Strange Particle Decay Processes and the Fermi Interaction*

R. H. DALITZ

Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

1. INTRODUCTION

IT recently has become apparent that the processes of beta decay, muon decay, and nuclear capture of μ^- mesons may each be well described in terms of a four-fermion coupling of the form $V-A$, each of these couplings having a strength $gm_\pi^2 \sim 10^{-7}$. Feynman and Gell-Mann[1] have suggested that these interactions may be parts of a more general weak interaction expressed in terms of a weak interaction current J_μ, this general interaction having the form

$$H_{wk}=J_\mu{}^*J_\mu, \tag{1.1}$$

where the current J_μ consists of the sum

$$J_\mu=g^{\frac{1}{2}}\bar{\nu}\gamma_\mu(1+\gamma_5)\mu^+ + g^{\frac{1}{2}}\bar{\nu}\gamma_\mu(1+\gamma_5)e^+ + g^{\frac{1}{2}}\bar{n}\gamma_\mu(1+\gamma_5)p+\cdots. \tag{1.2}$$

These authors, and earlier Gershtein and Zeldovich,[2] have suggested further that the terms of J_μ which involve strongly interacting particles, but allow no change of strangeness, may obey a conservation principle.

As a result of these successes of the four-fermion interaction, attention is confined here to a survey of the possibility of accounting for strange particle decays by the addition of further terms to the current J_μ, terms which have the same form as those of Eq. (1.2) but which do not conserve strangeness. There are many possibilities for additional terms of this kind, for example $\bar{\Lambda}\gamma_\mu(1+\gamma_5)p$, $\bar{\Sigma}^-\gamma_\mu(1+\gamma_5)n$, $\bar{\Xi}^-\gamma_\mu(1+\gamma_5)\Lambda$, etc. The coupling of these terms to those of (1.2) which involve strongly interacting particles and no change in strangeness then lead directly to the pionic decay modes which are observed for the hyperons; their coupling to the leptonic terms of (1.2) leads to leptonic decay modes for the hyperons. The decay of K mesons is then interpreted as due to their coupling to virtual hyperon-nucleon pairs which allow weak decays for the K mesons through the four-fermion couplings of the expression (1.2), for example

$$K^+ \xrightarrow[\text{strong}]{} p+\bar{\Lambda}\begin{cases} \xrightarrow[\text{weak}]{} \mu^+ + \bar{\nu}, & (1.3a) \\ \xrightarrow[\text{weak}]{} p+\bar{n} \xrightarrow[\text{strong}]{} \pi^+ + \pi^- + \pi^+. & (1.3b) \end{cases}$$

The possibility of a qualitative account of the decay modes of hyperons and K particles on the basis of such four-fermion couplings has been known for some time. It was first put forward by Dallaporta[3] and by Gell-Mann,[4] and has been discussed in some detail by Gell-Mann and Rosenfeld[5] in their recent review article.

At this stage it is necessary to inquire to what extent this scheme offers the possibility of accounting quantitatively[6] for the branching ratios and detailed characteristics of these decay modes. However, such a complete program is not carried through here. Our present purpose is simply to discuss the degree of agreement between the data and those expectations from this model which do not depend on detailed theoretical calculations of its consequences.

2. PIONIC DECAY MODES FOR STRANGE PARTICLES

First *the* Λ *decay modes*,

$$\Lambda \rightarrow \begin{cases} p+\pi^-, & (2.1a) \\ n+\pi^0, & (2.1b) \end{cases}$$

are considered. These modes are regarded as the result of weak interactions connecting $(\bar{\Lambda}p)$ and $(\bar{n}p)$. With a parity nonconserving weak interaction, each of these modes requires two parameters (s_-,p_-) and (s_0,p_0), which denote the amplitudes for emission of s- and p-wave pions in the π^-- and π^0-decay processes (2.1), respectively. Assuming time-reversal invariance to be valid for both strong and weak interactions, the phases of these amplitudes arise only from the scattering in the final pion-nucleon state, as first pointed out by Takeda.[7] Since the relevant pion-nucleon phase shifts are known to be small at the energy of Λ decay, the parameters s

* Work done under the auspices of the U. S. Atomic Energy Commission.

[1] R. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[2] S. Gershtein and J. Zeldovich, Soviet Phys. JETP **2**, 576 (1957).

[3] N. Dallaporta, Nuovo cimento **1**, 962 (1953); G. Costa and N. Dallaporta, Nuovo cimento **2**, 519 (1955).

[4] M. Gell-Mann, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1956).

[5] M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nuclear Sci. **7**, 407 (1957).

[6] The absence of evidence for the beta decay of the Λ particle appeared a stumbling block for this scheme for a long time. However this process has now been observed by Crawford, Cresti, Good, Kalbfleisch, Stevenson, and Ticho [Phys. Rev. Letters **1**, 377 (1958)] and by Nordin, Orear, Reed, Rosenfeld, Solmitz, Taft, and Tripp [Phys. Rev. Letters **1**, 380 (1958)]. The observed rate suggests (cf. Sec. 4) that the term of J_μ associated with this process may have an amplitude of order 0.3 relative to that for beta decay of the neutron.

[7] G. Takeda, Phys. Rev. **101**, 1547 (1956).

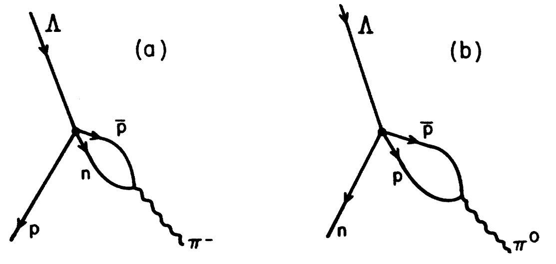

FIG. 1. The simplest graphs leading to pionic Λ decay through the Fermi interaction.

and p will be assumed real. The experimental information bearing on these parameters is at present limited to the following[8]:

$$(s_0^2+p_0^2)/(s_-^2+p_-^2)=0.59\pm0.07, \qquad (2.2a)$$

$$\alpha_-=2s_-p_-/(s_-^2+p_-^2)\geqslant 0.73\pm0.14, \qquad (2.2b)$$

the first representing the branching ratio between the modes (2.1), the second representing the information available on the up-down asymmetry in the decay of polarized Λ particles. The condition (2.2b) limits p_-/s_- to lie between the limits

$$0.45\leqslant |p_-/s_-|\leqslant 2.25. \qquad (2.3)$$

Calculation of the amplitudes for the processes (2.1) from the four-fermion weak couplings of the type $(\bar{\Lambda}p)(\bar{p}n)$ would involve consideration of many complicated radiative corrections arising from the strong pion couplings of these fermions, a task beyond our present ability. For the purpose of orientation, we confine attention to the simplest possible graphs leading to these decay processes, those shown in Fig. 1. For this graph the amplitude for process (2.1a) takes the form

$$\sqrt{2}C(\bar{\Lambda}\gamma_\mu(1+\gamma_5)p)\partial\phi_\pi/\partial x_\mu$$
$$=\sqrt{2}C\{(m_\Lambda-m)+(m_\Lambda+m)\boldsymbol{\sigma}\cdot\mathbf{q}/2m\}, \qquad (2.4)$$

in the nonrelativistic limit appropriate to the low-energy release in Λ decay, when $\mathbf{q}$ denotes the pion momentum. This crude estimate leads to a ratio $p_-/s_-=(m_\Lambda-m)q/2m(m_\Lambda+m)=0.64$, corresponding to a value $\alpha_-=+0.9$ for the polarization coefficient in Λ decay. This value of p_-/s_- lies within the limits (2.3) from the up-down asymmetry observed[8] in polarized Λ-particle decay and corresponds to the sign recently[9] deduced for α from the polarization observed for protons resulting from the decay of unpolarized Λ particles. Such a large value of p_-/s_- is characteristic of a coupling which involves V coupling with the chirality factor $(1+\gamma_5)$; in the same approximation, S coupling (with this chirality factor) would lead to a ratio $p_-/s_-\sim q/2m=0.05$ and the large lower limit observed for α_- would then be difficult to understand in terms of a chirality factor.

