Long-Range Forces and the Diffusion Coefficients of a Plasma

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1. INTRODUCTION

THIS paper is concerned with a problem that lies at the heart of the kinetic theory of transport processes in an ionized gas, the nature of the effective interaction between particles when correlation effects are taken into account. The usual discussion of transport processes in a diffuse gas starts from Boltzmann's equation for the distribution function $f(\mathbf{x},\mathbf{v},t)$, which may be written

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}}+\mathbf{F}\cdot\frac{\partial f}{\partial \mathbf{v}}=\frac{df}{dt}\bigg|_c, \tag{1}$$

where $\mathbf{F}$ represents the acceleration of a particle by a macroscopic force field and $(df/dt)|_c$ represents the rate of change in f produced by collisions between pairs of particles. Serious difficulties arise if an attempt is made to describe a fully ionized gas by this method. In particular, the collision term introduces into the solution factors of the form

$$Q=\int\sigma(v,\theta)(1-\cos\theta)d\Omega, \tag{2}$$

the momentum transfer cross section.[1] For the Coulomb interaction σ has the Rutherford form

$$\sigma=(e^2/mv^2)^2\csc^4(\theta/2),$$

and the integral Q diverges for small values of the scattering angle θ. The significance of this divergence becomes obvious if the integrand in Q is expressed in terms of the impact parameter b,

$$\begin{aligned}Q&=\int[1-\cos\theta(b)]2\pi bdb\\&=\lim_{b_{\max}\to\infty}4\pi\left(\frac{e^2}{mv^2}\right)^2\int_0^{b_{\max}}\frac{bdb}{b^2+(2e^4/m^2v^2)}\\&=\lim_{b_{\max}\to\infty}4\pi\left(\frac{e^2}{mv^2}\right)^2\log\left(1+\frac{b^2{}_{\max}}{2e^4/m^2v^4}\right)\\&\simeq 8\pi\left(\frac{e^2}{mv^2}\right)^2\lim_{b_{\max}\to\infty}\log\frac{b_{\max}}{b_0},\end{aligned} \tag{3}$$

where $b_0=\sqrt{2}(e^2/mv^2)$. Clearly, the divergence as $b_{\max}\to\infty$ arises from large values of the impact parameter b, and has been avoided in the past by cutting off the range of integration at some large value $b_{\max}$ of the impact parameter. In the earliest treatments this was taken as the interparticle separation $n^{-\frac{1}{3}}$, n being the number density,[2] but more recently as the Debye shielding length,

$$\lambda_D=(kT/4\pi ne^2)^{\frac{1}{2}}. \tag{4}$$

The cutoff procedure has been justified by arguing that beyond $b_{\max}$, many particles interact, and the electric field of a particle is screened by its neighbors, so that particles do not interact if their separation exceeds the screening length. Our major problem is to discuss the significance of this approximation.

A number of authors[3] have observed that encounters are not binary, since there are many particles inside the screening sphere. In fact, the number of particles within the screening sphere is

$$\lambda_D{}^3n=[(kT/4\pi e^2)n^{-\frac{1}{3}}]^{\frac{3}{2}}\sim\Lambda\sim10^3$$

for typical gas discharge plasmas, e.g., $T=20\,000°$ $n=10^{12}$, or $T=200\,000°$, $n=10^{18}$. A method of treating such many-particle encounters was developed by Chandrasekhar[4] in a study of stellar dynamics. This was based on Jean's[5] demonstration that the effect on the trajectory of a star of close binary encounters is negligible when compared with the cumulative effect of interactions with many distant stars. This holds for encounters between charged particles in a diffuse plasma, since the deflection is large only if the impact parameter $b\simeq b_0\simeq e^2/kT\sqrt{2}$, and since interparticle spacings are $\sim n^{-\frac{1}{3}}$, the fraction of particles making such encounters at any instant is

$$\sim[(e^2/\sqrt{2}kT)n^{-\frac{1}{3}}]^3\sim\Lambda^{-2}\sim10^{-6},$$

while at any instant each particle is interacting with $\sim10^3$ others.

The change in the distribution function produced by these small overlapping encounters is random, but almost continuous, the particles diffusing in velocity space. An appropriate representation of this diffusion

[1] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, New York, 1952), 2nd ed., p. 157.

