Theory of Muon Capture*

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THE present paper deals with various aspects of the theory of muon capture with emphasis on the relation between theory and experiment. The theory is based on an effective Hamiltonian, $H_{\text{eff}}^{(\mu)}$, which describes muon capture with subsequent neutrino emission by an aggregate of A dressed nucleons:

$$\mu^- + \begin{Bmatrix} Z \text{ protons} \\ A-Z \text{ neutrons} \end{Bmatrix} \to \nu + \begin{Bmatrix} Z-1 \text{ protons} \\ A-Z+1 \text{ neutrons} \end{Bmatrix}.$$

The treatment is subdivided as follows:

1. Effective Hamiltonian.
2. Total Muon Capture Rate in Closure Approximation—"Isotope" Effect.
3. Comparison of Closure Approximation Expression for Total Muon Capture Rate with Experiment.
4. Muon Capture to Particular Final States of Daughter Nucleus.
5. "Hyperfine" Effect in Muon Capture Rate—Muon Capture in Hydrogen.
6. Radiative Muon Capture: Total Rate and Photon-Neutrino Angular Correlation.
7. Parity Non-conservation Effects:

(a) Angular Distribution of Recoil Nuclei in Capture of Polarized Muons.
(b) Angular Distribution of Photons in Radiative Capture of Polarized Muons.
(c) Polarization of Recoil Nuclei in Muon Capture.
(d) Polarization of Photons in Radiative Muon Capture.

1. EFFECTIVE HAMILTONIAN

The appropriate expression for $H_{\text{eff}}^{(\mu)}$, as discussed in some detail by Fujii and Primakoff,[1] is, in a configuration space representation,

$$H_{\text{eff}}^{(\mu)} = \frac{1}{\sqrt{2}}\tau^{(+)}\frac{(1-\boldsymbol{\sigma}\cdot\mathbf{v}_1)}{\sqrt{2}}\sum_{i=1}^{A}\tau_i^{(-)}\{G_V^{(\mu)}1\cdot 1_i + G_A^{(\mu)}\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_i - G_P^{(\mu)}\boldsymbol{\sigma}\cdot\mathbf{v}_1\boldsymbol{\sigma}_i\cdot\mathbf{v}_1\}\delta(\mathbf{r}-\mathbf{r}_i) \quad (1a)$$

with

$$G_V^{(\mu)} \equiv g_V^{(\mu)}\left(1+\frac{\nu}{2m_p}\right);$$
$$G_A^{(\mu)} \equiv g_A^{(\mu)} - g_V^{(\mu)}(1+\mu_p-\mu_n)\frac{\nu}{2m_p}; \quad (1b)$$
$$G_P^{(\mu)} \equiv [g_P^{(\mu)} - g_A^{(\mu)} - g_V^{(\mu)}(1+\mu_p-\mu_n)]\frac{\nu}{2m_p}$$

and

$$g_V^{(\mu)} \cong g_V^{(\beta)}\times 0.972; \quad g_A^{(\mu)} \cong g_A^{(\beta)}\times 0.999; \quad g_P^{(\mu)} \cong 8g_A^{(\mu)} \cong 8g_A^{(\beta)}. \quad (1c)$$

This $H_{\text{eff}}^{(\mu)}$ corresponds,[1] in a nonrelativistic approximation for the muon and for the nucleons, to the most general Lorentz covariant transition matrix element for the reaction $\mu^- + p \to \nu + n$ in a theory where the lepton-bare nucleon coupling is V and A, where the neutrinos are emitted with unit negative helicity, where time reversal invariance holds, and where any bare hyperon, bare kayon "currents" which interact with the lepton current, have the same transformation property under the charge symmetry operation as the bare nucleon, bare pion currents.[2,3] However we neglect in Eq. (1) "many body" terms in $H_{\text{eff}}^{(\mu)}$ arising from the possibility of exchange of virtual pions, kayons, etc., among the nucleons. Such many body terms depend on the relative space coordinates of pairs, triplets, . . ., of nucleons and are believed, on the basis of a rough analysis of the corresponding beta decay situation, to be relatively small.[1]