Further evidence concerning this ratio p_-/s_- may be obtained from the data on the frequency of the two-body decay mode

$${}_\Lambda H^4 \rightarrow \pi^- + He^4, \qquad (2.5)$$

relative to that for all the three-body π^--decay modes of ${}_\Lambda H^4$, for example π^-+p+H^3 or π^-+n+He^3. Systematic investigation by the EFINS–NU emulsion groups[10] at Chicago has established 35 examples of the two-body mode (2.5), compared with a maximum of 27 examples for other π^--mesonic modes of ${}_\Lambda H^4$ decay. This last figure includes six events which probably represent three-body modes of ${}_\Lambda H^4$ decay but which could not be uniquely established as representing ${}_\Lambda H^4$ decay events. From this evidence the proportion R of π^--mesonic ${}_\Lambda H^4$ decays which proceed through the two-body mode (2.5) is rather high, in fact,

$$R\geqslant 0.6\pm0.1. \qquad (2.6)$$

If the ground state of ${}_\Lambda H^4$ has spin $J=1$, the process (2.5) must involve the emission of a p-wave pion. This can take place only through the p channel of Λ decay since the initial and final nuclear systems consist predominantly of s states. According to a recent estimate,[11] the value R_1 for this case cannot exceed 0.25, reaching this value only with the upper limit (2.3) for p_-/s_-. On the other hand, with $J=0$, the pion emission can proceed through the s channel of the Λ-decay interaction and the corresponding estimate R_0 can reach a value of 0.45 with the use of the lower limit (2.3) for p_-/s_-. There is now some reason[12] to believe that these calculations may involve some overestimate of the probability of the three-body modes, and correction for this will allow correspondingly larger estimates (by as much as 20%) for R. However, the qualitative conclusion appears definite, that it is difficult to account for the value (2.6) observed for R unless $J=0$ for ${}_\Lambda H^4$ and $p_-/s_-<1$. With this last conclusion,

[8] D. Glaser, *Proceedings of the 1958 Annual International Conference on High Energy Physics at CERN*, p. 265; Glaser, Good, and Morrison, *Proceedings of the 1958 Annual International Conference on High Energy Physics at CERN*, p. 270.

[9] Boldt, Bridge, Caldwell, and Pal, Phys. Rev. Letters **1**, 256 (1958).

[10] Levi-Setti, Ammar, Slater, Limentani, Roberts, Schlein, and Steinberg (Nuovo cimento, to be published).

[11] R. H. Dalitz, Phys. Rev. **112**, 605 (1958).

[12] R. H. Dalitz and L. Liu (to be published). [*Note added in proof.*—Re-examination of the calculation reported in reference 11 has shown that the conclusions drawn above are too strong. With $J=0$, the values calculated for R are consistent with the experimental result (2.6), within the stated error, for values of p_-/s_- less than 1.5. With $J=1$, the calculated value of R is within two standard deviations of (2.6) for $p_-/s_->1$. Consequently, no conclusion concerning the value of p_-/s_- can be reached from these data alone until the spin of ${}_\Lambda H^4$ is known independently. Conversely, if a value of p_-/s_- could be obtained directly (for example, from observations on the proton polarization resulting from polarized Λ decay), this argument may allow the ${}_\Lambda H^4$ spin to be deduced. The evidence on the nonmesonic decay rates for Λ hypernuclei mentioned below does suggest that it is rather unlikely that p_-/s_- should exceed unity.]

it appears that the p_-/s_- ratio of 0.64 obtained from this crude model of Λ decay does not differ widely from the actual situation.

A similar crude calculation may be made for the mode (2.1b) on the basis of the $V-A$ interaction $(\bar{\Lambda}p)(\bar{p}n)$, with the result[13] $C(\bar{\Lambda}\gamma_\mu(1+\gamma_5)n)\partial\phi_0/\partial x_\mu$. This estimate predicts that

$$s_0/s_- = p_0/p_- = 1/\sqrt{2}. \tag{2.7}$$

This is consistent with the observed ratio (2.2b) between the (π^0+n) and (π^-+p) modes of Λ decay, and it further requires that the polarization properties of these two modes should be identical, a prediction of current interest, since the ratio α_0/α_- may soon be determined from Λ particles by experiments in bubble chambers with working fluid of high density.

However, these experimental predictions would also follow directly from the $\Delta T=\frac{1}{2}$ selection rule proposed by Gell-Mann and Pais[14] for strange particle decays. With $T=0$ for the Λ particle, this would require $T=\frac{1}{2}$ for the $(\pi+N)$ system and therefore that

$$s_0/s_- = p_0/p_- = -1/\sqrt{2}. \tag{2.8}$$

It is obviously difficult to distinguish between the cases (2.7) and (2.8), since they differ only in the relative sign of the matrix elements for two different physical processes. However, this relative sign is, in principle, a physical observable and can have physical consequences, as shown in the following. At present there is no theoretical basis to favor the existence of such a $\Delta T=\frac{1}{2}$ selection rule. The proposal of this selection rule is based entirely on the empirical evidence that is discussed in the next section. It is possible to construct a four-fermion interaction which leads to this $\Delta T=\frac{1}{2}$ selection rule; in fact, for an interaction involving only the Λ hyperon, this would have the unique $(V-A)$ form

$$\{(\bar{\Lambda}p)(\bar{p}n)+(\bar{\Lambda}n)(\bar{n}n)\}. \tag{2.9}$$

However, this interaction does not fit with the viewpoint of Feynman and Gell-Mann as expressed in Eq. (1.1), since the only terms which they permit for the current J_μ involve a charge change ΔQ of one unit. This restriction appears necessary in order to forbid certain types of decay process (e.g., $\mu^+\rightarrow e^+ + e^- + e^+$, $K^+\rightarrow\pi^+ + \nu + \bar{\nu}$) whose occurrence has not been observed, and excludes terms of the type $(\bar{\nu}\nu)$, $(\bar{\Lambda}n)$ or $(\bar{n}n)$ from J_μ. The four-fermion interaction allowed by these considerations is composed of the current terms $(\bar{\Lambda}p)$ and $(\bar{p}n)$, leading to the form

$$(\bar{\Lambda}p)(\bar{p}n). \tag{2.10}$$

This interaction allows isotopic spin changes of $\Delta T=\frac{1}{2}$ and $\frac{3}{2}$. The crude calculation of the Λ-decay amplitudes given in the foregoing, which is based on this interaction, therefore leads to both $T=\frac{1}{2}$ and $T=\frac{3}{2}$ components in the final pion-nucleon state. These combine fortuitously to give the result (2.7) and the ratio 2:1 for the $(\pi^-+p)/(\pi^0+n)$ modes, in agreement with experiment. Inclusion of radiative corrections may be expected to modify the (π^0+n) and (π^-+p) amplitudes in quite different ways. On this view, the agreement between (2.2a) and the prediction of the $\Delta T=\frac{1}{2}$ rule would be regarded as fortuitous and would give no basis for expecting α_0/α_- to be unity, as would follow if the $\Delta T=\frac{1}{2}$ rule held. Observation of the polarization properties of $\Lambda\rightarrow n+\pi^0$ decay will therefore have considerable relevance for understanding of the $(\pi^-+p)/(\pi^0+n)$ ratio in Λ decay and of the basic mechanism underlying Λ decay.

