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Magnetic Compression of Plasmas

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I. INTRODUCTION

ONE method for the production of high-temperature plasmas makes use of a rapidly rising magnetic field directed along the axis of a low-inductance single-turn coil containing preionized deuterium at an initially low pressure (typically $P_0 \sim 0.01-0.5$ mm Hg). If the initial rise time of the magnetic field is sufficiently short, and if a current sheet is established defining a boundary between the plasma and the field, then the plasma implodes toward the tube axis and a shock wave is formed.[1–4] It has been demonstrated[3,4] that shock velocities corresponding to several hundred electron volts per ion can be achieved.

There are still many problems associated with the initial ionization and shock preheating that remain to be investigated[5]; there are, for example, complicated effects associated with ion-electron relaxation, ionization and excitation by precursor radiation, and shocks moving into magnetic fields. However, it remains that it is possible to reach initial temperatures $T_0 \sim 10^6$ °K by magnetically driven shock waves. It is, therefore, of interest to attempt calculations of the additional heating that might be expected from the adiabatic compression of a shock-preheated plasma by a slowly rising axial magnetic field.

In the following discussion we consider the compression problem in the limit of high electrical conductivity and distinguish between the loss mechanisms for rarefied and dense plasmas in the magnetic-mirror geometry.[6] This model might be expected to describe the gross features of the discharge because the main source of particle loss is out the ends rather than across the field lines, and also because the compressional heating is much larger than the joule heating at high temperatures. We are, therefore, concerned here primarily with the modifications to the usual adiabatic equations due to end losses and finite $\beta = 8\pi nkT/B^2$; $0 < \beta \leq 1$. One of the main results is that one can obtain scaling laws that can be used to design experiments and also to estimate more precisely plasma densities and temperatures from magnetic-compression data. It turns out experimentally[7] that end losses drastically influence the state and volume of the plasma, and this provides us with some additional justification for presenting a simplified theory of compressional heating with losses. The main purpose here is to delineate the region where particle losses lead to an additional heating of the confined plasma and to distinguish between the confinement of low- and high-density plasmas in a magnetic-mirror device.

A discussion of some experimental difficulties and limitations of the theory is contained in the last section. For a high-density plasma the factors which influence the effective mirror ratio and heating due to field mixing[8] (when there is an initial reverse field opposite to the external confining field) are poorly understood.

II. HEATING EQUATIONS

It is assumed that a plasma has been created by some means[9] in an axial confining field generated by a coil array of length l. For simplicity we neglect spatial gradients in the temperature and density so that the energy of the compressed plasma with a uniform trapped magnetic field is

$$E = (f/2)NkT + (B_p^2/8\pi)V, \qquad (1)$$

where $N = nV = n\pi R_p^2 l$ is the total number of ions and electrons in the volume V and B_p is the field trapped in the plasma. For simplicity, the ion and electron temperatures are taken to be equal,[10] so that all relaxation effects are neglected. For generality, we consider a plasma where the particles have f degrees of freedom. For low densities $f \sim 2$ and for high densities $f \sim 3$.

Energy conservation leads to

$$dE/dt = \langle \epsilon_i dN_i/dt \rangle_{av} + \langle \epsilon_e dN_e/dt \rangle_{av} - P_m dV/dt, \qquad (2)$$

[1] S. Colgate, University of California Lawrence Radiation Laboratory Rept. UCRL-4829 (1957).

[2] G. S. Janes and R. M. Patrick, *Conference on Extremely High Temperatures*, H. Fischer and L. C. Mansure, Editors (John Wiley & Sons, Inc., New York, 1958), pp. 3–12.

[3] A. C. Kolb, in *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy*, (United Nations, New York, 1958), Vol. 31, p. 332; Phys. Rev. **107**, 345 (1957).

[4] W. C. Elmore, E. M. Little, and W. E. Quinn, in *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy* (United Nations, New York, 1958), Vol. 32, p. 337.

[5] Further discussion of some of these problems is contained in A. C. Kolb, "Recent progress in shock wave research," in *Proceedings of the Fourth International Conference on Ionization Phenomena in Gases*, R. Nilsson, Editor (North-Holland Publishing Company, Amsterdam, 1959), Vol. IVC, p. 1021.

[6] R. F. Post, in *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy* (United Nations, New York, 1958), Vol. 32, p. 245.

[7] A. C. Kolb, H. R. Griem, and W. R. Faust, "Dense plasmas confined by external fields," in *Proceedings of the Fourth International Conference on Ionization Phenomena in Gases*, R. Nilsson, Editor (North-Holland Publishing Company, Amsterdam, 1959), Vol. IVC, p. 1037.

[8] A. C. Kolb, C. B. Dobbie, and H. R. Griem, Phys. Rev. Letters **3**, 1 (1959).

[9] Shock preheating is only one way for generating a high temperature plasma. The plasma could also be introduced by injection from an ion source or plasma gun, for example.[6]

[10] Cases where $T_e \gg T_i$ or $T_i \gg T_e$ can also be discussed semi-quantitatively by the present theory. See, for example, the discussion in Sec. III.

where dN_i/dt and dN_e/dt are the flux of ions and electrons out the ends; ϵ_i and ϵ_e are the ion and electron energies for particles leaving the system, which for the moment, we take to be arbitrary; the last term represents the work done on the plasma and trapped field by the external magnetic pressure

$$P_m \equiv B^2/8\pi = NkT/V + B_p^2/8\pi. \quad (3)$$

In the infinite conductivity approximation there is no dissipation of the energy in the trapped field so that

$$B_p = B_0(V_0/V), \quad (4)$$

where B_0 and V_0 are the initial trapped field strength and plasma volume.

On invoking the condition that the plasma is quasi-neutral due to the electrostatic forces that would be set up by charge separation, we have

$$dN_i/dt = dN_e/dt = \tfrac{1}{2}(dN/dt). \quad (5)$$

With Eqs. (3)–(5), the energy equation becomes

$$\frac{f}{2}\frac{1}{NT}\frac{dNT}{dt} - \zeta\frac{1}{N}\frac{dN}{dt} + \frac{1}{V}\frac{dV}{dt} = 0, \quad (6)$$

where for convenience the parameter ζ is defined in terms of quantities $\bar{\epsilon}_i$ and $\bar{\epsilon}_e$

$$\left\langle \epsilon_i \frac{dN_i}{dt}\right\rangle_{\text{av}} + \left\langle \epsilon_e \frac{dN_e}{dt}\right\rangle_{\text{av}} \equiv \frac{\bar{\epsilon}_i + \bar{\epsilon}_e}{2}\frac{dN}{dt} \equiv \zeta kT\frac{dN}{dt} \quad (7)$$

and is specified later for various experimental conditions.[11]

The influence of end losses on the temperature is obtained from Eq. (6), thus

$$\frac{T}{T_0} = \left(\frac{N_0}{N}\right)^{1-(2\zeta/f)}\left(\frac{V_0}{V}\right)^{2/f} = \left(\frac{N_0}{N}\right)^{1-(\gamma-1)\zeta}\left(\frac{V_0}{V}\right)^{\gamma-1}, \quad (8)$$

where $f = 2/(\gamma-1)$ is expressed in terms of the usual specific heat ratio and $N/N_0 \leq 1$.

