{Re}_m\rightarrow\infty$ that is different from the theory $\mathrm{Re}_m=\infty$ (an axially symmetric discharge, for instance, with varying density of the current $\mathbf{j}$ in the direction perpendicular to the axis).

In other cases motion can be regarded according to the theory for $\mathrm{Re}_m=\infty$ in principle, but can be completely realized only on the basis of the magnetohydromechanics of a finitely conducting medium; as is the case with the study of the effect of magnetic "pressing-off," some kinds of shock waves, etc.

Thus, the situation in magneto-fluid dynamics resembles greatly that of in usual hydromechanics, where one should refer to the theory of viscious fluid for treating and choosing a solution obtainable within the limits of the theory of a perfect fluid (in explaining the problem of lift for instance). This is the leading point of the present paper.

The preceding analogy has obvious physical reasons, as in both cases we have to do with diffusion processes (in one case the current flows across the conductor, in the other we have inner friction) having one common point, the mechanism of collision of particles, and therefore they are formally similar.

Now we consider the main features of the interaction between the outside magnetic field and the flow of a conducting gas.

Let a plate be situated perpendicularly to the plane of the chart in Fig. 1 and immersed in a conducting fluid; and let the current be passed perpendicularly to the plane of the chart, inside of the plate; the character of the magnetic lines of force being shown in Fig. 1. If the fluid is set in motion in the direction parallel to the plate, the magnetic field given for the surface of the plate by large $\mathrm{Re}_m(\mathrm{Re}_m\gg1)$ is localized in a layer with the thickness $L/(\mathrm{Re}_m)^{\frac{1}{2}}$ (see Fig. 2). We call this layer a *magnetic boundary layer of the first kind*. (One should not confuse this magnetic boundary layer with a conventional viscous boundary layer in a plasma where electrodynamic forces are considered.)

Let us have next a plate as in Fig. 1, its leading and trailing edges being electrodes, and let us have an emf (electromotive force) inside of the plate and the plate again immersed in a conducting fluid at rest; the character of current lines is given in Fig. 3. If the fluid moves with the velocity $\mathbf{U}$, the electric current in the case of $\mathrm{Re}_m\gg1$ passes in the near vicinity of the plate in a layer having a thickness of the order $L/(\mathrm{Re}_m)^{\frac{1}{2}}$. This we call a *magnetic boundary layer of the second kind* (see Fig. 4).

The equations of the magnetic boundary layers have been given by the author.[1] These equations yield a class of similar solutions, viz.,

$$\psi=x^{\frac{1}{4}(\delta+2)}f(\zeta)$$
$$W=x^{\frac{1}{4}(\delta+2)}g(\zeta),\ \text{for the layer of the first kind}$$
$$H=x^{\frac{1}{2}\delta}h(\zeta),\ \text{for the layer of the second kind}$$
$$p_m=x^{\delta}$$
$$\rho=l(\zeta)$$
$$T=x^{\delta}t(\zeta)$$
$$s=m(\zeta)$$
$$\zeta=x^{\frac{1}{4}(\delta-2)}y,$$

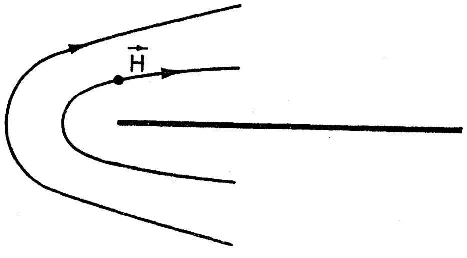

Fig. 1. Magnetic force lines pattern near the edge of the plate with a current. *Note.* Vectors indicated by overhead arrows in the figures are equivalent to boldface letters in the text.

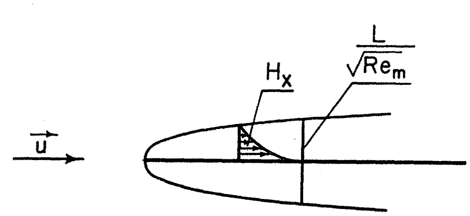

Fig. 2. Magnetic boundary layer of the first kind.

[1] V. N. Zhigulev, Doklady Akad. Nauk S. S. S. R. **124**, 1001 (1959); Soviet Phys. Doklady **4**, 57 (1959).

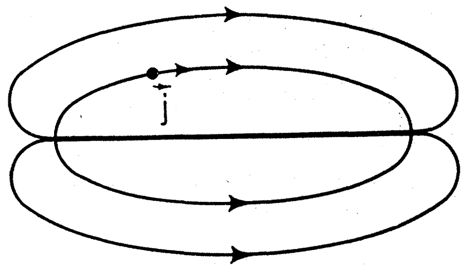

FIG. 3. Pattern of the lines of the current density in a conducting medium.

where ψ is the current function; W the z component of the magnetic field potential; $\mathbf{H}$ the magnetic field vector; ρ, p, T, and s the density, pressure, temperature, and entropy per unit mass of the medium; and p_m denotes $p+H^2/8\pi$. This class is likely to describe quantitatively the main features of the interaction of the outside boundary layer with the flow of a conducting gas.