[2] Reference 1, p. 179.

[3] R. S. Cohen, L. Spitzer, and P. M. Routley, Phys. Rev. **80**, 230 (1950).

[4] S. Chandrasekhar, *Principles of Stellar Dynamics* (The University of Chicago Press, Chicago, 1951).

[5] J. Jeans, *Astronomy and Cosmogony* (Cambridge University Press, New York, 1929), p. 319.

can be borrowed from the theory of Brownian motion, and is the Fokker-Planck equation,

$$(\partial f/\partial t)|_c = -(\partial/\partial v_j)(D_j f) + \tfrac{1}{2}(\partial^2/\partial v_i \partial v_j)(D_{ij} f) + \cdots,$$

where the friction coefficients are $D_j = (d/dt)\langle\Delta v_j\rangle$, and the diffusion coefficients are $D_{ij} = \langle(d/dt)\Delta v_i \Delta v_j\rangle$, the quantities in angular brackets representing the mean rate of change of products of the fluctuating velocity components of a particle produced by the interaction with its neighbors. These quantities have in the past been calculated by forming the appropriate sums of the accelerations of a chosen particle as its neighbors move by it in rectilinear orbits, but even in this many-particle treatment a cutoff is needed in the interaction at long distance, otherwise the expressions for the diffusion coefficients D_{ij} diverge.

In spite of these difficulties, the interaction between any pair of particles is weak, since the ratio of the mean potential energy between a pair of neighboring particles to the mean kinetic energy is small, i.e., $(e^2 n^{-\frac{1}{3}}/kT) = \Lambda^{-\frac{2}{3}} \ll 1$, and the probability of large angle scattering is small; thus, the acceleration experienced by a particle is usually small, and the velocity at any instant is almost a constant. The remainder of this paper is devoted to a calculation of the Fokker-Planck diffusion coefficients D_{ij}, which makes use of these features, is valid when $\Lambda(\sim 10^3)$ is large, and is suggested by the electrical behavior of a plasma.

2. DIELECTRIC BEHAVIOR OF A PLASMA

If the electric field about any charge is screened by its neighbors, and if in the Debye sphere there are many particles (Λ large), it should be possible to ascribe the screening to the macroscopic dielectric properties of the plasma. In studying the dielectric behavior of a plasma, we can to a sufficient approximation neglect entirely the effect of particle interaction and use Vlasov's equation, in which the interaction term is omitted altogether from the transport equation. To avoid carrying initial conditions, however, and to define a singular integral, it is preferable to retain a crude representation of the collision terms, $-\tau^{-1}(f-f_0)$, where f_0 is the distribution function unperturbed by the electric field. Then, as has often been shown,[6] the charge induced by an harmonic electric field $\mathbf{E}(\mathbf{k},\omega) \times \exp i(\mathbf{k}\cdot\mathbf{x}+\omega t)$ is, to a first approximation,

$$q(\mathbf{k},\omega) = \lim_{\tau\to\infty} -\frac{e^2}{m}\mathbf{E}\cdot\int \frac{\partial f_0/\partial \mathbf{v}}{i(\omega+\mathbf{k}\cdot\mathbf{v})+\tau^{-1}} d^3\mathbf{k}, \tag{5}$$

and the dielectric coefficient,

$$\epsilon(\mathbf{k},\omega) = 1-(\omega_0^2/\omega^2)\phi(-\omega/kv_\theta), \tag{6}$$

where $\omega_0^2 = 4\pi ne^2/m$ is the square of the plasma frequency, v_θ is the thermal velocity $\frac{1}{2}mv_\theta^2 = kT$. For a Maxwellian distribution, f_0, and real x, ϕ is the complex function

$$\phi(x) = \frac{x^2}{\pi^{\frac{1}{2}}}\left\{\int dt \frac{t\exp(-t^2)}{x-t} - i\pi x \exp(-x^2)\right\} = x^2\{\zeta + i\eta\}. \tag{7}$$

3. FIELD OF A TEST PARTICLE

We are now in a position to discuss the field of a test particle, and assess the validity of the screened Coulomb approximation. The potential produced by an impressed charge f is determined by the relevant form of Poisson's equation,

$$\mathrm{div}(\epsilon\mathbf{E}) = 4\pi f(x,t),$$

or, writing $\mathbf{E} = -\mathrm{grad} V$ and Fourier analyzing,

$$V(\mathbf{k},\omega) = 4\pi f(\mathbf{k},\omega)/k^2\epsilon(\mathbf{k},\omega). \tag{8}$$