[11] At the Williamsburg Symposium the case $f=3$, $\beta=1$, and $\zeta=\frac{5}{2}$ was discussed. This corresponds to a dense plasma with three degrees of freedom with no internal fields and assumes that the particles leave the ends with their internal energy $\frac{5}{2}kT$. In that case the heating equation (7) reduces to

$$(1/T)(dT/dt) = -\tfrac{4}{3}(1/R_p)(dR_p/dt)$$

and leads to the result that since particle losses cause the plasma to overcompress relative to an adiabatic compression without losses; then

$$(1/R_p)(dR_p/dt) > [(1/R_p)(dR_p/dt)]_{\text{adiabatic (no losses)}},$$

and the final temperature is higher *due* to the losses. It was questioned by members of the Symposium whether this conclusion is valid because of the particular assumptions. In collaboration with W. B. Thompson, the analysis was then extended to include the case when $\zeta \neq \frac{5}{2}$ and $\beta=1$, and a similar conclusion was reached for $0<\zeta<\frac{5}{2}$, namely, particle losses provide additional heating for the plasma that remains. The present theory for arbitrary β modified the quantitative conclusions and yields the range of parameters for which additional heating occurs.

It follows from Eqs. (3), (4), and (8) that the ratio of the gas pressure to the external magnetic pressure is *not* a constant in time if there are losses:

$$\begin{aligned}\beta &= 8\pi NkT/VB^2 = (1+B_p^2V/8\pi NkT)^{-1}\\ &= \beta_0/[\beta_0 + (1-\beta_0)(N_0/N)^{2\zeta/f}(V/V_0)^{(2/f)-1}]\\ &= \beta_0/[\beta_0 + (1-\beta_0)(N_0/N)^{1+\zeta-(f/2)}(T/T_0)^{(f/2)-1}].\end{aligned} \quad (9)$$

For $f=2$ the quantity β is independent of T and V and remains constant only if there are negligible losses during the compression. However, if $f=3$ as in a dense plasma, then β decreases even when there are no losses. This is to be expected since an axial magnetic field behaves like a two-dimensional fluid when radially compressed. If a plasma with three degrees of freedom is confined by the field, then relatively more work is done on the trapped field than on the plasma during the compression.

In order to obtain the dependence of the density and temperature on the external magnetic field we differentiate the pressure balance Eq. (3), and use Eqs. (4) and (6);

$$\frac{1}{P_m}\frac{dP_m}{dt} = \left[\frac{2\beta+(2-\beta)f}{2}\right]\frac{1}{T}\frac{dT}{dt} + \left[\frac{2\beta+f(2-\beta)-2(2-\beta)\zeta}{2}\right]\frac{1}{N}\frac{dN}{dt}. \quad (10)$$

For arbitrary initial $\beta(t=0)\equiv\beta_0$, one must specify the loss mechanism; i.e., the functional dependence of dN/dt on P_m, β, T must be known, and a first-order differential equation with time-dependent coefficients must be solved. The general solution will be discussed elsewhere. In the following discussion we take β equal to unity and only indicate the qualitative influence of $\beta<1$. For $\beta=1$, the general heating equation reduces to

$$\frac{1}{P_m}\frac{dP_m}{dt} = \left(1+\frac{f}{2}\right)\frac{1}{T}\frac{dT}{dt} + \left(1+\frac{f}{2}-\zeta\right)\frac{1}{N}\frac{dN}{dt}. \quad (11)$$

If there were no particle losses ($dN/dt=0$), the solution of Eq. (11) would be

$$\begin{aligned}T &= gP_m^{\alpha};\\ \alpha &\equiv 2/(2+f),\end{aligned} \quad (12)$$

where g is a constant. If particle losses cannot be neglected, then one obtains a solution of the same form as Eq. (12) by allowing g to be time dependent. Then g is determined by

$$(1/g)(dg/dt) = -\delta(1/N)(dN/dt),$$

where

$$\delta \equiv 1-[2\zeta/(2+f)], \quad (13)$$

and the temperature is given by

$$T/T_0 = (g/g_0)(P_m/P_0)^{\alpha} = (N_0/N)^{\delta}(B/B_0)^{2\alpha}. \quad (14)$$

For a high-β two-dimensional plasma confined in a magnetic-mirror geometry, we find with $\beta=1$, $f=2$, $\zeta=\frac{1}{2}$ (see discussion in the next section),

$$T/T_0=(N_0/N)^{\frac{3}{4}}B/B_0; \tag{15}$$

and the end losses markedly influence the temperature.

A high-β plasma with $f=2$ could perhaps be produced by generating a very high velocity (collisionless) radial shock in a preionized and preheated low-density plasma. If the energy of the ions due to the radial implosion is sufficiently high (say, 100 ev at $N_e\cong 10^{15}/\text{cm}^3$) and if the initial (before the shock implosion) conductivity is large enough due to some form of preionization and preheating, then a current sheet will be formed and the ions will move in a $\beta\sim 1$ region with infrequent collisions. The confinement of the radially moving ions would then be determined by the ion-ion scattering time and Eq. (15) could have some utility. Under such circumstances the electrons might well have much less energy than the ions. In that case the temperature T in Eq. (15) is the ion temperature and N is the ion density. The cooling due to energy transfer between the ions and electrons would require a more elaborate theory and limits the validity of the present calculation to times short compared to the characteristic time for such an energy exchange.

In the high-density collision-dominated regime where $f\sim 3$ and $\beta=1$, we have

$$T/T_0=(N_0/N)^{1-(2\zeta/5)}(B/B_0)^{4/5}. \tag{16}$$

This leads to the interesting result that for $\zeta<\frac{5}{2}$ the particle losses result in additional heating since $N_0/N>1$. The factor $(B/B_0)^{4/5}$ is the usual adiabatic result for $\beta=1$. If the mean free path of the plasma ions is much less than the characteristic size of the "holes" at the ends, then the ordinary fluid equations for steady compressible adiabatic flow in a channel give $\frac{1}{2}(\bar{\epsilon}_i+\bar{\epsilon}_e)=\frac{5}{2}kT$ with $P/\rho^\gamma=\text{constant}$. Then $\zeta=\frac{5}{2}$, $\delta=0$ [see Eq. (13)], and there is no additional heating due to losses.

However, if the mean free path is greater than the size of the mirror aperture, then the particles effuse into a low pressure region and do no work in passing through the mirror. In that case, the mean particle energy can be calculated from kinetic theory for a Maxwell-Boltzmann velocity distribution. This gives $\bar{\epsilon}_i=2kT$ if the effective mirror aperture is independent of the particle velocities. The electrons, on the other hand, are constrained by electrostatic forces and are lost at a rate determined by the mean ion velocity. We take, therefore, $\bar{\epsilon}_e\sim\frac{3}{2}kT$ in *estimating* $\zeta=\frac{1}{2}(\bar{\epsilon}_i+\bar{\epsilon}_e)/kT\approx 7/4$. A more complete theory for ζ would include the possible influence of the electrostatic fields from ambipolar diffusion on the particle energies.