Note especially that in the case analyzed for the plate ($\delta=0$), the preceding solution is valid for the case of a gas with arbitrary equations of state and with coefficients of magnetic viscosity ν_m, heat conductivity k, and viscosity μ, these being arbitrary functions of temperature and density.

The main property of the magnetic boundary layers of both the first and the second kind is the constancy of the value $p+(H^2/8\pi)$ across the layer ($\partial p_m/\partial y=0$); hence it follows that the pressure of a gas is likely to change greatly across the magnetic boundary layer; for large values $H|_{y=0}$, in particular, there appear zones of negative pressure in the vicinity of the plate. Since real media cannot possess negative pressures, the flow detaches from the plate and forms streamlines about a certain equivalent body including the plate and the magnetic field (see Fig. 5). The field in zone A is subjected to the usual principles of electrodynamics and therefore at the moment of detaching of the flow from the plate the increase of the thickness of the streamline body with the increase of the magnetic field on the plate is rather great.

We call this effect magnetic "pressing-off" of the flow. As the previous analysis shows, the volume separating the flow from the magnetic field is the magnetic boundary layer. If the current in the plate flows perpendicularly to the plane of the chart there appears a layer of the first kind; if the current flows in the direction of the flow there occurs a layer of the second kind. In the first case the Joule losses take place at the expense of the energy of the main flow, thus leading to drag applied to the electric currents inside of the plate; in the second case the Joule losses take place at the expense of the outside emf.

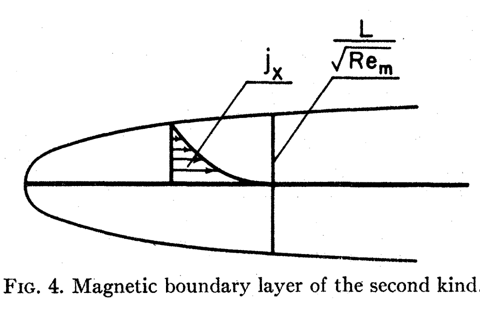

FIG. 4. Magnetic boundary layer of the second kind.

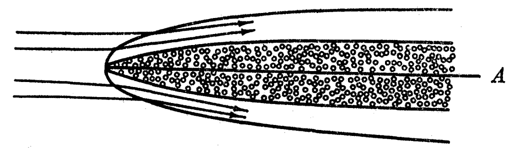

FIG. 5. Picture of magnetic "pressing-off."

The effect of pressing-off exists when we have motions with finite magnetic Reynolds number. In this case the volume separating the magnetic field from the flow reaches infinity and embraces the total flow.

When $\mathrm{Re}_m \gg 1$ the problem of magnetic pressing-off can be divided as follows: (a) the problem of pressing-off itself for $\mathrm{Re}_m=\infty$, (b) the problem of evaluation of a magnetic boundary layer, and (c) evaluation of a set of linear corrections to problems (a) and (b).

The first examples of the solution of the problem (a) were given by Kulikovsky.[2] The general formulation of the problem (a), the method of solution of the plane problem, and also the examples of streamlining one linear current and a flat dipole by a hypersonic flow of a gas are given by the author,[3] and the author together with Romishevsky.[4]

Since the magnetic field tends to press-off the flow of a conducting gas, the gas in its turn tends to oust the magnetic field; that leads to the effect of *screening*. If the screening layer is a magnetic boundary layer of the second kind, the screening leads to the compression of a discharge channel in a gas (Fig. 6). This effect is analyzed by the author.[1,5]

In conclusion we point out one more effect that is an illustration of the thesis given at the beginning of the paper. We analyze an axially symmetric discharge of a diverging kind (i.e., where $\partial H/\partial x\neq 0$) (see Fig. 7). One can readily show that the discharge of this type on the

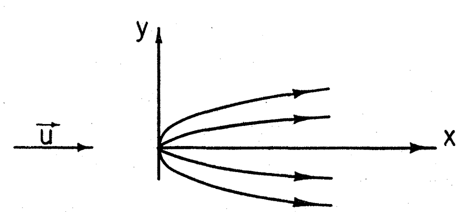

FIG. 6. Electric discharge in a flow of a conducting medium.