The process of *nonmesonic decay of Λ hypernuclei* is of special interest concerning the mechanism underlying Λ decay, since the elementary four-fermion interactions (2.9) and (2.10) mentioned above actually correspond to processes of nonmesonic de-excitation of the Λ particle,[15] thus

$$\Lambda + p \rightarrow n + p \tag{2.11}$$

from interaction (2.10). The strength $\sqrt{f}$ appropriate to a current term $(\bar{\Lambda}p)$ in J_μ is not well-known empirically. The hypothesis of universality for the four-fermion weak interactions would require that f should equal g, the coupling strength appropriate to the terms $(\bar{\nu}e^+)$ or $(\bar{\nu}\mu^+)$; in fact, as shown later (cf. §4), there is reason to believe that f may be an order of magnitude smaller than g. To a sufficient approximation (of order 10%), it is sufficient to consider only the non-relativistic approximation to the $(V-A)$ interaction for (2.10), namely,

$$(fg)^{\frac{1}{2}}(\bar{\psi}_\Lambda\gamma_\mu(1+\gamma_5)\psi_p)(\bar{\psi}_p\gamma_\mu(1+\gamma_5)\psi_n) \simeq (fg)^{\frac{1}{2}}\{\bar{\psi}_\Lambda\psi_p\bar{\psi}_p\psi_n - (\bar{\psi}_\Lambda\boldsymbol{\sigma}_\Lambda\psi_p)\cdot(\bar{\psi}_p\boldsymbol{\sigma}_N\psi_n)\}. \tag{2.12}$$

This dominant term is parity conserving (conventionally, the Λ parity has been defined to be even); parity nonconserving terms are of order $v/c\sim 0.25$ relative to this term. There are many radiative corrections due to the strong pion and K-meson couplings of these particles, which modify the form of the amplitude for the nonmesonic capture process (2.11) from this form (2.12). Several of the many possibilities are sketched in Fig. 2. Terms corresponding to Figs. 2(b) and 2(c) are of special importance and have been discussed in detail by Karplus and Ruderman.[16] They may be

[13] With the $V-A$ form of interaction, the interactions $(\bar{a}b)(\bar{c}d)$ and $(\bar{a}d)(\bar{b}c)$ are identical. Consequently, with $(\bar{\Lambda}n)(\bar{p}p)$ replacing $(\bar{\Lambda}p)(\bar{p}n)$, the same factor C appears here, but without the factor $\sqrt{2}$ since the emission is now of a π^0 meson instead of a π^- meson.

[14] M. Gell-Mann and A. Pais, *Proceedings of the International Conference on High Energy Physics* (Pergamon Press, London, 1955).

[15] S. B. Treiman, *Proceedings of the 1958 Annual International Conference on High Energy Physics at CERN*, p. 276.

[16] M. Ruderman and R. Karplus, Phys. Rev. **76**, 1458 (1949).

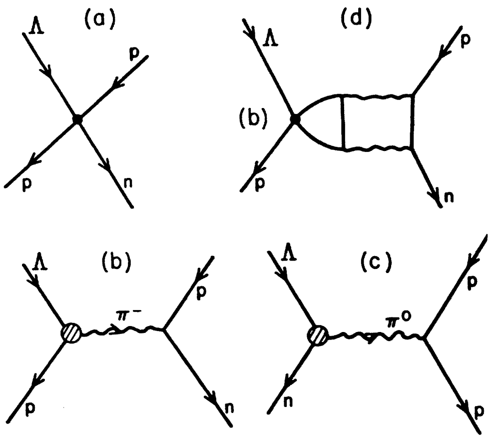

FIG. 2. The elementary four-fermion interaction (a) and some examples (b), (c), and (d) of pionic radiative corrections to this.

expressed phenomenologically in terms of the amplitudes for Λ decay and for pion-nucleon coupling. In physical terms, they represent the process of internal conversion of the pion field generated by the $\Lambda \to N+\pi$ decay interaction, due to the presence of a neighboring nucleon. The sum of these terms and the basic interaction (2.12) is

$$(fg)^{\frac{1}{2}}(1-\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N)+\left\{\frac{\sqrt{2}G}{2m}\frac{4\pi(s_- - p_-\boldsymbol{\sigma}_\Lambda\cdot\mathbf{q})(-\boldsymbol{\sigma}_N\cdot\mathbf{q})}{\frac{1}{4}(m_\Lambda-m)^2-q^2-m_\pi{}^2}\right.$$
$$\left.+\frac{-G}{2m}P_{\Lambda N}{}^{\sigma}\frac{4\pi(s_0+p_0\boldsymbol{\sigma}_\Lambda\cdot\mathbf{q})(\boldsymbol{\sigma}_N\cdot\mathbf{q})}{\frac{1}{4}(m_\Lambda-m)^2-\mathbf{q}^2-m_\pi{}^2}\right\}. \quad (2.13)$$

where $\mathbf{q}$ is the momentum of the outgoing proton, G denotes the pion-nucleon coupling parameter ($G^2/\hbar c \simeq 13.5$), $\boldsymbol{\sigma}_\Lambda$ is the spin vector of the initial Λ particle or final proton, $\boldsymbol{\sigma}_N$ is that of the initial proton or final neutron, and $P_{\Lambda N}{}^{\sigma}$ denotes the spin exchange operator $(1+\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N)/2$. Since these last terms are coherent, as was realized by Cerulus,[17] the relative signs of (s_-,p_-) and (s_0,p_0) affect the rate computed for the nonmesonic capture process. The direct amplitude (2.12) interferes only with the p_- and p_0 terms of (2.13) in the total nonmesonic capture rate. The expression (2.13) now serves to illustrate two points:

(i) that the relative sign between s_- and s_0, and between p_- and p_0 does have physical consequences, although these cannot be computed completely in the present case, and

(ii) that the rate of nonmesonic capture generally depends on the relative spin orientation of the Λ particle and the nucleons present. The direct term (2.12) actually vanishes for a triplet spin configuration although its value is quite appreciable $[4(fg)^{\frac{1}{2}}]$ for a singlet configuration. This may be of importance in the light hypernuclei. For example, in $_\Lambda H^4$, the (Λp) spin orientation appears to be singlet, from the arguments given above on the decay modes of $_\Lambda H^4$, whereas in the mirror nucleus $_\Lambda He^4$, there are two protons and the (Λp) spin orientations are randomly distributed. The ratio of the nonmesonic capture process (2.11) in these systems therefore reflects the spin dependence of the amplitude for this process. To illustrate this effect, we give in Table I correction coefficients F_s and F_p for the Ruderman-Karplus calculation of the internal conversion coefficient R, as function of the (Λp) spin state. In terms of F_s, F_p, the internal conversion coefficient $R=$ (nonmesonic capture rate)/(π^--mesonic capture rate) is given by

$$R=(F_s s_-{}^2R_s+F_p p_-{}^2R_p)/(s_-{}^2+p_-{}^2), \quad (2.14)$$

where R_s, R_p are the coefficients computed for $l=0$ and 1 by Ruderman and Karplus. This estimate includes only the "internal conversion" graphs of Figs. 2(b) and 2(c) and neglects the direct term (2.12) as well as all other radiative corrections. It is intended only to illustrate the order of magnitude of these effects.

No example of nonmesonic decay for $_\Lambda H^4$ has yet been reported although many examples are known of nonmesonic decay for $_\Lambda He^4$ and $_\Lambda He^5$. This contrast may possibly be the result of experimental bias, since $_\Lambda H^4$ nonmesonic decay can lead only to a one-pronged star.

To sum up, we emphasize again the difficulty of making a quantitative estimate of the rate of nonmesonic Λ-hypernuclear decay in terms of the elementary four-fermion interaction, owing to the complications of the radiative corrections possible. However, there is one conclusion from the hypernuclear decay evidence which appears unlikely to be modified by the effect of further radiative corrections; with the estimates $R_s \simeq 1$, $R_p \simeq 17$ of Ruderman and Karplus for $_\Lambda$He hypernuclei, the observed value of ~ 1.5 for $R(_\Lambda\mathrm{He})$ obtained by Schlein[18] and by Silverstein[19] still requires that the amplitude p_- should be smaller than s_-. If p-channel emission were dominant relative to s-channel emission in free Λ decay, it would require very detailed cancellations to account for such a low value of R as that observed.

TABLE I. Correction factors to the internal conversion coefficient in hypernuclear decay.[a]

Spin state	(a) $\Delta T=\frac{1}{2}$ F_s	F_p	(b) Amplitudes (2.7) F_s	F_p
$S=0$	9/4	9/4	1/4	1/4
$S=1$	19/12	1/4	11/12	9/4
Spin average (appropriate for $_\Lambda He^4$ and $_\Lambda He^5$)	7/4	3/4	3/4	7/4

[a] These correction factors have been obtained in joint work with S. Eckstein.

[17] F. Cerulus, Nuovo cimento 5, 1685 (1957).

[18] P. E. Schlein, Phys. Rev. Letters 2, 220 (1959).

[19] E. Silverstein, Suppl. Nuovo cimento, 10, 41 (1959).