For a free expansion, the temperature is then given by

$$T/T_0=(N_0/N)^{3/10}(B/B_0)^{4/5};\quad \beta=1. \tag{17}$$

If the plasma pressure is low compared with the external magnetic pressure ($\beta\ll 1$), then the particle losses can lead to a cooling relative to the pure adiabatic case. The sign of the ratio of the coefficient of $(1/N)dN/dt$ and $(1/T)dT/dt$ in Eq. (10) determines whether there is additional heating. The critical β at which the particle losses do not influence the temperature is obtained by putting this ratio equal to zero. For $f=2$

$$\beta_c=2(\zeta-1)\zeta^{-1},$$

and for $f=3$

$$\beta_c=(4\zeta-6)/(2\zeta-1). \tag{18}$$

For $f=2$ and $\zeta\leq 1$, the critical β_c is always negative and there is additional heating for all β between zero and unity. However, for $f=3$ and $\zeta=7/4$, the end losses result in heating only if $\beta>\beta_c=0.4$.

If the end losses are negligible and β is small ($\beta\ll 1$) for a plasma with $f=3$, the heating equation (10) is approximately

$$(1/P_m)(dP_m/dt)\simeq 3(1/T)(dT/dt)$$

or compared to $T/T_0\sim(B/B_0)^{0.8}$ for $\beta=1$,

$$T/T_0\sim(P_m/P_0)^{\frac{1}{3}}=(B/B_0)^{0.67} \tag{19}$$

and the heating due to compression is less efficient than for a high-β. It can also be shown that much higher compression ratios are required to heat a dense plasma if particle losses are important in a low $\beta(\beta<\beta_c)$ experiment, because then the losses result in a cooling of the remaining plasma. The general conclusion is that for $T_0\sim 10^6$ °K (from shock heating, for example) it is difficult to reach the thermonuclear range of temperatures in a dense plasma without an enormous compression ratio unless the β is made high or some other heating mechanism is employed, e.g., field mixing. This demonstrates the importance of carefully controlled preionization and preheating so that diamagnetic surface currents will flow during the initial stages of the discharge to exclude the external field, due to a high conductivity.

III. RADIAL COMPRESSION OF HIGH-TEMPERATURE LOW-DENSITY PLASMAS

In a low-density plasma where the mean free path of the ions is much greater than the distance between the magnetic mirrors, the loss rate is determined by the scattering time τ_s:

$$(1/N)(dN/dt)=-(bn/T^{\frac{3}{2}})\sim-(1/\tau_s); \tag{20a}$$

or with Eqs. (3), (10), and (12), $\beta\leq 1$, and $f=2$:

$$\begin{aligned} &g^{\frac{3}{2}}(dg/dt)=(\delta\beta b/P_m^{\frac{1}{4}});\\ &\delta\equiv[\beta+(2-\beta)(\tfrac{1}{2}f-\zeta)]/2, \end{aligned} \tag{20b}$$

where b is a parameter of order unity, which depends weakly on the mirror ratio and on the precise form of the ion velocity distribution function.[6,12]

Equations (20b) and (13) with $f=2$ for a radial compression yield with $\beta\approx\beta_0$ as a first approximation

$$\frac{T}{T_0}\approx\frac{B}{B_0}\left\{1+\frac{5}{4}\frac{1}{\tau_s(0)}\times[2-(2-\beta_0)\zeta]\int_{t_0}^{t}\left[\frac{P_m(0)}{P_m(t')}\right]^{\frac{1}{2}}dt'\right\}^{2/5},\quad(21)$$

where $\tau_s(0)$ is the characteristic confinement time when the temperature is T_0 at the beginning of the slow compression, $t=t_0$. It has been shown[12] for a Maxwell distribution, that the average energy of the particles which are scattered into the loss cone and out the mirrors is about $\frac{1}{2}kT$. This result follows because the low-energy particles have the largest cross section for Coulomb scattering into the loss cone and is equivalent to choosing $\zeta\sim\frac{1}{2}$, so that Eq. (21) becomes

$$\frac{T}{T_0}\approx\frac{B}{B_0}\left\{1+\frac{5}{8}\frac{(2+\beta_0)}{\tau_s(0)}\int_{t_0}^{t}\left[\frac{B(0)}{B(t')}\right]^{\frac{1}{2}}dt'\right\}^{2/5}.\quad(22)$$

To estimate the additional heating due to such end losses we consider a plasma confined by a constant external field $B(t)=B(0)$; then

$$T\approx T_0\left\{1+[\tfrac{5}{8}(2+\beta_0)]\frac{t-t_0}{\tau_s(0)}\right\}^{2/5}.\quad(23)$$

With $\beta_0\sim1$, in one scattering time a temperature increase of $\sim53\%$ is indicated. For $\beta_0\ll1$, we have

$$T\approx T_0\left[1+\frac{5}{4}\frac{(t-t_0)}{\tau_s(0)}\right]^{2/5},\quad(24)$$

independent of β_0, and the temperature rises by 38% in one scattering time.

The time dependence of the density can also be obtained from $nkT/B^2\approx\beta_0/8\pi$ in this approximation; i.e., $\beta\sim\beta_0$

$$\frac{n}{n_0}\approx\left(\frac{B}{B_0}\right)\left\{1+\frac{5}{8}\frac{(2+\beta_0)}{\tau_s(0)}\int_{t_0}^{t}\left[\frac{B(0)}{B(t')}\right]^{\frac{1}{2}}dt'\right\}^{-2/5}.\quad(25)$$

If one estimates the loss rate by neglecting the effect of the particle loss on the temperature and plasma volume, the density is given by[6]

$$dn/dt=-(n/\tau_s);\quad\text{with}\quad\tau_s\sim T_0^{\frac{3}{2}}/n=\tau_s(0)n_0/n,$$

whose solution is

$$n=n_0\Big/\left[1+\frac{t-t_0}{\tau_s(0)}\right].\quad(26)$$

This should be compared to

$$n\approx n_0\Big/\left[1+\frac{5}{4}\frac{t-t_0}{\tau_s(0)}\right]^{2/5}\quad(27)$$

obtained from Eq. (25) with $B(t)=B(0)$ and $\beta_0\ll1$.

According to Eq. (26), we find that in one scattering time, 50% of the particles are lost instead of 28% predicted from Eq. (27).