[2] A. G. Kulikovskii, Doklady Akad. Nauk S. S. S. R. **117**, (1957).
[3] V. N. Zhigulev, Doklady Akad. Nauk S. S. S. R. **126**, 521 (1959); Soviet Phys. Doklady **4**, 514 (1959).
[4] V. N. Zhigulev and E. A. Romishevskii, Doklady Akad. Nauk S. S. S. R., **127**, 1001 (1959); Soviet Phys. Doklady **4**, 859 (1959).
[5] V. N. Zhigulev, Doklady Akad. Nauk S. S. S. R. **124**, 1226 (1959); Soviet Phys. Doklady **4**, 61 (1959).

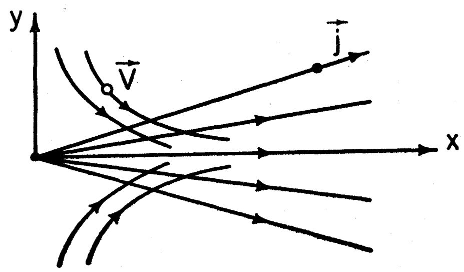

FIG. 7. Ejection of a gas by a discharge.

basis of the equation of conservation of impulse is always accompanied by motion of the medium, even if the latter is at rest at infinity.[6]

The main parameter that is an analogy of the magnetic Reynolds number for this type of discharge is the value $R=I/\nu_m c\rho^{\frac{1}{2}}$ (I denotes the total current in a discharge).

In case of large values R ($R\gg 1$), the discharge has the character of a boundary layer with the thickness L/R, and the y component of the momentum equation is

$$\partial p_m/\partial y = -(H^2/4\pi y),$$

hence

$$p_m = p_{m\infty} + \int_y^\infty \frac{H^2 dy}{4\pi y}$$

taking into consideration that $\partial|H|/\partial x<0$ for the discharge in Fig. 7, we obtain for a case of an inviscid gas

$$du/dt>0,$$

where u is the horizontal component of the velocity vector. The pattern of the flow is as shown in Fig. 7.

Thus, a diverging axially symmetric discharge ejects the gas.

The equations of ejection have been given by the author.[6] In case of incompressible fluid they admit a class of similar solutions of the kind

$$H=x^\gamma h(\zeta);\quad \psi=xf(\zeta);\quad p_m=x^\delta g(\zeta);\quad \zeta=x^\alpha y;$$
$$T=x^q t(\zeta);\quad (\delta=2+4\alpha;\quad \gamma=-\alpha^{-1}-2;\quad q=-2\alpha\gamma).$$

The most interesting solution of this class is the case (Fig. 7) in which the total current in a discharge is independent on x; then $\gamma=\alpha=-1$; $\delta=q=-2$; $\zeta=y/x$.

Note especially that in the last case the kind of solution remains the same for a compressible viscous heat-conducting fluid. Thus for the Navier-Stokes equations, taking into consideration magnetic terms, we have the following axially symmetric (and plane) solution:

$$\psi=xf(\zeta);\quad H=h(\zeta)/y;\quad p_m=g(\zeta)/x^2;\quad T=t(\zeta)/x^2;$$
$$\rho=l(\zeta);\quad s=m(\zeta);\quad \zeta=y/x.$$

[6] V. N. Zhigulev, Doklady Akad. Nauk. S. S. S. R. **130**, 280 (1960).

REVIEWS OF MODERN PHYSICS VOLUME 32, NUMBER 4 OCTOBER, 1960

Reducible Problems in Magneto-Fluid Dynamic Steady Flows*

HAROLD GRAD

Institute of Mathematical Sciences, New York University, New York, New York

1. INTRODUCTION

THE magnitude and degree of complexity of the phenomena encompassed in the subject of nondissipative magneto-fluid dynamics is perhaps best illustrated by the presence of three distinct and strongly anisotropic modes of signal propagation. The linearized problem of the propagation of small disturbances in an unbounded medium is fairly well understood (but is by no means complete).[1] Boundary value problems, even when linearized, are considerably more abstruse. One reason is that, although the various modes of propagation are inherently coupled even in an unbounded domain, they may be decoupled (somewhat artificially, to be sure) by introducing Fourier components. In a boundary-value problem, a higher-order system is, generally speaking, solvable in useful terms only when the boundary conditions as well as the differential equations separate into subsystems.

An alternative technique which is very useful in the early development of a new subject is the discovery of special classes of flows which yield conventional fluid-dynamical or classical second-order mathematical structures. This paper lists and to some extent develops a number of such reducible fluid-magnetic boundary-

* This work was supported by the U. S. Atomic Energy Commission under contract.

[1] For example, see (a) A. Baños, Phys. Rev. **97**, 1435 (1955); (b) A. Baños, Proc. Roy. Soc. (London) **A233**, 350 (1955); (c) J. Bazer and O. Fleischman, Phys. Fluids **2**, 366 (1959); (d) H. Grad, in *The Magnetodynamics of Conducting Fluids*, D. Bershader, Editor (Stanford University Press, Stanford, California, 1959).