The decay processes of the Σ particles,

$$\Sigma^+ \rightarrow \begin{cases} n+\pi^+, & (2.15a) \\ p+\pi^0, & (2.15b) \end{cases}$$

$$\Sigma^- \rightarrow n+\pi^-, \qquad (2.15c)$$

could already take place through the interaction (2.10), since the Σ and Λ particles are strongly coupled, but probably occur also through additional four-fermion couplings involving the Σ particles. The most general such coupling would allow isotopic spin changes ΔT up to $\frac{5}{2}$ in strange particle decays. However, the isotopic spin changes may be limited to $\Delta T \leqslant \frac{3}{2}$ by considering the interaction

$$\left\{(\bar{\Sigma}^- n)+\frac{1}{\sqrt{2}}(\bar{\Sigma}^0 p)\right\}(\bar{p}n). \qquad (2.16)$$

It is attractive to combine this with $(fg/2)^{\frac{1}{2}}$ times the term (2.10) to give the form

$$(fg)^{\frac{1}{2}}(\bar{Z}N)(\bar{p}n) = (fg)^{\frac{1}{2}}\left\{(\bar{\Sigma}^- n)+\left(\frac{\bar{\Lambda}+\bar{\Sigma}^0}{\sqrt{2}}p\right)\right\}(\bar{p}n), \qquad (2.17)$$

where Y, Z denote the combinations

$$Z=\begin{pmatrix} (\Lambda+\Sigma^0)/\sqrt{2} \\ \Sigma^- \end{pmatrix}, \quad Y=\begin{pmatrix} \Sigma^+ \\ (\Lambda-\Sigma^0)/\sqrt{2} \end{pmatrix} \qquad (2.18)$$

of Σ and Λ states appropriate to the hypothesis of a universal pion-hyperon coupling, advanced by Gell-Mann[20] and by Schwinger.[21] The interaction (2.17) appears naturally on the hypothesis that the strangeness nonconserving current J_μ' consists of the terms

$$J_\mu' = f^{\frac{1}{2}}(\bar{Z}\gamma_\mu(1+\gamma_5)N+\bar{\Xi}\gamma_\mu(1+\gamma_5)Y), \qquad (2.19)$$

each of which is a spinor under isotopic spin rotations. The hypothesis of a $T=\frac{1}{2}$ form for the strangeness nonconserving current has been put forward especially by Okubo *et al.*[22]

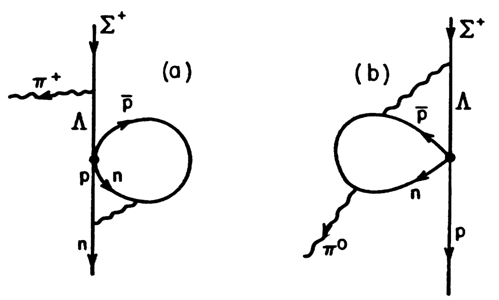

FIG. 3. Some of the lowest order graphs which lead to the Σ^+ decay processes through the four-fermion interaction $(\bar{Z}N)(\bar{N}N)$.

[20] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).
[21] J. S. Schwinger, Ann. Phys. **2**, 407 (1957).
[22] Okubo, Marshak, Sudarshan, Teutsch, and Weinberg, Phys. Rev. **112**, 665 (1958).

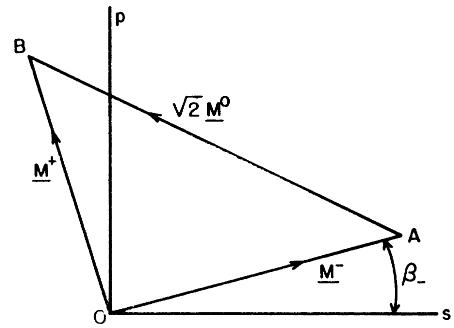

FIG. 4. The Gell-Mann—Rosenfeld diagram for representation of the matrix-elements for Σ^+ and Σ^- pionic decay modes.

The interaction current (2.19) is a $T=\frac{1}{2}$ form which corresponds to the rule $\Delta Q/\Delta s=+1$, whose empirical basis has been discussed recently by Gell-Mann and Feynman.[1] The main argument in support of this rule is the absence of evidence for $\Delta s=\pm 2$ decay interactions of strength comparable to those observed for $\Delta s=\pm 1$, since the combination of two interaction currents with $\Delta Q/\Delta s=+1$ and $\Delta Q/\Delta s=-1$ would generate a four-fermion interaction of amplitude $\sim g$ which gives rise to processes $\Delta s=\pm 2$. In accord with this rule, expression (2.19) does not include any term corresponding to decay of the Σ^+ particle. This means that the beta-decay process $\Sigma^+ \rightarrow n+e^+ +\bar{\nu}$ is forbidden with these interactions (however the beta decay $\Sigma^+ \rightarrow \Lambda+e^+ +\bar{\nu}$ is still an allowed process). The pionic processes (2.15) of Σ^+ decay are forbidden in the lowest approximation corresponding to the graphs of Fig. 1 for Λ decay, but take place through processes involving pionic radiative corrections, examples of which are shown in Fig. 3. The $\Sigma^- \rightarrow n+\pi^-$ decay, on the other hand, can take place already through the lowest order process analogous to that shown for Λ decay in Fig. 1.

For the phenomenological discussion of the decay processes (2.15), Gell-Mann and Rosenfeld[5] point out that it is convenient to represent each of the decay amplitudes for these processes as a vector $\mathbf{M}$ in an (s,p) space. With the assumption of time-reversal invariance, the smallness of $s_{\frac{1}{2}}$ and $p_{\frac{1}{2}}$ pion-nucleon phase shifts implies that the vectors $\mathbf{M}^+$, $\mathbf{M}^0$, $\mathbf{M}^-$ representing the processes (2.15) may be taken to be real. The existence of a $\Delta T=\frac{1}{2}$ rule would imply the following relationship between these real vectors,

$$\mathbf{M}^- = \mathbf{M}^+ + \sqrt{2}\mathbf{M}^0. \qquad (2.20)$$

The decay rates observed for the three processes (2.15) are very nearly equal, which requires about equal magnitudes for M^-, M^+, and M^0. This requires that the vectors $\mathbf{M}^-$, $\mathbf{M}^+$, and $\sqrt{2}\mathbf{M}^0$ should form a right-angled triangle, as in Fig. 4. Carrying through the same lowest order calculation for Σ^- decay on the basis of the interaction (2.16) as was done for Λ decay leads to a value $p/s \sim 0.9$ for M^-, which would correspond to an

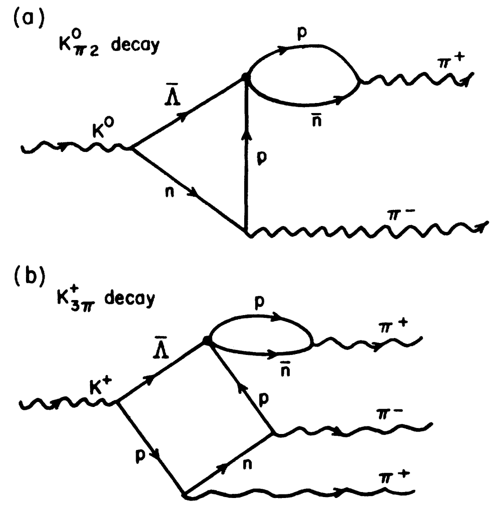

FIG. 5. Graphs illustrating the complexity of $K_{\pi 2}$ and $K_{3\pi}$ decay processes occurring through an elementary Fermi interaction.

angle $\beta_- \sim 40°$. As remarked, there is no corresponding lowest order calculation for Σ^+ decay. However the $\Delta T=\frac{1}{2}$ rule would then require that β_+ should have the value $-50°$ or $130°$, and that $\mathbf{M}^0$ should be parallel to either s or p axis.