The general conclusion is that the actual confinement time of a low-density plasma in a magnetic mirror device is longer than is predicted from a simple estimate of the scattering time at the early stages of the confinement. This can be clearly seen since the scattering time also increases as particles are lost. Since $\tau_s\sim T^{\frac{3}{2}}/n$, one finds

$$\tau_s(t)\approx\tau_s(0)\left[\frac{B(t)}{B_0}\right]^{\frac{1}{2}}\times\left\{1+\frac{5}{4}\frac{[2-(2-\beta_0)\zeta]}{\tau_s(0)}\int_{t_0}^{t}\left[\frac{B(0)}{B(t')}\right]^{\frac{1}{2}}dt'\right\}.\quad(28)$$

Therefore, the scattering time increases with increasing field strength and time and accounts for the more effective confinement of the remaining energetic particles. These conclusions are in qualitative accord with experimental observations[6] in which it is found that high-energy components of the plasma in a low-density mirror device are apparently confined for times an order of magnitude longer than the mean scattering time at the peak of the compression.

The particle flux out the ends is given by Eq. (28) with (14), (20a), and (21):

$$\frac{dN}{dt}\approx\frac{-N}{\tau_s(t)}=-\frac{N_0}{\tau_s(0)}\left(\frac{B_0}{B}\right)^{\frac{1}{2}}\times\left\{1+\frac{5[2-\zeta(2-\beta_0)]}{4\tau_s(0)}\int_{t_0}^{t}\left(\frac{B_0}{B(t')}\right)^{\frac{1}{2}}dt'\right\}^{-[1+(2/5\delta)]}.\quad(29)$$

This equation implies that the loss rate decreases with time if the plasma loses particles by the preferential scattering of low energy ions into the loss cone. This is the qualitative result expected and observed[13] after the time of maximum compression.

However, at the beginning of the compression cycle the loss rate dN/dt also decreases with increasing field strength according to Eq. (29). It has been found[6] that just the opposite occurs; i.e., the loss rate increases

[12] D. Judd, W. McDonald, and M. Rosenbluth, U. S. Atomic Energy Commission Rept. WASH-289, p. 158, Conference on Controlled Thermonuclear Reactions, Berkeley, California, February, 1955.

[13] See, for example, Fig. 5 of reference 6.

during the compression. This suggests that during the early stages of the compression when B is small and the orbits are large, that nonadiabatic losses are also important[14] or that with low initial temperatures the plasma losses are collision dominated. (See Sec. IV where it is shown that the loss rate increases with increasing temperature when collisions keep the loss cone filled. In the foregoing analysis the loss rate decreases with increasing temperature.)

In these calculations we took the electron and ion temperatures to be equal. However, the equations are not directly applicable to these experiments[15] since the electron temperature is much larger than the ion temperature. Here $T=T_e$ and $N=N_e$ in the preceding equations. If relaxation effects are neglected, these equations would also apply to a two-dimensional ion plasma where the ion-ion scattering time determines the loss rate and the electron temperature is negligible (then $N=N_i$ and $T=T_i$).

However, if the electron and ion temperatures are comparable, the electron-electron scattering time is a factor 60 less than the ion-ion scattering time, and it is more reasonable to take $f_{\text{electron}}=3$ and $f_{\text{ion}}=2$; so that $\bar{f}=(f_e+f_i)/2=\frac{5}{2}$. In addition, due to ambipolar (electrostatic) effects when $T_e=T_i$, the electron plasma is clamped to the ion component so that the average energy of the electrons in leaving the system will be of the order $\frac{3}{2}kT$ while the preferential scattering of the ions into the loss cone gives $\frac{1}{2}kT$, as before. Therefore, $\frac{1}{2}(\bar{\epsilon}_e+\bar{\epsilon}_i)=kT$ and the quantity ζ in our equations is ~ 1. We, therefore, conclude that depending on whether the ratio T_e/T_i is much less than, equal to, or much greater than unity, there will be marked changes in the form of the equation for the loss rate for particular experiments. Caution should be exercised in predicting the behavior of one of these regions on the basis of observations taken in another region.

In any practical case where nonadiabatic losses are negligible, β is time dependent, and the magnetic field increases with time in a prescribed way, then the heating and loss rate can be found by integrating Eq. (20b) numerically. The actual confinement time will be longer than we have estimated by taking $\beta=\beta_0$ since $\beta\leq\beta_0$ due to end losses. This is evident by inspecting Eqs. (27) or (20b) which show the essential functional dependence of the particle density and flux out at the ends on β.

IV. MAGNETIC COMPRESSION OF A DENSE PLASMA

In a high-density plasma where the mean free path is less than the distance between the magnetic mirrors, the particles are not confined according to the usual reflection condition[6] in the mirror region. As in the cusp geometry,[16] the loss cone in the velocity space is kept filled by collisions and the particles stream out the geometrical aperture at the open ends of the system. The rate at which particles effuse out at the ends is given by

$$-dN/dt=2A_m n\bar{v}_{iz}=\tfrac{1}{2}N(A_m/A_p)(\bar{v}_i/l)\equiv N/\tau_e(t), \quad (30)$$

where $A_p l$ is the volume of the confined plasma, $\bar{v}_i$ is the average velocity of the ions, $2A_m$ is the effective area of the two open ends, and the quantity $\tau_e(t)$ is called the characteristic escape time in a collision-dominated plasma. This equation is derived from kinetic theory and assumes that the mean free path is much greater than the characteristic size of the "hole." In this case no work is done by the free-streaming particles as they pass through the plasma constriction caused by the mirror field. Consistent with this approximation, a similar kinetic calculation gives $\bar{\epsilon}_i=2kT$ for the mean ion energy for the escaping particles.

Here we took the mirror ratio A_p/A_m to be constant and independent of $\bar{v}_i$ and B. Actually, the effective area A_m has been shown[17] to be intimately connected with the thickness of the boundary between plasma and vacuum field. In addition, a reverse field in the plasma, where the flux lines close on themselves, could reduce the loss rate.

To obtain an *estimate* of the dependence of the temperature and other relevant quantities on the field strength and loss rate we take the ratio A_m/A_p to be of the order of the geometrical mirror ratio, for simplicity. With Eqs. (12), (13), and (30) [see also the discussion preceding Eq. (17)], we obtain

$$\frac{T}{T_0}=\left(\frac{B}{B_0}\right)^{2\alpha}\left\{1-\frac{\delta}{2\tau_e(0)}\int_{t_0}^{t}\left[\frac{B(t')}{B_0}\right]^{\alpha}dt'\right\}^{-2}$$

$$\equiv(B/B_0)^{2\alpha}[1/G^2(t)], \quad (31a)$$

$$n/n_0=(B/B_0)^{2(1-\alpha)}G^2(t) \quad (31b)$$

$$R_p/R_p(0)=(B_0/B)^{1-\alpha}G^{(1-\delta)/\delta} \quad (31c)$$

$$N/N_0=G^{2/\delta};\quad \alpha=\tfrac{2}{5}\quad\text{and}\quad \delta=\tfrac{3}{10}\quad \text{for}\quad \beta=1,\quad \zeta=7/4,\quad f=3. \quad (31d)$$

From these equations it follows that the particle losses heat or cool the remaining plasma depending on whether δ is positive or negative. The quantity $G(t)$, defined by Eq. (31a), represents the correction factor to the usual adiabatic relations that results from end losses.