We may now consider the field due to a charged particle moving with velocity $\mathbf{v}$, whereupon the Fourier transform of the induced charge density is $f \sim \delta(\omega+\mathbf{k}\cdot\mathbf{v})$, and the function ϕ need be evaluated only at the point $\omega+\mathbf{k}\cdot\mathbf{v}=0$, i.e., $-(\omega/k) = v\cos\theta$, where θ is the angle between $\mathbf{k}$ and $\mathbf{v}$. If the particle is at rest, the denominator reduces to $k^2+k_D^2$ and the potential $V(r) \sim e^{-k_D r}/r$, where $k_D = (2\pi ne^2/kT)^{\frac{1}{2}}$ is the reciprocal of the Debye screening length; thus, for particles at rest, or moving sufficiently slowly, the field is screened as required. However, if particles are moving rapidly, the denominator tends to $k^2 - \frac{1}{2}k_D^2 v_\theta^2/v^2 \cos^2\theta$, and as Bohm and Pines[7] have indicated, the field propagates away from the source as a wake.

This happens because the screening cloud does not have time to form, the group velocity of the oscillations being small. Since the critical speed above which this happens is roughly the mean thermal speed of the electrons, the screened Coulomb approximation is valid for the ion interactions, but for the electron interactions some new justification of screening is required. Since the interaction is nonlocal and retarded, the discussion of the modified binary interactions does not appear hopeful.

4. DIFFUSION COEFFICIENTS IN TERMS OF THE MICROFIELD

To escape from this difficulty we may avoid discussion of the direct interactions of the particles, and use instead the Fokker-Planck approach, calculating the diffusion coefficients in terms of the fluctuating electric microfield produced by the partially correlated motion of the electrons, and then attempt to calculate this microfield. A similar attempt was made by Gabor.[8] Here we use this method to calculate the electron contribution to the two independent quadratic diffusion coefficients $D_{\parallel}(\mathbf{v}) = (v_i v_j/v^2)D_{ij}(\mathbf{v})$ and $D_{\perp}(\mathbf{v}) = [\delta_{ij} - (v_i v_j/v^2)]D_{ij}$ for a system in thermal equilibrium.

[6] L. Landau, J. Phys. (U.S.S.R.) **10**, 25 (1946).

[7] D. Bohm and D. Pines, Phys. Rev. **85**, 338 (1952).

[8] D. Gabor, Proc. Roy. Soc. (London) **A213**, 73 (1952).

It is first necessary to express these diffusion coefficients in terms of the fluctuating electric microfield, and it is easily seen that the correlation function of the microfield is required. The fluctuating components of velocity Δv_i for a particle of charge e and mass m are obtained from

$$\dot{v}_i = -(e/m)E_i(\mathbf{x},t),$$

$$\Delta v_i = -\frac{e}{m}\int_{t-\tau}^{t} E_i[\mathbf{x}(t'),t']dt',$$

where $\mathbf{x}(t')$ is the position at time t' of a particle with velocity $\mathbf{v}$ which at t is at $\mathbf{x}$. From this we obtain the diffusion coefficient

$$D_{ij}(\mathbf{v}) = (d/dt)\langle\Delta v_i\Delta v_j\rangle = \langle\dot{v}_i\Delta v_j + \dot{v}_j\Delta v_i\rangle$$
$$= \frac{e^2}{m^2}\left\langle\int_0^{\tau} ds\{E_i(\mathbf{x},t)E_j[\mathbf{x}(t-s),\, t-s] + E_j(\mathbf{x},t)E_i[\mathbf{x}(t-s),\, t-s]\}\right\rangle, \quad (9)$$

where the brackets indicate average values. If the field correlations persist for a time τ_c^* much less than the time τ_s for the trajectory of a particle to be much altered, two simplifications to (9) are possible: The range of integration can be extended from 0 to infinity, and $\mathbf{x}(t-s)$ may be replaced by $\mathbf{x}(t)-\mathbf{v}s$, where v is the constant mean velocity of the particle. These substitutions are reasonable approximations if the product of the plasma frequency $\omega_0 = (4\pi ne^2/m)^{\frac{1}{2}}$ and the scattering time $\tau_s = 1/nQv$ is large, i.e., using (3), if $(kT/e^2n^{\frac{1}{3}})^{\frac{3}{2}}[\log(kT/e^2n^{\frac{1}{3}})]^{-1} \simeq \Lambda$ is large.