The observations of Cool *et al.*[23] on up-down asymmetry in Σ^+ and Σ^- decay are completely opposite to these predictions, for they found that evidence for a polarization effect appears only in the (π^0+p) mode (2.15b), no polarization effect being observed for the $(\pi^+ + n)$ mode nor for Σ^- decay. Their observations are not inconsistent with the $\Delta T=\frac{1}{2}$ rule but imply that the triangle of Fig. 4 should have its sides OA, OB quite close to the two axes, with $\mathbf{M}^0$ at an angle of order $\pm 45°$ to the axes. This failure of the lowest order calculation for Σ^- decay to agree with the data certainly casts doubt on its relevance in the case of Λ decay; there is certainly no reason to believe that the pionic radiative corrections should not have a large effect and the degree of agreement noted above for Λ decay may well be fortuitous. Alternatively, it may be that the $\Delta T=\frac{1}{2}$ rule does not hold here and that no up-down asymmetry has been observed for Σ^- decay because the relevant Σ^- production processes happen to give little polarization; if $\Delta T=\frac{3}{2}$ transitions are also effective, the present data certainly allow many other interpretations.

In terms of the interaction (2.19), $K_{\pi 2}$ and $K_{\pi 3}$ decay modes occur through even more complicated sequences of virtual processes, such as those of Fig. 5. With this situation, it is reasonable to expect the ratio of the matrix elements for $K_{\pi 2}{}^0$ and $K_{3\pi}{}^+$ decays to be of the order of magnitude $1/M$, M being the nucleon mass. Taking into account the ratio of phase space for 2π and 3π systems, the observed ratio $P(\theta_1{}^0)/P(\tau^+)=3\times 10^3$ is in reasonable accord with this expectation. The situation for $K_{\pi 2}{}^+$ decay appears exceptional and provided the first evidence suggesting the possibility of an isotopic spin selection rule in strange particle decays, the topic discussed in the next section.

3. ISOTOPIC SPIN RELATIONSHIPS IN STRANGE PARTICLE DECAY

The possibility of a $\Delta T=\frac{1}{2}$ selection rule for strange particle decays was first suggested by the empirical data, especially by the large ratio between the partial lifetimes for the $K_{\pi 2}{}^+$ and $K_1{}^0$ decays. More recently, the possibility of isotopic spin relationships in weak decay processes has been considered in connection with the isotopic spin character of the weak interaction currents. Gell-Mann and Feynman[1] have suggested that the strangeness-conserving current involving strongly interacting particles may bear a close relationship to the $m_T=1$ component of the isotopic spin vector for these particles; in fact, the current $(\bar{p}n)$ of expression (1.2) is already the $m_T=1$ component of a $T=1$ operator. On the other hand the strangeness nonconserving current (2.19) has a spinor character in isotopic spin. This would mean that the interaction (1.1) formed from $(\bar{Z}N)$ and $(\bar{p}n)$ would be a combination of $T=\frac{1}{2}$ and $\frac{3}{2}$ terms, so that $\Delta T=\frac{1}{2}$ and $\Delta T=\frac{3}{2}$ interactions would contribute to pionic decay processes for strange particles. On the other hand, the leptonic modes, which would result from combinations such as $(\bar{Z}N)(\bar{\nu}e)$, would allow only an isotopic spin change $\Delta T=\frac{1}{2}$, as far as concerns the strongly interacting particles among the decay products. The empirical evidence bearing on these possibilities is in the following.

$K_{\pi 2}$ Decay

A strict $\Delta T=\frac{1}{2}$ rule would forbid the $K_{\pi 2}{}^+$ decay since the final pions have even angular momentum and therefore isotopic spin $T=2$ ($T=0$ being excluded since the system has nonzero charge). Existence of $K_{\pi 2}{}^+$ decay implies the presence of some $\Delta T=\frac{3}{2}$ interaction, with an amplitude of about 4% of the $\Delta T=\frac{1}{2}$ amplitude leading to $K_1{}^0$ decay. Presence of this $\Delta T=\frac{3}{2}$ amplitude will also affect the $K_1{}^0$ branching ratios; a strict $\Delta T=\frac{1}{2}$ rule implies the value 0.33 for the ratio of decay probabilities,

$$r_0=\frac{R(K_1{}^0\to\pi^0+\pi^0)}{R(K_1{}^0\to\pi^0+\pi^0)+R(K_1{}^0\to\pi^++\pi^-)}. \quad (3.1)$$

Inclusion[24,25] of a $\Delta T=\frac{3}{2}$ amplitude which accounts for the $K_{\pi 2}{}^+$ partial lifetime allows this ratio r_0 to be between the limits

$$0.33\{1\pm 4(2T_+/3T_0)^{\frac{1}{2}}\}, \quad (3.2)$$

[23] Cool, Cork, Cronin, and Wenzel, Bull. Am. Phys. Soc. Ser. II, 4, 83 (1959).

[24] M. Gell-Mann, Nuovo cimento **5**, 758 (1957).

[25] R. H. Dalitz, Proc. Phys. Soc. (London) **A69**, 527 (1956).

where T_+ and T_0 are the $K_{\pi 2}{}^+$ and $K_1{}^0$ partial lifetimes, that is between 0.29 and 0.37. The empirical value of this ratio is still rather uncertain. The observation of γ rays from the neutral $K_1{}^0$ mode leads to the value 0.14 ± 0.06, whereas the observation of Λ particles produced in the π^-+p reaction, but unaccompanied by $K_1{}^0$ decay in the charged mode, leads to the value† 0.30 ± 0.06.

Λ and Σ Decay

The branching ratio $R(\Lambda\to p+\pi^-)/(R(\Lambda\to p+\pi^-)+R(\Lambda\to n+\pi^0))$ is observed to have the value 0.63 ± 0.03 in close agreement with expectation ($\frac{2}{3}$) from $\Delta T=\frac{1}{2}$. As emphasized by Okubo *et al.*,[22] this agreement could also be fortuitous, since there is a certain combination of final $T=\frac{1}{2}$ and $\frac{3}{2}$ states which also corresponds to the observed ratio.

The Σ-decay situation was discussed in the foregoing. The present evidence is compatible with the $\Delta T=\frac{1}{2}$ rule but provides no strong argument for it.

$K_{3\pi}$-Decay Modes

From the spectrum observed for τ^+ decay,[26] it is reasonable to conclude that the final 3π state is predominantly symmetrical between the three pions; this is also the theoretical expectation for the decay of a K particle of spin zero.[26] There are only two totally symmetric configurations for three pions, one with $T=1$ and the other with $T=3$. If the amplitudes of these states in $K_{3\pi}$ decay are denoted by I_1 and I_3, the ratio of the τ' and τ modes in K^+ decay is then given by[25]

$$R(\tau')/R(\tau)=1.295(I_1-2I_3)^2/(2I_1+I_3)^2. \quad (3.3)$$

The observed ratio 0.32 ± 0.05 obtained from the present data[27] agrees well with the value 0.325 expected for a pure $T=1$ state; however there is also a nonzero solution for I_3/I_1 which gives the observed value. Accepting the solution $I_3/I_1\approx0$, this agreement with experiment still does not provide any support for the $\Delta T=\frac{1}{2}$ hypothesis, since both $\Delta T=\frac{1}{2}$ and $\Delta T=\frac{3}{2}$ interactions can reach only the $T=1$ state. The experimental ratio will be attained with any interaction which produces pions predominantly in a state of total symmetry and which allows only $\Delta T=\frac{1}{2}$ and $\Delta T=\frac{3}{2}$.