In magnetic compression experiments with high plasma densities ($n>10^{16}$ cm^{-3}), high fields ($B\gtrsim 10^5$ gauss) would be required to maintain temperatures in

[14] See Fig. 2(b) of reference 6.

[15] For these experiments[6] the ion-ion and electron-electron scattering times are comparable since T_e/T_i is reported to be between 10 and 20.

[16] J. Berkowitz, K. O. Friedrichs, H. Goertzel, H. Grad, J. Killeen, and H. Rubin, in *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy* (United Nations, New York, 1958), Vol. 31, p. 171; H. Grad, New York University Rept. No. NYO-7969 (1957).

[17] J. Berkowitz, New York University Rept. No. NYO-2536 (1959).

the kilovolt range. Capacitor discharges are used to obtain the required high currents and we may write approximately

$$B=B_m \sin[(\pi/2)(t/\tau_{1/4})], \qquad (32)$$

where $\tau_{1/4}=(\pi/2)(LC)^{\frac{1}{2}}=\pi/2\omega$ is the quarter period of the discharge, C is the capacitance, L the circuit inductance, and B_m is the maximum external field strength. The variation of the inductance due to the motion of the plasma is neglected, because at high compression ratios this is not important. With Eq. (32), $G(t)$ becomes

$$G(t)=1-(\delta/\pi)[\tau_{1/4}/\tau_e(0)][S(\omega t)-S(\omega t_0)], \qquad (33)$$

where

$$S(\omega t)\equiv\int_0^{\omega t}(\sin z)^\alpha dz, \qquad (34)$$

and is tabulated in Table I for $f=3$, $\beta=1$, $\alpha=\frac{2}{5}$.

For $t_0\ll\tau_{1/4}$, $B_0\approx B_m\pi t_0/2\tau_{1/4}$ so that

$$S(\omega t_0)\approx 5/7[(\pi/2)(t_0/\tau_{1/4})]^{7/5}; \qquad (35)$$

and at the time of maximum field strength ($t=\tau_{1/4}$)

$$S(\omega\tau_{1/4})=S\left(\frac{\pi}{2}\right)=\frac{\pi^{\frac{1}{2}}}{\alpha+1}\frac{\Gamma\frac{1}{2}(\alpha+1)}{\Gamma(\frac{1}{2}\alpha+1)}=1.253. \qquad (36)$$

The temperature ratio at the time of the maximum magnetic field is

$$T(\tau_{1/4})/T_0=(2\tau_{1/4}/\pi t_0)^{4/5}/G^2(\tau_{1/4}). \qquad (37)$$

If at some early stage of the discharge (at a time t_0) one is able to estimate the initial temperature (from shock-velocity measurements, for example) then $\tau_e(0)$ can be calculated and Eq. (36) can be used to estimate the influence of end losses on the temperature, density, and plasma radius at the time of maximum field strength. In the present experiments at NRL, the coils are designed so that $G(\tau_{1/4})$ is large enough so that $\gtrsim 50\%$ of the plasma is contained at the maximum field strength.

If in fact the effective mirror ratio is much greater than the geometrical mirror ratio, then the confinement time is longer than predicted by the foregoing equations where A_m is taken to be a constant. This possibility for enhanced confinement is discussed further in the last section. The above estimates are, therefore, regarded as conservative in predicting the confinement time for dense plasmas in experiments where instabilities are not the dominating factor in the loss of particles.

TABLE I. $S(\omega t)$ for $\alpha=\frac{2}{5}$.

ωt	$S(\omega t)$	ωt	$S(\omega t)$	ωt	$S(\omega t)$	ωt	$S(\omega t)$
0.001	0.0000	0.8	0.512	1.8	1.48	2.8	2.35
0.005	0.0004	0.9	0.602	1.9	1.58	2.9	2.41
0.02	0.003	1.0	0.694	2.0	1.68	3.0	2.46
0.1	0.028	1.1	0.788	2.1	1.77	3.1	2.49
0.2	0.075	1.2	0.884	2.2	1.86	π	2.5058
0.3	0.131	1.3	0.982	2.3	1.95		
0.4	0.196	1.4	1.08	2.4	2.04		
0.5	0.268	1.5	1.18	2.5	2.12		
0.6	0.345	1.6	1.28	2.6	2.20		
0.7	0.427	1.7	1.38	2.7	2.28		

TABLE II. τ_e/τ_s for $A_p/A_m=3$, $T=10^7$ °K.

n[cm^{-3}]	l[cm]=10	50	100
10^{14}	0.003	0.015	0.03
10^{15}	0.03	0.15	0.3
10^{16}	0.3	1.5	3
5×10^{16}	1.5	6.5	15
10^{17}	3	65	150

V. CRITERIA FOR HIGH- AND LOW-DENSITY REGIONS

An essential point in comparing various experiments utilizing the basic magnetic mirror geometry is to distinguish between the predominant mechanisms which determine the confinement time.[18] The parameter of interest here is the ratio of the characteristic times τ_e and τ_s.

$$\frac{\tau_e}{\tau_s}\sim\frac{2A_p}{A_m}\frac{l}{\bar{v}_i}\Big/\frac{16T^{\frac{3}{2}}}{n\ln\Lambda}\sim10^{-4}\frac{nl}{T^2}\frac{A_p}{A_m} \qquad (38)$$

with $\bar{v}_i=10^4T^{\frac{1}{2}}$ for deuterium plasma and $\ln\Lambda\sim10$.

Table II gives the ratio τ_e/τ_s for $T=10^7$ °K and a typical geometric mirror ratio $A_p/A_m=3$ (see discussion in the final section).

From Table II we see that for $T\lesssim10^7$ °K with densities above 10^{16} ions/cm^3 and for coils of moderate length, the confinement time is determined by τ_e; τ_s is not the important factor. For temperatures of a few hundred electron volts or less the plasma is collision-dominated, i.e., $f=3$ and $\tau_e/\tau_s\gg1$, down to densities of 10^{14} or 10^{15} cm^{-3}, or lower if the effective mirror ratio is actually larger than the geometric mirror ratio.

On the other hand, for temperatures greater than 10^7 °K and for densities less than 10^{15} cm^{-3}, the scattering time is always the determining factor in the confinement (neglecting instabilities, charge exchange to neutral gas outside the plasma, etc.).

The parameters in the NRL magnetic compression experiments are chosen so that $\tau_e/\tau_s>1$. In this regime it is necessary to scale up the coil length in order to increase both the temperature and confinement time. If $\tau_e/\tau_s\ll1$, then the confinement time depends only on the density and temperature. So for a given density it is necessary to increase the temperature in order to increase the confinement time. In the collision-dominated case, the confinement time can be increased at constant temperature, i.e., in a dense plasma one can obtain confinement times which are long compared to the relaxation times by scaling up the length of the

[18] This discussion is included here only for emphasis and completeness. The delineation of different regions for particle confinement was discussed earlier by H. Grad.[16]

coil. This is the direction of our present experiments and represents the basic difference between this approach and experiments[6] at low density.