It is possible to simplify (9) by introducing the Fourier transform of the field

$$\mathbf{E}(\mathbf{k},\omega) = \int d^3\mathbf{x}\int dt\mathbf{E}(\mathbf{x},t)\exp[-i(\mathbf{k}\cdot\mathbf{x}+\omega t)], \quad (10)$$

writing (9) as

$$D_{ij} = \frac{e^2}{m^2}\left(\frac{1}{2\pi}\right)^8\cdot\int d^3\mathbf{x}\int_{-\infty}^{\infty} dt\cdot\int d^3\mathbf{k}\int d^3\mathbf{k}'$$
$$\times\int d\omega\int d\omega'\int_0^{\infty} ds\langle[E_i(\mathbf{k},\omega)E_j(\mathbf{k}',\omega') + E_j(\mathbf{k},\omega)E_i(\mathbf{k}',\omega')]\rangle\exp[i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}]$$
$$\times\exp[i(\omega+\omega')t]\exp[-i(\mathbf{k}\cdot\mathbf{v}+\omega)s]$$
$$= \frac{e^2}{m^2}\left(\frac{1}{2\pi}\right)^3\frac{1}{2}\cdot\int d^3\mathbf{k}\int d\omega\langle[E_iE_j^*(\mathbf{k},\omega) + E_i^*E_j(\mathbf{k},\omega)]\rangle\delta(\omega+\mathbf{k}\cdot\mathbf{v}), \quad (11)$$

where $\mathbf{E}^* = \mathbf{E}(-\mathbf{k}, -\omega)$ is complex conjugate to $\mathbf{E}(\mathbf{k},\omega)$.

Since the field E is produced by charges, it may be written as the derivative of a potential, and $E_i(\mathbf{k},\omega) = (k_i/k)|E(\mathbf{k},\omega)|$, therefore

$$D_{\parallel} = \frac{e^2}{m^2}\left(\frac{1}{2\pi}\right)^3\cdot\int d^3\mathbf{k}\int d\omega\langle\mathbf{E}\cdot\mathbf{E}^*(\mathbf{k},\omega)\rangle\frac{(\mathbf{k}\cdot\mathbf{v})^2}{k^2v^2}\times\delta(\omega+\mathbf{k}\cdot\mathbf{v}) \quad (12)$$

and

$$D_{\perp} = \frac{e^2}{m^2}\left(\frac{1}{2\pi}\right)^3\int d^3\mathbf{k}\int d\omega\langle\mathbf{E}\cdot\mathbf{E}^*(\mathbf{k},\omega)\rangle\left(1-\frac{(\mathbf{k}\cdot\mathbf{v})^2}{k^2v^2}\right)\times\delta(\omega+\mathbf{k}\cdot\mathbf{v}). \quad (13)$$

5. FLUCTUATING FIELD

It is now necessary to calculate the energy spectrum of the fluctuating microfield, a problem which is formidable in general, but which may be tackled in a number of ways for systems near thermal equilibrium. The simplest of these invokes the fluctuation-dissipation or generalized Nyquist noise theorem which has been proved in some generality by Callen and Welton.[9] It enables us to express the energy spectrum in terms of the resistivity $R(\mathbf{k},\omega)$:

$$\langle\mathbf{E}\cdot\mathbf{E}^*(\mathbf{k},\omega)\rangle = 2\kappa T\, R(\mathbf{k},\omega). \quad (14)$$

The resistivity in turn can be represented in terms of the complex dielectric constant as

$$R(\mathbf{k},\omega) = (4\pi/\omega)\,\mathrm{Im}[\epsilon(\mathbf{k},\omega)]^{-1}, \quad (15)$$

where Im means imaginary part or, using (6) and (7),

$$R(\mathbf{k},\omega) = 4\pi\, k_D^2\cdot(k^2/\omega)\cdot\{\eta/[(k^2+k_D^2\zeta)^2+k_D^2\eta^2]\}. \quad (16)$$