However this situation does not hold for the comparison of the partial lifetimes for 3π decay of the K^+ and $K_2{}^0$ mesons. With the $\Delta T=\frac{1}{2}$ rule these partial lifetimes should be equal[25,28] and $\frac{2}{5}$ of the $K_2{}^0\to3\pi$ decay processes should be of the mode $(\pi^++\pi^-+\pi^0)$. With the known decay probability $R(K_{3\pi}{}^+)=6.0\times10^6$ sec^{-1}, the $\Delta T=\frac{1}{2}$ rule therefore predicts that this mode should have decay probability

$$R(K_2{}^0\to\pi^++\pi^-+\pi^0)=2.4\times10^6\ \text{sec}^{-1}. \quad (3.4)$$

In contrast, the empirical lifetime of the $K_2{}^0$ meson is $8.1^{+3.2}_{-2.4}\times10^{-8}$ sec and it is stated by Bardon *et al.*[29] that not more than 15% of the $K_2{}^0$ decay events giving charged particles can represent the $(\pi^++\pi^-+\pi^0)$ mode. This corresponds to the empirical statement

$$R(K_2{}^0\to\pi^++\pi^-+\pi^0)<1.8(\pm0.6)\times10^6\ \text{sec}^{-1}, \quad (3.5)$$

which certainly provides a considerable overestimate for the empirical value of this decay probability since it includes events for which this interpretation is doubtful and omits the possibility of unobserved neutral modes of decay. As a result, there is some degree of disagreement at this point which may provide evidence of the influence of $\Delta T=\frac{3}{2}$ interactions on this decay mode. If I_1 and I_1' denote the contributions of $\Delta T=\frac{1}{2}$ and $\Delta T=\frac{3}{2}$ interactions, respectively, to the final $T=1$ state, the amplitude of the $T=1$ configuration is $(I_1-\frac{1}{2}I_1')$ for $K_{3\pi}{}^+$ decay, but (I_1+I_1') for $K_2{}^0\to3\pi$ decay. Generally, assuming a totally symmetric state for the final three pions, the ratio of all 3π modes for $K_2{}^0$ and K^+ particles is

$$\frac{R(K_2{}^0\to3\pi)}{R(K^+\to3\pi)}=\frac{(I_1+I_1')^2+\frac{3}{2}I_3{}^2}{(I_1-\frac{1}{2}I_1')^2+I_3{}^2}. \quad (3.6)$$

With $I_3=0$ (absence of $\Delta T=\frac{5}{2}$ interactions), this ratio can fall to a value of 0.5 for a $\Delta T=\frac{3}{2}$ admixture as small as $I_1'/I_1=-0.2$. The ratio $(3\pi^0)/(\pi^++\pi^-+\pi^0)$ for $K_2{}^0$ decay should still be 1.5, and it would be of interest to have this prediction of the $K_2{}^0\to3\pi^0$ decay rate checked, for example by exposure of a dense bubble chamber in a $K_2{}^0$ beam.

Leptonic Modes

Here the hypothesis is that $\Delta T=\frac{1}{2}$ holds for the strongly interacting products of this decay. In this sense, interactions for which $\Delta T=\frac{1}{2}$ certainly do exist, as shown by the existence of the $K_{\mu2}{}^+$ mode, for which the only strongly-interacting particle taking part is the K^+ meson itself; the question is whether these are the only interactions. At present the main physical consequence of this rule is a relation between the amplitudes for $(\pi\mu\nu)$ or $(\pi e\nu)$ decay for K^+ and K^0

† *Note added in proof.*—Crawford, Cresti, Douglass, Good, Kalbfleisch, Stevenson, and Ticho [Phys. Rev. Letters **2**, 266 (1959)] have now obtained the value 0.32 ± 0.04. Their observation of γ rays from three $K_1{}^0$ decay events raises the former value to 0.18 ± 0.05, still rather low.

[26] See R. H. Dalitz, Repts. Progr. in Phys. **20**, 163 (1957).

[27] Birge, Perkins, Peterson, Stork, and Whitehead, Nuovo cimento **4**, 834 (1956); Alexander, Johnston, and O'Ceallaigh, Nuovo cimento **6**, 478 (1957).

[28] This has also been pointed out by Okubo and Sudarshan (private communication). See also A. Pais and S. B. Treiman, Phys. Rev. **106**, 1106 (1957).

[29] Bardon, Fuchs, Lande, Lederman, Chinowsky, and Tinlot, Phys. Rev. **110**, 780 (1958); Bardon, Lande, Lederman, and Chinowsky, Ann. Phys. (N. Y.) **5**, 156 (1958).

states, namely

$$M(K^0 \to \pi^- + \mu^+ + \bar{\nu}) = \sqrt{2} M(K^+ \to \pi^0 + \mu^+ + \bar{\nu}), \quad (3.7)$$

and a similar relation for the electron modes. Okubo *et al.*[22] point out that, irrespective of whether or not time-reversal invariance holds for these interactions, this result implies a definite relation between the K_2^0 and K^+ decay rates

$$R(K_2^0 \to \pi^\pm + \mu^\mp + \nu) = 2R(K^+ \to \pi^0 + \mu^+ + \bar{\nu}), \quad (3.8)$$

and a similar relation for the electron modes. With the decay rates 3.4×10^6 sec^{-1} and 3.3×10^6 sec^{-1} for the $K_{\mu 3}^+$ and K_{e3}^+ modes, as given by Gell-Mann and Rosenfeld,[5] a total decay probability of 13.5×10^6 sec^{-1} is therefore predicted for the leptonic modes of K_2^0 decay. Since there are observed some $K_2^0 \to 3\pi$-decay events (the $\Delta T = \frac{1}{2}$ rule would require a total of 6.0×10^6 sec^{-1} for charged and neutral modes) in addition to these leptonic modes, the observed K_2^0 decay probability, given as $12.3_{-3.5}^{+5.2} \times 10^6$ sec^{-1} by Bardon *et al.*,[29] is somewhat lower than these remarks would suggest. However this does not necessarily vitiate the $\Delta T = \frac{1}{2}$ rule for the leptonic modes for it may simply indicate some degree of cancellation between I_1 and I_1' for the 3π modes of K_2^0 decay. The empirical lower limit for $K_2^0 \to (\pi^+ + \pi^- + \pi^0)$ events is 5% of all charged modes, and the addition of 7.5% for $K_2^0 \to 3\pi^0$ modes (required if $T=1$ holds for the final 3π state) would then bring the predicted probability only up to 15.3×10^6 sec^{-1}, well within the experimental errors of the observed value.

To sum up, the only evidence pointing strongly to the $\Delta T = \frac{1}{2}$ selection rule consists of the long partial lifetime for the $K_{\pi 2}^+$ decay and the $(\pi^- + p)/(\pi^0 + n)$ branching ratio in Λ decay. In these cases, further tests of the $\Delta T = \frac{1}{2}$ rule will be possible in the comparison of the polarization properties of the two Λ-decay modes, and in a definitive measurement of the branching ratio in K_1^0 decay. On the other hand, none of the data at present available offers any strong conflict with the requirements of the $\Delta T = \frac{1}{2}$ rule, and all of the data is in accord with the hypothesis of a $T = \frac{1}{2}$ weak interaction current associated with the strangeness nonconserving decays.

4. LEPTONIC MODES OF STRANGE PARTICLE DECAY

K-Meson Decays

The existence of a four-fermion coupling involving a hyperon-nucleon term and a leptonic term, of the same strength whether the lepton be electron or muon, and consisting only of V and A terms, is at present in qualitative accord with the evidence on leptonic modes of K decay.

First, such an interaction accounts very naturally for the absence of evidence for K_{e2}^+ decay. The foregoing assumptions lead to an interaction of the form

$$C_K [\bar{\psi}_\nu \gamma_\mu (1+\gamma_5) \psi_L] \partial \phi_K / \partial x_\mu = C_K m_L (\bar{\psi}_\nu (1+\gamma_5) \psi_L) \phi_K \quad (4.1)$$

for K_{L2} decay, which implies a $K_{e2}^+/K_{\mu 2}^+$ ratio of 2.5×10^{-5}. This corresponds to the prediction $\pi_{e2}^+/\pi_{\mu 2}^+ = 1.25 \times 10^{-4}$ for the pion, now borne out by experiment, and is to be compared with the experimental upper limit, $K_{e2}^+/K_{\mu 2}^+ < 0.01$, at present.

The decay probability for the $K_{\mu 2}^+$ mode (48×10^6 sec^{-1}) is only about 20% greater than that for $\pi_{\mu 2}^+$ decay, despite the much greater energy release. These lifetimes correspond to a ratio $(C_K/C_\pi)^2 \sim 1/15$. However, the relation between C_K and C_π is not necessarily simple but may depend on the following factors:

(a) The parity of the K meson (relative to even parity for the Λ particle). The expressions for the intermediate baryon loops between the initial meson and the point at which the four-fermion weak interaction is effective are quite different for a scalar K meson from those for π-meson decay. Even for a pseudoscalar K meson, the graphs for π-meson and K-meson leptonic decays do not generally correspond in detail unless the Λ and Σ particles have the same parity, their virtual K-meson interactions are neglected and their pion interactions have the global symmetry, and their weak interactions are closely correlated in a similar manner.