The scaling law for the confinement of a dense plasma in a mirror device has been verified in compression experiments[7] where the plasma radius and density are measured as a function of time for coils of varying length and geometric mirror ratio. lA_p/A_m was varied from 7–75 and will be extended to 300–400 in a larger apparatus with currents of $\sim 20\times 10^6$ amp. It is also observed that the time duration of the neutron emission from a deuterium plasma scales properly with the confinement time. This does not prove that we deal with a Maxwellian ion plasma, but only that the confinement of the energetic deuterons which cause D-D reactions depends on the confinement time of the bulk of the plasma.

The confinement time during a particular experiment is not always determined by the same mechanism. Consider, for example, the compression of a high-density plasma at a moderate initial temperature. In the early stages of the compression the escape time τ_e governs the confinement, but as the density falls due to losses, and the temperature increases, the scattering time τ_s may become the characteristic confinement time.

VI. REACTION RATES IN DENSE PLASMAS

At sufficiently high temperatures in a deuterium plasma there will be D-D reactions, and the reaction rate is given by

$$P_{\mathrm{DD}}=\tfrac{1}{2}n^2\langle\sigma v\rangle_{\mathrm{DD}}\pi R_p{}^2 l \text{ reactions/sec.} \qquad (39)$$

Substituting Eqs. (31a) and (31b) into this expression gives an estimate of the effect of end losses on the reaction rate[19]

$$P_{\mathrm{DD}}=\tfrac{1}{2}[\pi R_p{}^2(0)l]n_0{}^2(B/B_0)^{6/5}\langle\sigma\bar{v}\rangle_{\mathrm{DD}}G^7(t). \qquad (40)$$

However, in the kilovolt temperature range the quantity

$$\langle\sigma\bar{v}\rangle_{\mathrm{DD}}\approx dT(t)^m, \qquad (41)$$

where $m\sim 7$ and $d\sim 2\times 10^{-22}$ cm^3/sec (kev)^7. Therefore, near the threshold for producing an observable thermonuclear reaction, the reaction rate depends inversely on $G(t)$. Using Eqs. (31a) and (41), Eq. (40) becomes

$$P_{\mathrm{DD}}\cong\tfrac{1}{2}[\pi R_p{}^2(0)l]n_0{}^2 d(B/B_0)^{34/5}[1/G(t)^7]. \qquad (42)$$

The factor in this expression exclusive of $G(t)$ is the reaction rate as a function of the field strength when there are no end losses. The sensitive dependence on the field strength is evident. However, since the factor $G(t)$ is always less than unity and because of the strong dependence of the cross section on the particle energy, the net effect of additional heating due to end losses is to increase the reaction rate for temperatures in the kilovolt range. This is the case in spite of the reduced plasma densities and volume resulting from the losses. For example, when $G(t)$ is 0.7 and 0.5 the reaction rate can be one and two orders of magnitude larger, respectively, than for a pure adiabatic compression at $\beta=1$.

At temperatures above a few kev the cross section depends less sensitively on the temperature, and the reaction rate is proportional to some small power of $G(t)$. In that case the reaction rate decreases monotonically to zero because of the end losses. It is also clear that at very high compression, when the plasma radius approaches the penetration depth of the magnetic field or the ion Larmor radius, then the assumptions which led to the reaction rate given by Eq. (42) are no longer valid because then $\beta\ll 1$. This effect leads to a decrease in the reaction rate which eventually compensates for the factor $G(t)^{-7}$ in Eq. (42).

Further examination of Eq. (40) shows that the reaction rate is not necessarily a maximum at the time of maximum field strength. These two maxima would be coincident only if there were no losses and the plasma β remains constant. In practice, however, one might expect that with short coils (short confinement times) the maximum rate would occur before the field maximum, while for longer coils the maximum might well occur after the field maximum, i.e., because $G(t)^{-7}$ can increase faster than $(B/B_0)^{34/5}$ decreases.

In any event, it is clear that one must have a rather complete theory for the effect of losses in order to determine temperatures from measured compressions (plasma radius) and neutron yield. Therefore, it is evident that this complication precludes the possibility of magnetic compression neutrons being used as an accurate "thermometer" unless end losses are proven to be negligible or the plasma density, as well as the plasma radius and β, are also measured with some precision.

This presupposes that the neutron emission has been demonstrated to be of thermal origin. Because of the difficulties outlined in the foregoing, this in itself is difficult without a direct measurement of the ion energies (as they escape through the mirrors, for example) or without an extensive body of experimental data over a wide range of parameters (coil length, confinement time, field strength, density, etc.) to determine whether the neutron emission scales in a predictable (and reasonable) fashion.

VII. EXPERIMENTAL OBSERVATIONS AND PROBLEMS

Stability and Neutron Emission

It appears that the macroscopic stability of a plasma compressed by an axial magnetic field is somewhat better than for the familiar dynamic pinch without a stabilizing B_z field. This is the general conclusion that

[19] For the purposes of discussion we take $\beta=1$ and $\zeta=7/4$. The more general rate equations for $\beta<1$ are obtained from Eq. (10). At lower values of β the reaction rate does not depend as strongly on the losses.

one can draw from time-resolved photographs and spectra of the compressed plasma.[3,7,20]

It has also been shown experimentally[8] that to produce neutrons by magnetic compression in a dense plasma, for the range of parameters studied, it is necessary to initially establish a magnetic field in the plasma (before the discharge of the main capacitor bank) whose direction is opposite to the main confining field. One possible effect of this initial reverse field is that it may cancel the external field which might penetrate the plasma during the early stages of the discharge when the plasma has a poor conductivity because of its low temperature. The field mixing at the plasma boundary Ohmic heats the surface, creating a current sheet which prevents the penetration of the external field and traps the remaining reverse field. It has been observed with a streak camera that the shock implosion is more energetic with an initial reverse field than it is if the external field penetrates the plasma with no field cancellation. This results in a relatively high initial temperature. The higher shock temperature with an initial reverse field could lead to a higher final temperature after adiabatic compression than if there were no reverse field at the beginning of the main discharge, and could also lead to thermonuclear reactions. However, it can also be plausibly argued that the reverse field configuration is unstable, and that this causes neutron production by some process related to spurious neutron emission in low temperature pinches. If some instability (or resonance phenomena) causes rapid field mixing, then the voltages arising from a very high local dH/dt could conceivably accelerate ions to high energies. Indeed, dH/dt of the order 10^{11} gauss/sec has been observed[8,21,22] in the plasma during magnetic compression. The associated electric field is probably large enough to account for the energetic deuterons. However, it is not clear whether the observed large time variation in the internal field strength is a real effect or is due to the influence of the magnetic probe on the plasma.

Assuming for a moment that a rapid dissipation of the reverse field is responsible for the high deuteron energies, then this might provide an important heating mechanism which will not necessarily destroy the over-all macroscopic confinement. It can also be argued that at high densities, with long confinement times, and after the disappearance of the reverse fields, a nonthermal energy distribution could relax to a nearly equilibrium distribution where the production of neutrons may not be insignificant.