Thus we have

$$\langle\mathbf{E}\cdot\mathbf{E}^*\rangle = 32\pi^3\frac{ne^2}{\pi^{\frac{1}{2}}v_\theta}\cdot\frac{k\exp[-(\omega/kv_\theta)^2]}{[k^2+k_D^2\zeta(-\omega/kv_\theta)]^2+k_D^4\eta^2(-\omega/kv_\theta)}. \quad (17)$$

Note particularly that the resistivity (15) has nothing to do with the loss of momentum by collisions; indeed, we wish to calculate the collisional loss. Instead this resistivity arises from the Landau damping term[6] in ϵ and represents the effect on the harmonic component $E(\mathbf{k},\omega)$ of those particles moving in phase with that component. This permits a physical interpretation of (17); the numerator represents the source of the component $E(\mathbf{k},\omega)$, those charges which remain in phase, while the denominator represents the effect of the plasma on the propagation of the field. It is this propagator that contains the correlation effects including Debye shielding since these are neglected in the numerator.

This procedure is possible only because the Landau damping exceeds collisional damping for the significant range of frequency and wave number in (11).

[9] H. B. Callen and T. A. Welton, Phys. Rev. **83**, 34 (1951).

6. EVALUATION OF THE DIFFUSION COEFFICIENTS

We are now in a position to evaluate the transport coefficients, $D_{\|}$, $D_{\perp}$. Inserting (17) into (12), we have

$$D_{\|}=\left\langle\frac{d}{dt}\Delta v_{\|}^2\right\rangle=\frac{ne^4}{m^2v_\theta}\frac{32\pi^3}{(2\pi)^3}\frac{1}{\pi^{\frac{1}{2}}}\int d^3\mathbf{k}$$

$$\times\int d\omega\frac{k\exp[-(\omega/kv_\theta)^2]}{(k^2+k_D{}^2\zeta)^2+k_D{}^2\eta^2}\frac{(\mathbf{k}\cdot\mathbf{v})^2}{k^2v^2}\delta(\omega+\mathbf{k}\cdot\mathbf{v}). \quad (18)$$

On introducing spherical polar coordinates (k,θ,ϕ) for $\mathbf{k}$ with pole along $\mathbf{v}$, we may write the argument of the δ function as $\omega+kv\cos\theta$, and by performing the ω integration replace the argument $-\omega/kv_\theta$ of the functions ζ and η by $(v/v_\theta)\cos\theta$. The integration over k then leads to

$$D_{\|}=\lim_{k_{\max}\to\infty}\frac{8\pi ne^4}{m^2v_\theta}\frac{1}{\pi^{\frac{1}{2}}}$$

$$\times\int_0^{2\pi}d\theta\sin\theta\cos\theta\exp\{-[(v/v_\theta)\cos\theta]^2\}$$

$$\times\left[\log\frac{k_{\max}}{k_D}+\frac{1}{4}\log\frac{[1+(k_D/k_{\max})^2\zeta]^2+(k_D/k_{\max})^2\eta^2}{\zeta^2+\eta^2}\right.$$

$$\left.-\frac{\zeta}{2|\eta|}\left\{\frac{\pi}{2}-\tan^{-1}\frac{\zeta}{|\eta|}\right\}\right]. \quad (19)$$

At this point a difficulty introduced by use of the Fokker-Planck equation must be faced; we have not properly considered the effect of close encounters, and consequently discover that (19) diverges as $k_{\max}\to\infty$. This divergence, however, represents the effect of close encounters, for which the binary collision theory is satisfactory. The k integration may be cut off at the value $k_{\max}$ suggested by (3), $k_{\max}=1/b_0=mv^2/\sqrt{2}e^2$. The first term in the integrand of (19) can thus be written

$$\log(kT/e^2n^{\frac{1}{3}})^{\frac{3}{2}}+\log(mv^2/kT). \quad (20)$$

The first term here, $\log\Lambda$, is larger than the remaining terms in the integrand of (19) and has been called the dominant term by Chandrasekhar. Following Chandrasekhar we now neglect the nondominant terms and obtain from (19)