(b) The amplitude $f^{\frac{1}{2}}$ of the strangeness nonconserving current need not be the same as that for the strangeness conserving current. The radiative corrections to the former current have quite different structure from those for the latter, so that, even if the weak interaction current strengths are the same in the approximation where the strong pion and K-meson interactions are turned off, these coupling strengths may be modified by different effective renormalizations. In fact, the experimental data on the beta decay of the hyperons (see the following) require that f should be of an order of magnitude smaller than g.

(c) The K-hyperon coupling strength $G_K^2/\hbar c$ appears to be about an order of magnitude weaker than the pion-nucleon coupling strength $G^2/\hbar c \simeq 13.5$.

Consider now the three-body leptonic modes $K_{\mu 3}$ and K_{e3}. Assuming only V and A interactions to be effective, the matrix elements for these processes may generally be written in the form‡

$$\{R p_{K\alpha} + S(p_{K\alpha} - p_{\pi\alpha})\} (\bar{\psi}_\nu \gamma_\alpha (1+\gamma_5) \psi_L) = \bar{\psi}_\nu \{R m_K \gamma_0 (1+\gamma_5) + S m_L (1+\gamma_5)\} \psi_L. \quad (4.2)$$

R and S are scalar functions of $p_K \cdot p_\pi / M^2 = m_K \omega_\pi / M^2$, which are independent of m_L. If time-reversal invariance

‡ *Note added in proof.*—See, for example, A. Pais and S. B. Treiman, Phys. Rev. **105**, 1616 (1957), R. Gatto, Phys. Rev. **111**, 1426 (1958), and references cited there. A recent preprint, "Decay of Hyperons and Mesons from the Universal Fermi Interaction," by A. Fujii and M. Kawaguchi is also relevant.

holds for both strong and weak interactions, the ratio R/S will be real.[30] In K_{e3} decay, the term proportional to m_L may be neglected and the only contribution is from the R term of (4.2). Since the variation of $p_K \cdot p_\pi/M^2$ over the allowed range in K_{L3} decay is only about 0.05 it appears a reasonable first approximation to neglect the energy dependence of R, in which case the shape predicted for the electron spectrum is unique. This spectrum[31] is shown in Fig. 6, where comparison is made with the available data[27,32] on K_{e3}^+ decay. The agreement is rather poor, but the data are subject to experimental bias whose effect is difficult to estimate. If the hypothesis of a $T=\frac{1}{2}$ weak interaction current is valid, the expression (4.2) holds for both $K_{e3}^\pm$ decay and for the K_2^0 modes $(e^\pm+\nu+\pi^\mp)$.

In $K_{\mu3}$ decay, both R and S terms contribute and there are therefore a range of theoretical possibilities for the $K_{\mu3}$ spectrum, of which two examples[31] are shown in Fig. 7. This figure also shows the data obtained on $K_{\mu3}^+$ decay in the emulsion investigation of Alexander *et al.*[27] who attempted to assess the effects of various empirical biases on this distribution and who conclude that these are so many and so little understood that no detailed comparison with the theoretical distributions is justified at present. With neglect of the energy variation of R and S, the ratio of the decay probabilities

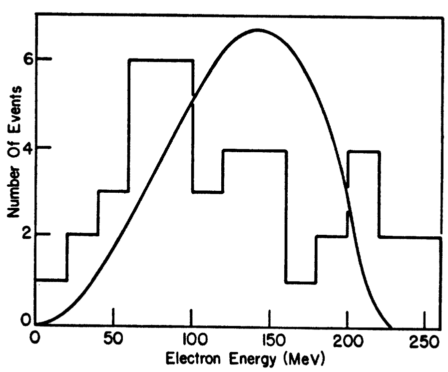

FIG. 6. The electron energy spectrum in K_{e3}^+ decay, compared with the theoretical prediction.

[30] In the literature, it has frequently been stated that this ratio will be real as long as time-reversal invariance is valid for the strong interactions alone. When the strong interactions have this property, this conclusion is generally correct only if the K_{L3} mode arises from just one four-fermion interaction [for example, $(\bar{p}\Lambda)(\bar{\nu}L)$]. When there are several such four-fermion interactions, for example $(\bar{p}\Lambda)(\bar{\nu}L)$ and $(\bar{\Lambda}\Xi)(\bar{\nu}L)$, the K_{L3}-decay amplitude will consist of several coherent terms, thus

$$K^+ \to \left\{\begin{matrix}\bar{\Lambda}+p \\ \bar{\Xi}^-+\Lambda\end{matrix}\right\} \to L^+ + \bar{\nu} + \pi^0,$$

and reality will hold for R/S only if the ratio of the coupling coefficients of these four-fermion interactions is also real; that is, if the weak interaction also satisfies time-reversal invariance.

[31] These K_{e3} and $K_{\mu3}$ spectra are taken from the paper by Furuichi, Kodama, Ogawa, Sugahara, Wakasa, and Yonezawa, Progr. Theoret. Phys. 17, 89 (1957).

[32] Bruin, Holthuizen, and Jongejans, Nuovo cimento 9, 422 (1958), have collected together the data obtained with the same selection criteria.

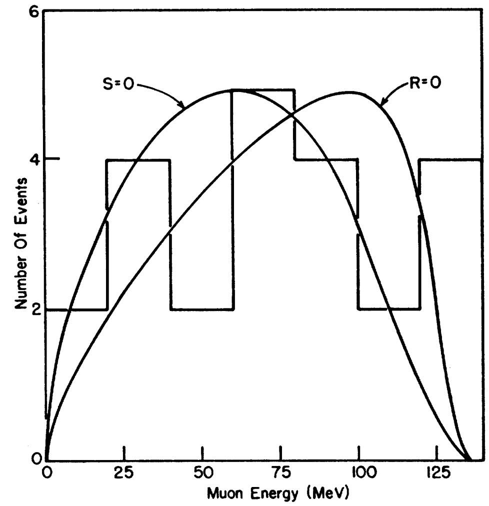

FIG. 7. The muon energy spectrum for $K_{\mu3}^+$ decay according to Alexander *et al.*,[27] and two examples of the $K_{\mu3}^+$ spectrum predicted by the interaction (4.2).

for K_{e3} and $K_{\mu3}$ decay is

$$R(K_{e3})/R(K_{\mu3}) = R^2/(0.80R^2+0.33RS+0.075S^2). \quad (4.3)$$

This ratio is limited to values less than 2.3, which does not disagree with the present empirical ratio of about unity. The ratio of the decay probabilities for K_{e3}^+ and $K_{\mu2}^+$ modes is about 0.07, which corresponds to a value $R/C_K \sim 2/(3M)$, where M is the nucleon mass and C_K the coefficient in expression (4.1). This ratio is therefore in reasonable accord with qualitative expectation on the basis of the Fermi coupling model, since the additional pion is then emitted from the intermediate baryon pairs and the effective radius of the system is $\hbar/Mc$.

Hyperon Decays

One of the most direct consequences of Gell-Mann's tetrahedral scheme of four-fermion interactions is the prediction of a beta-decay process for the Λ and Σ^- hyperons, arising from the couplings $(\bar{\Lambda}p)(\bar{e}^+\bar{\nu})$ and $(\bar{\Sigma}^-n)(\bar{e}^+\bar{\nu})$ generated by expressions (1.1) and (1.2). With the strength $f^{\frac{1}{2}}$ of the interaction currents (2.19) equal to $g^{\frac{1}{2}}$, the expectation is that the beta decay and muon decay of the Λ particle,

$$\Lambda \to \begin{cases} p+e^-+\nu, & (4.4\text{a}) \\ p+\mu^-+\nu, & (4.4\text{b}) \end{cases}$$

should have rates 0.8% and 0.15%, respectively, of the total Λ-decay rate, and that Σ^- beta decay and muon decay

$$\Sigma^- \to \begin{cases} n+e^-+\nu, & (4.5\text{a}) \\ n+\mu^-+\nu, & (4.5\text{b}) \end{cases}$$

should have rates 5.7% and 2.5% that for normal Σ^-

decay. Recently, several events of the type (4.4a) have been reported,[6] but their rate appears to be significantly smaller than these expectations. Including an earlier Σ event[33] probably representing the mode (4.5a), or possibly (4.5b), the present situation is that, with $f=g$, 12 Λ_e events and 2.5 Λ_μ events would have been expected in the experiments to date compared with the observation of 2 Λ_e events and no Λ_μ events, and that 11 Σ_e^- events and 5 Σ_μ^- events would have been expected compared with one Σ_μ or Σ_e event. These results are compatible with a four-fermion coupling based on the interaction (2.19) only for a value $f^{\frac{1}{2}}/g^{\frac{1}{2}}\sim 0.3$. The calculation of leptonic decay probabilities for K mesons in terms of an elementary four-fermion interaction involves divergences and many other uncertainties, so that the conclusion that the strangeness nonconserving interaction current is weaker than the strangeness conserving current by a factor of order 3 does not conflict with any evidence on K-meson decay.