There could also be rapid field mixing simply due to the high compressions. This could occur when there are not enough electrons to carry the reverse current without exceeding the mean electron thermal velocity. In that case, there may be some dissipative mechanism, i.e., a two-stream instability, turbulence or collisions at high densities, which limits currents due to runaway electrons (hard x rays which accompany electron runaway are not observed at higher densities). Then the consequent limitation in the magnitude of the electron drift velocity (which limits the reverse current and trapped field) results in a transfer of the reverse field magnetic energy to the plasma during compression. This dissipation of the internal magnetic energy could then heat the electrons, which in turn could transfer energy to the ions by collisions. There may well be other mechanisms which remain obscure due to the lack of quantitative information, i.e., ion heating may result from turbulence or a resonance phenomena.

Another interesting effect which has been observed[3,7] is the sudden onset (after a period of slow compression of a quiescent plasma) of large amplitude oscillations of the plasma cylinder when the plasma radius is ~ 1 mm (which is comparable to the ion Larmor radius). The frequency of these oscillations is of the order of the ion cyclotron frequency and also of the order of the natural hydromagnetic frequency $\sim(H^2/M)^{\frac{1}{2}}$ of the plasma cylinder (M is the mass/unit length of plasma column). These oscillations seem to be associated with the presence of the reverse field. However, the experiments are not sufficiently complete to distinguish between the following mechanisms as the primary cause of the oscillations: (a) an ion resonance; (b) rapid cancellation of the reverse field due to some obscure mechanism; or (c) Ohmic dissipation of the reverse field. This latter process would be rapid when the plasma radius is small and comparable to the skin depth. There could also be other possibilities that would explain the observed oscillations, which are sometimes accompanied by a rotational motion.[23]

In experiments[7] with short coils (10–15 cm) with a geometrical mirror ratio of 1 to 2.5, the neutron emission was concurrent with the oscillations and lasted about 1–2 μsec for confinement times of 3–5 μsec as observed with a streak camera.

With a longer coil (30 cm) having a geometric mirror ratio of 2.5, the plasma confinement time was increased to 5–9 μsec and the neutron emission was also extended by the same factor (average duration of neutron emission 5 μsec). The confinement times were in accord with expectations of the present theory. However, with the long coil, the plasma oscillations were delayed for several μsec, building up in amplitude and then damping out before the plasma was lost out

[20] I. F. Kvartshava, K. N. Kervalidze, and J. S. Gvaladze, "Some magnetohydrodynamic effects of pulsed plasma confinement," in *Proceedings of the Fourth International Conference on Ionization Phenomena in Gases*, R. Nilsson, Editor (North-Holland Publishing Company, Amsterdam, 1959).

[21] H. Fay, E. Hintz, and H. Jordan, "Experiments on shock compression of plasmas," in *Proceedings of the Fourth International Conference on Ionization Phenomena in Gases*, R. Nilsson, Editor (North-Holland Publishing Company, Amsterdam, 1959).

[22] W. E. Quinn, F. L. Ribe, and J. L. Tuck, Bull. Am. Phys. Soc. Ser. II, **5**, 328 (1960).

[23] *Note added in proof.* Following a discussion with N. Rostoker, stereoscopic time-resolved observations were made recently and show that the apparent radial oscillations [see Fig. 8(b) of reference 7] are primarily due to an instability associated with plasma rotation (period 0.3 to 1 μsec).

the ends. The neutron emission began before the start of the oscillations and terminated after they were damped. The delay in the onset of the oscillations due to the rotational instability appears to be correlated with the longer time to reach a critical radius (of the order of the ion Larmor radius); that is, associated with a longer confinement time.[7]

An analysis has been made of the NRL experiments according to the present theory, assuming shock preheating, and adiabatic compression with $\beta<1$ due to the initial reverse field. It appears that the estimated final temperature is only marginal in accounting for thermonuclear reactions. If the energy in the trapped field is partially transferred to the plasma, the required high energy deuterons could be produced. These statements only represent tentative speculations which are based on a limited amount of evidence. An evaluation of the potentialities of the use of the reverse fields to heat a plasma to thermonuclear temperatures without destroying the macroscopic stability over long times is not possible at the present time because of the uncertainties about the relevant mechanisms involved. The observation of neutron emission from a quiescent column of plasma is only suggestive that field mixing may be a useful heating mechanism and not *necessarily* the symptom of an undersirable process as in many pinch experiments.

Mirror Ratio in High-Density Plasmas

In the theoretical considerations the mirror ratio (A_p/A_m) was taken to be a constant and of the order of the geometric mirror ratio. The actual mirror ratio is determined by the experimental method. For example, when there is a reverse field the flux lines must close on themselves inside the plasma, perhaps inhibiting the end losses, provided that the configuration is not unstable; the reverse field is maintained by a current sheet similar to that proposed for the "Astron."[24] However, in the mirror region, magnetic probes show the absence of such a reverse field and indicate a low β. Because of the presence of a trapped field parallel to the external field, the additional internal magnetic pressure prevents the plasma near the mirrors from compressing (the sheath thickness near the ends can be large). This results in a large value of the effective aperture A_m and increases the loss rate. Framing camera photographs[25] of a magnetically compressed argon plasma show that the ratio A_p/A_m is close to the geometrical mirror ratio. If by some method the plasma β is made high, then, according to Grad and his co-workers[17,18,26] one might expect that the area of the aperture at the open ends decreases (because of the high magnetic pressure in the mirror region) to the order of the Larmor radius, R_L times the plasma radius for both the point and line cusp. If such a situation were established, then very long containment times might be expected with relatively short coils (a few meters). The heating and loss rates can be calculated from the present theory using the Larmor radius for a deuteron in estimating the effective aperture A_m.[27] In that case the effective area A_m depends on the particle velocities v_i and v_e, and this must be taken into account in evaluating

$$\tfrac{1}{2}(dN/dt)(\bar{\epsilon}_i+\bar{\epsilon}_e)\cong 2n\langle A_m(\mathbf{v}_i,\mathbf{v}_e)v_{iz}[\epsilon_i(v_i)+\epsilon(v_e)]\rangle_{\text{av}}.$$

The effect[28] is that for a given B, the effective aperture is the largest for the more energetic particles, so that they are lost more rapidly than the low energy ions in the plasma. This is only true if the plasma is collision dominated; if not, the opposite is true. In the collision-dominated case, where the aperture depends on the Larmor radius, the effective value of ζ is increased and the additional heating due to end losses is reduced accordingly.

If the radius of the plasma cylinder is large compared to the ion Larmor radius and/or the sheath thickness, then the loss rate of particles could, in principle, be made very small. The experimental techniques necessary for achieving this situation are not at all clear. The technical problems are similar to obtaining the theoretically possible long confinement times in the proposed cusp geometry.[16–18]

A better understanding of the role of reverse fields as a heating mechanism and of the factors which influence the mirror ratio is essential for the further development of the magnetic compression technique. The present lack of understanding of these factors and limited experimental data is the major cause of uncertainties in the validity of the present theory for estimating plasma temperatures and confinement times.