$$D_{\|}=\frac{8\pi ne^4}{m^2v_\theta}\log\Lambda\frac{2}{\pi^{\frac{1}{2}}}\int_0^1 dt\,t^2\exp\left(-\frac{v^2}{v_\theta{}^2}t^2\right)$$

$$=\frac{8\pi ne^4}{m^2v}\log\Lambda\,G\left(\frac{v}{v_\theta}\right), \quad (21)$$

where

$$G(x)=\frac{2}{\pi^{\frac{1}{2}}}\left[\int_0^t\exp(-t^2)dt-x\exp(-x^2)\Big/2x^2\right] \quad (22)$$

is the function tabulated by Chandrasekhar. In a similar way,

$$D_{\perp}=(8\pi ne^4/m^2v)\log\Lambda\,H(v/v_\theta), \quad (23)$$

where the function

$$H(x)=\frac{2}{\pi^{\frac{1}{2}}}\int_0^x\exp(-t^2)dt-G \quad (24)$$

is also tabulated by Chandrasekhar.

Thus, by using Vlasov's method which neglects particle interactions, we have been able to calculate the propagation of an electric field in a plasma, and from this the Nyquist theorem has enabled us to calculate the fluctuating microfield, including the main effects of correlation between particles. In turn, from this we have calculated the Fokker-Planck transport coefficients, and in the limit of small $e^2n^{\frac{1}{3}}/kT=\Lambda^{-\frac{2}{3}}$ have recovered the results of Spitzer *et al.* without artificially introducing a long-range cutoff.

There are several obvious further developments of this theory. One is to calculate $D_i=\langle(d/dt)\Delta v\rangle$, which requires a second-order treatment of the motion of a particle in the fluctuating microfield but is fairly straightforward.

The effects of a steady magnetic field can be treated at the price of much algebraic and analytical complication, ϵ being woefully complicated, but with no change in principle. Of rather more interest is the generalization to systems not in thermal equilibrium, which presents no great difficulty if the system is homogeneous in space and time, although the Nyquist theorem can no longer be used to calculate the field. In this case, there does exist the possibility that the nondominant terms may become large and introduce novel phenomena. Finally, it is important to consider more carefully the effects of close encounters, and develop a formalism that will include these in a logically consistent way. Some of these problems will be considered in publications which we hope will appear in the near future.

DISCUSSION

Session Reporter: W. B. Riesenfeld

R. Lüst, *Max-Planck-Institut für Physik und Astrophysik, Munich, Germany:* I want to ask a question on the last point you raised. The noise theory you mentioned was originally intended only as a thermal noise theory.

W. B. Thompson: Yes, it is true that the noise theorem applies only in thermal equilibrium; however, with the assumption that the electric field produces only a small perturbation in the motion of the particles, and that the correlation phenomena induced by the interactions are adequately summed up in the dielectric coefficient, the fluctuating fields can be calculated whether there is thermal equilibrium or not. First, the dielectric coefficient $\epsilon(\omega,\mathbf{k})$ may be calculated for any steady spatially homogeneous distribution function f_0, since the usual solution of the Vlasov equation can be given for any function of the velocities. Next, if we neglect correlations, an approximation to the charge fluctuation can be calculated by using the Bohm and Pines expression

$$\rho(\mathbf{k},\omega)=\Sigma_i \exp[-i\mathbf{k}\cdot(\mathbf{x}_i+\mathbf{v}_i t)],$$

where the $\mathbf{v}_i$ are constant. Finally, by using Poisson's equation for a dielectric medium, we obtain

$$\mathbf{E}(\mathbf{k},\omega)=-4\pi e[i\mathbf{k}\rho(\mathbf{k},\omega)/k^2\epsilon(\mathbf{k},\omega)]$$

$$\langle \mathbf{E}\cdot\mathbf{E}^*(\mathbf{k},\omega)\rangle=(4\pi e)^2\frac{\langle\rho(\mathbf{k},\omega)\rho^*(\mathbf{k},\omega)\rangle}{k^2|\epsilon(\mathbf{k},\omega)|^2}=\frac{(4\pi e)^2 n}{k^3|\epsilon(\mathbf{k},\omega)|^2}f_0\left(-\frac{\omega}{k}\right).$$

In thermal equilibrium this agrees with the noise theorem result.