5. TIME-REVERSAL INVARIANCE FOR WEAK INTERACTIONS

It has frequently been assumed that the weak interactions are invariant under time-reversal. In the present framework, this may be expressed as the assumption that the interaction constants for each term in the interaction current (1.2) may all be chosen to have the same phase.

There is very little direct information available at present on the validity of this assumption. In Λ decay, if the amplitudes s_- and p_- had relative phase ϕ, the expression for the polarization parameter α would have an additional factor $\cos\phi$. The experimental limitation (2.2b) shows that the angle ϕ cannot deviate by more than $\sim 45°$ from 0 or π, the values allowed with time-reversal invariance (neglecting the small pion-nucleon scattering phase shifts). No test is possible from the study of $K_{\mu 2}$-, K_{e3}-, or τ-decay modes. For $K_{\mu 3}^+$ decay, Sakurai[34] has pointed out that the violation of time-reversal for the weak interactions would generally imply that the muon polarization would generally have a component perpendicular to the (π^0,ν) plane. This possibility has yet to be examined experimentally. However, Sakurai's formulas show that, when the $K_{\mu 3}$ interaction is limited to the form (4.2), the existence of a relative phase between the coefficients R and S, which can arise if the strangeness nonconserving current (2.19) is not invariant under time-reversal, does not imply a normal component for the muon polarization.

Weinberg[35] has suggested recently that the most severe test of time-reversal at present may be the absence of 2π modes for K_2^0 decay. As pointed out by Lee *et al.*,[36] the K_1^0, K_2^0 states are generally expressible in terms of the K^0, $\bar{K}^0$ states by the relations

$$|K_1^0\rangle = p|K^0\rangle + q|\bar{K}^0\rangle, \tag{5.1a}$$

$$|K_2^0\rangle = q|K^0\rangle - p|\bar{K}^0\rangle, \tag{5.1b}$$

where p and q are generally complex numbers and $|p|^2+|q|^2=1$. If time-reversal invariance holds, which means that CP invariance is valid, then $p=q$ and both may be chosen real. Weinberg remarked that, for $j=0$, there are two final 2π states, one with $T=0$, the other $T=2$. If δ_0 and δ_2 denote the s-wave scattering phases for the pion-pion systems of $T=0$ and $T=2$, then the amplitudes for 2π decay of the K^0 system may be written

$$\langle K^0|2\pi, T=0\rangle = a_0 e^{i\delta_0}, \quad \langle K^0|2\pi, T=2\rangle = a_2 e^{i\delta_2}. \tag{5.2}$$

The corresponding amplitudes for $\bar{K}^0$ decay are directly related to these,

$$\langle \bar{K}^0|2\pi, T=0\rangle = a_0^* e^{i\delta_0}, \quad \langle \bar{K}^0|2\pi, T=2\rangle = a_2^* e^{i\delta_2}. \tag{5.3}$$

From these and Eqs. (5.1), the amplitudes for K_1^0 and K_2^0 decay to $2\pi^0$ and $\pi^+ + \pi^-$ states may then be deduced, for example,

$$\langle K_2^0|2\pi^0\rangle = \sqrt{\tfrac{1}{3}}(qa_0 - pa_0^*)e^{i\delta_0} + \sqrt{\tfrac{2}{3}}(qa_2 - pa_2^*)e^{i\delta_2}, \tag{5.4a}$$

$$\langle K_2^0|\pi^+ + \pi^-\rangle = \sqrt{\tfrac{2}{3}}(qa_0 - pa_0^*)e^{i\delta_0} - \sqrt{\tfrac{1}{3}}(qa_2 - pa_2^*)e^{i\delta_2}. \tag{5.4b}$$

From the experimental work of Bardon *et al.*,[29] the decay probability for $K_2^0 \to \pi^+ + \pi^-$ is known to be less than 10^{-5} that for K_1^0 decay, so that

$$|\langle K_2^0|\pi^+ + \pi^-\rangle| \lesssim 0.3\times 10^{-2}\{|pa_0 + qa_0^*|^2 + |pa_2 + qa_2^*|^2\}^{\frac{1}{2}}. \tag{5.5a}$$

An upper limit for the decay probability $K_2^0 \to \pi^0 + \pi^0$ is not as well known. It is certainly less than 10^{-3} of the K_1^0-decay probability, and Weinberg[35] gives an argument that this ratio is actually less than 2×10^{-4}. From this

$$|\langle K_2^0|\pi^0 + \pi^0\rangle| \lesssim 1.5\times 10^{-2}\{|pa_0 + qa_0^*|^2 + |pa_2 + qa_2^*|^2\}^{\frac{1}{2}}. \tag{5.5b}$$

If the right-hand sides of these inequalities were zero, these conditions would require, according to (5.4), that

$$a_0^*/a_0 = q/p = a_2^*/a_2, \tag{5.6}$$

and that a_0 and a_2 have the same phase, just the relationship which time-reversal invariance for the interaction $K^0 \to 2\pi$ would require.

However, owing to the uncertainty in the K_1^0 branching ratio, it is not clear exactly what restriction on the relative phases of a_2 and a_0 is implied by the empirical inequalities (5.5). If the $\Delta T=\frac{1}{2}$ rule held exactly, then the amplitude a_2 would be zero; the

[33] J. Hornbostel and E. O. Salant, Phys. Rev. **102**, 502 (1956).
[34] J. J. Sakurai, Phys. Rev. **109**, 980 (1958).
[35] S. Weinberg, Phys. Rev. **110**, 782 (1958).
[36] Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957).

inequalities (5.5) would then imply only that $a_0^*/a_0 = q/p$, a statement about the phase of a_0 which carries no implications concerning the time-reversal invariance of the decay interaction. If the ratio of $|(pa_2+qa_2^*)|$ and $|(pa_0+qa_0^*)|$ is taken from the ratio of the $K_{\pi 2}^+$ and K_1^0 decay probabilities, then it is still true that the phases of a_0^{*2} and q/p are very close; however the limitation on the phase of a_2 is given by

$$\left|\frac{qa_2-pa_2^*}{pa_2+qa_2^*}\right| \lesssim 0.014 \left|\frac{pa_0+qa_0^*}{pa_2+qa_2^*}\right| \sim \frac{1}{3}, \qquad (5.7)$$

which involves almost no restriction on the relative phase of a_2^{*2} and q/p. This argument would lead to a significant restriction on the relative phase of a_2 and a_0 if the present upper limit for the $K_2^0 \rightarrow \pi^0+\pi^0$ decay probability were improved by an order of magnitude or if the K_1^0 branching ratio were confirmed to lie in the range 0.1–0.2, as indicated by some of the present experiments.

So far, the only serious test of the assumed property of time-reversal invariance for the weak interactions has been provided by the experiments on the beta decay of polarized neutrons reported recently by Clark *et al.*[37] and by Burgy *et al.*[38] These experiments have shown that the Fermi and Gamow-Teller matrix-elements for neutron decay do not differ in phase by more than $\pm 8°$. For the strange particle decays, the only tests which have been available to the present have been rather inconclusive or have provided only very weak evidence in support of this property for the strangeness non-conserving weak interactions.

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[37] Clark, Robson, and Nathans, Phys. Rev. Letters 1, 100 (1958).

[38] Burgy, Krohn, Novey, Ringo, and Telegdi, Phys. Rev. Letters 1, 324 (1958).