ACKNOWLEDGMENTS

Discussions with Dr. Harold Grad, Dr. William Faust, and Dr. Hans Griem have been particularly helpful. The assistance of Dr. W. B. Thompson in formulating the more general equations where ζ is a parameter is gratefully acknowledged.

[24] N. C. Christofilos, in *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy* (United Nations, New York, 1958), Vol. 32, p. 279.

[25] R. C. Elton (private communication).

[26] H. Grad, Phys. Rev. Letters 4, 222 (1960).

[27] The hole size could, in principle, be determined by the electron Larmor radius[17,18] because of the large electrostatic fields which pull in the ion orbits to maintain quasi-neutrality in the plasma boundary. However, such a small loss aperture or depth of field penetration is probably unlikely because of the enormous current densities that would be required to maintain such a sharply defined plasma-field boundary. If the plasma density is high enough so that currents due to runaway electrons are not important, the minimum boundary thickness Δr required so that the electron drift velocity does not exceed the mean thermal velocity $\bar{v}_e$ is given by

$$\Delta r = c\Delta B/4\pi n_e e\bar{v}_e,$$

where n_e is the mean electron density at the boundary across which the field changes by ΔB. For $\Delta B=5\times10^4$ gauss, $n_e=10^{16}$, $T=5\times10^6$, then Δr is an order of magnitude larger than the electron Larmor radius but smaller than the ion Larmor radius. At high densities and moderate temperatures the penetration depth is also dependent on the ordinary diffusion rate which, in turn, depends strongly on the initial conductivity and rise time of the confining field.

[28] H. Grad (private communication).

DISCUSSION

Session Reporter: M. MITCHNER

H. Petschek, *Avco-Everett Research Laboratory, Everett, Massachusetts:* Why does the loss of particles result in a heating of the gas? I would have expected it to correspond to an expansion and therefore to result in cooling.

L. Spitzer, Jr., *Matterhorn Project, Princeton University, Princeton, New Jersey:* If the plasma radius is constant as the particles leak out the ends, the temperature falls adiabatically.

A. C. Kolb: That is the essential point. If there is a free expansion at constant volume, then the plasma which remains in the system is cooled; however, one effect of the end losses is that the plasma radius does not stay constant. It decreases continuously as a direct consequence of the particle loss, even at constant magnetic pressure. As a result of this additional motion of the plasma boundary, work is done on the system. In some cases, this additional work overcompensates for the cooling due to the expansion; and the net effect is that under particular circumstances the temperature of the plasma which remains in the system can be higher if there are losses than the temperature would be if there were no losses.

M. U. Clauser, *Space Technology Laboratories, Los Angeles, California:* The situation cannot be much different from thinking of the particles leaking out the ends as if there were movable pistons at the ends. The particles in the center of the confined region do not have any knowledge of how the particles leave the ends. Thermodynamically, the system is equivalent to the lengthening of a cylinder of decreasing diameter.

S. A. Colgate, *Lawrence Radiation Laboratory, University of California, Livermore, California:* This depends on whether there are a large number of collisions or just a few collisions during the expansion. If there were a channel flow out the ends, then one could use the adiabetic law and find that the particles coming out would have a higher velocity than the thermal velocity; however, the fact is here (Naval Research Laboratory experiment) that there may not be enough collisions to insure that there is an adiabatic channel flow with a throttling process at the throat (mirror constriction). There may only be a few collisions before a particle is lost. If the lost particles have a greater velocity than for an adiabatic expansion, the energy of the particles which remain go down; otherwise the residual energy per particle stay up or increase depending on the detailed loss mechanism.

L. Spitzer, Jr.: It then depends on whether the number of collisions is large or small compared to one.

A. C. Kolb: That is correct. If there are many collisions in passing through the mirror, then one must take into account the work done by the particles as they pass through the narrow channel in the mirror region, as suggested by Clauser; however, if the mean free path is much larger than characteristic dimensions of the mirror region, then there is a free expansion and the situation is somewhat different.

L. Spitzer, Jr.: Going on to other features, what happens in these experiments when the initial pressure, before the start of the magnetic compression, is substantially reduced?

A. C. Kolb: One can lower the pressure, but for practical reasons we have worked in the range 100–500 μ Hg. One reason is that the observations of electron temperature and density depend on the intensity of the bremsstrahlung radiation. Below about 80 μ the optical radiation is too weak to measure with our cameras. In addition to facilitating the diagnostic measurements, we work at high densities so that the mean free path of the ions and electrons is less than the length of the coil. The scaling laws for the confinement time depend on this. If the pressure is too low, the confinement time is governed by the ion-ion scattering time, while at high density the confinement time is determined by L/v_{ion}, where L is the characteristic length of the apparatus. At pressures less than $\sim$50 μ energetic x rays (few hundred kev) are observed, indicating a runaway effect of some sort.

M. U. Clauser: What is the ration of the frequencies of the preheater discharge and the compression discharge?

A. C. Kolb: Between 500 kc and 2 Mc for the preheater, and 20–50 kc for the main capacitor discharge.

M. U. Clauser: With this ratio of more than 10 to one, is it not difficult to know the phase of the preheater when the main bank is discharged?

A. C. Kolb: The time at which the main discharge is switched into the coil can be accurately controlled to within 10^{-7} sec. We use three-electrode spark-gap switches and delay cables for this purpose and have no difficulty in varying the relative phase of the two discharges. In fact, the time jitter in the switching can be made less than 10^{-8} sec if necessary.

O. Laporte, *University of Michigan, Ann Arbor, Michigan:* What is the shock configuration?

A. C. Kolb: A cylindrical shock wave is driven radially from the walls of the insulating chamber toward the axis. Also because the magnetic pressure is higher at the ends of the coil than in the central region, there are axial shock waves generated which move toward the central plane of the coil. These axial shocks are confined radially by the magnetic field and propagate in the plasma produced by the radial implosion. These axial shocks collide and provide some additional heating during the early phases of the discharge. The axial shock waves also seem to wash out the dynamic bouncing of the plasma which results from the collision of the radial shocks near the axis of the coil.

P. C. Thonemann: The reproducibility of magnetic probe traces are often taken as a measure of the stability of pinch devices. Will you comment on the reproducibility of your magnetic probe characteristics?

A. C. Kolb: During the preheater discharge, when a reverse field is established in the plasma, the probe traces are very reproducible without fluctuations. This suggests that the plasma is stable with a reverse field when the temperature and compression are low. At high compressions we have been unable to get direct information on the microstability because the size of the probe is comparable to the plasma radius; however, it might well be that due to the reverse field an instability does develop which leads to a turbulent situation with large local electric fields. This could be the origin of the neutrons observed. Microinstabilities, if they exist, do not seem to markedly affect the macroscopic stability and radial confinement, as observed with time-resolving streak cameras.