S. A. Colgate, *Lawrence Radiation Laboratory, University of California, Livermore, California:* Does this allow you to calculate diffusion coefficients for a nonthermal system?

W. B. Thompson: Yes.

S. A. Colgate: How does kT enter this situation?

W. B. Thompson: The fundamental energy spectrum does not necessarily involve kT, it only involves the unperturbed distribution function.

A. C. Kolb, *U. S. Naval Research Laboratory, Washington, D. C.:* Does your analysis yield Spitzer's result with Debye screening?

W. B. Thompson: Yes.

A. C. Kolb: You also asserted that in spite of the absence of any explicit Debye screening in the analysis you obtain this result. I suspect that this (the inference that there is no screening) may not be justified and may be connected with your treatment of the delta function which can also be written as a Fourier integral. One can then interchange orders of integration, and determine the spectrum from a Fourier analysis of the correlation function. If you then cut off the time integral by essentially the transit time across a Debye sphere, would not you obtain precisely Spitzer's result?

W. B. Thompson: There is no cutoff introduced here at all.

A. C. Kolb: Yes, according to your procedure, but could not yet handle the intergrals in your theory slightly differently, thereby putting the Debye cutoff in again automatically, and obtain the same result? The main point here is that I consider your statement that there is *no* Debye cutoff a little bit extreme.

W. B. Thompson: In this calculation no arbitrary cutoff is made. Instead, an approximate treatment of the dielectric properties of the plasma is used to derive the correct screening, hence the fields are screened, but the physical origin of this is given.

B. S. Liley, *Associated Electrical Industries, Ltd., Aldermaston, Berkshire, England:* Does not the cutoff amount to the question of the distance at which the two-particle distribution function becomes equal to the product of single-particle distribution functions?

W. B. Thompson: Not quite, since in the absence of correlation there would be no screening. Correlation effects are included in this calculation through the dielectric coefficient and it is possible to get at the charge correlation from the calculated fields; however, for the Fokker-Planck equation this is not necessary.

M. Mitchner, *Lockheed Aircraft Corporation, Sunnyvale, California:* Apparently, if you use either the Boltzmann collision integral or the Fokker-Planck equation to calculate the conductivity you obtain the same result, seemingly on the basis of different physical ideas. Why?

W. B. Thompson: Essentially because the integral of a sum of terms is equal to the sum of their integrals. In solving the Boltzmann equation the momentum and energy transfers are needed, and collisions can be treated in the impulse approximation and the rate of change of momentum written as

$$\Delta\mathbf{p}_i=\Sigma_j\int\mathbf{F}_{ij}dt,$$

where $\mathbf{F}_{ij}$ is the proportional to the force exerted on particle i by particle j as the latter moves along a trajectory unperturbed by the presence of particles. In the Fokker-Planck treatment, on the other hand, you calculate the change in momentum experienced by a particle as it moves under the force exerted by all the other particles moving along unperturbed trajectories, $\Delta\mathbf{p}_i=\int dt\,\Sigma_j\,\mathbf{F}_{ij}$. Of course, the Boltzmann equation can also describe close encounters, hence avoids the inner cutoff needed in the Fokker-Planck equation.

R. Balescu, *Université Libre de Bruxelles, Bruxelles, Belgium:* With regard to your statement about equilibrium situations, I think it should be stressed that the system consists of one particle out of equilibrium moving in a gas that is in equilibrium. The diffusion coefficient, after all, describes a nonequilibrium property. Secondly, you state that you rederive all the results of the binary collision approximation. I think that for very large velocities there are other corrections.

W. B. Thompson: This is correct; it is only in the dominant term that there is agreement between this and the Spitzer-Chandrasekhar result; that is, in the term $\sim\log k_{\max}$. For very large velocities the nondominant terms may become important; however, for a system in thermal equilibrium these terms are bounded for any finite velocity, and in the limit, $e^2n^{1/3}/kT\rightarrow 0$ become negligible. If the system is not in thermal equilibrium, however, these nondominant terms may become very large indeed.

T. Kihara, *Department of Physics, University of Tokyo, Tokyo, Japan:* Is it possible to prove that your method is equivalent to cutting off the time range of correlation functions at the plasma period?

W. B. Thompson: Not quite; however, because of its frequency dependence the dielectric constant does screen the interactions in time as well as in space.