In Eqs. (1a) to (1c), $g_V^{(\mu)}$, $g_A^{(\mu)}$, and $g_P^{(\mu)}$ are vector, axial vector and "induced" pseudoscalar muon-dressed nucleon coupling constants effective in muon capture while $g_V^{(\beta)}$ and $g_A^{(\beta)}$ are electron-dressed nucleon vector and axial vector coupling constants effective in beta decay. The numerical relations in Eq. (1c) between $g_V^{(\mu)}$, $g_V^{(\beta)}$; $g_A^{(\mu)}$, $g_A^{(\beta)}$ arise from the assumption of "universality" between the V, A muon-bare nucleon and electron-bare nucleon coupling constants which implies that $g_V^{(\mu)}$, $g_A^{(\mu)}$ differ from $g_V^{(\beta)}$, $g_A^{(\beta)}$ only because of the differing nucleon four-momentum transfers in the muon capture and in the beta decay. The numerical relation in Eq. (1c) between $g_P^{(\mu)}$ and $g_A^{(\mu)}$, due to Goldberger and Treiman[3] and to Wolfenstein,[4] is based on the assumption that a reaction such as: $\mu^- + \pi^+ \to \nu$ takes place predominantly via the sequence of "steps": $\mu^- + \pi^+ \to \mu^- + p + \bar{n} \to \nu$ which implies the possibility of muon capture via the "four-step process": $\mu^- + p \to \mu^- + \pi^+ + n \to \mu^- + p + \bar{n} + n \to \nu + n$. The quantities $\mu_p = 1.793$, $\mu_n = -1.913$ are the proton, neutron (static) anomalous magnetic moments (in units of $e/2m_p$ with e, m_p proton charge, mass and $\hbar = 1$, $c = 1$); these appear in the interaction effective in muon capture [Eq. (1b)] as a consequence of the Gell-Mann–

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[1] A. Fujii and H. Primakoff, Nuovo cimento (to be published).

[2] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

[3] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 355 (1958).

[4] L. Wolfenstein, Nuovo cimento **8**, 882 (1958).

Feynman assumption of a "conserved vector current"[5] which necessitates the existence, for example, of the reaction: $\mu^- + \pi^+ \rightarrow \nu + \pi^0$ and hence implies the possibility of muon capture via the "three-step process": $\mu^- + p \rightarrow \mu^- + \pi^+ + n \rightarrow \nu + \pi^0 + n \rightarrow \nu + n$. Also in Eqs. (1a) and (1b), $\mathbf{v} = \nu \mathbf{v}_1$ is the neutrino momentum; 1, 1_i and $\boldsymbol{\sigma}$, $\boldsymbol{\sigma}_i$ are 2×2 matrix unit operators and spin angular momentum operators for the lepton and the ith nucleon; $\mathbf{r}$ and $\mathbf{r}_i$ are space coordinates of the lepton and the ith nucleon; $\tau^{(+)}$, $\tau_i^{(-)}$ are isobaric-spin operators which transform a lepton muon state into a lepton neutrino state and an ith nucleon proton state into an ith nucleon neutron state; the factor $1/\sqrt{2}$ arises from the normalization of the neutrino relativistic wave function; the factor $(1-\boldsymbol{\sigma}\cdot\mathbf{v}_1)/\sqrt{2}$ is a consequence of the assumption of a maximum parity nonconserving two-component neutrino type muon-neutrino-nucleon coupling $(\sim(\psi_\nu^+[(1+\gamma_5)/\sqrt{2}]\gamma_4\gamma_\lambda\psi_\mu)\times(\psi_n^+\gamma_4\gamma_\lambda\psi_p)$, etc.). Though the muon and the nucleons are treated nonrelativistically in the derivation of $H_{\text{eff}}^{(\mu)}$ all first-order nucleon recoil corrections, i.e., all terms in $H_{\text{eff}}^{(\mu)} \sim \nu/m_p$, are nevertheless included.

2. TOTAL MUON CAPTURE RATE IN CLOSURE APPROXIMATION—"ISOTOPE" EFFECT

With the $H_{\text{eff}}^{(\mu)}$ of Eqs. (1a) to (1c) we can obtain the square of the muon capture transition matrix element, $\langle|\text{M.E.}^{(\mu)}|^2\rangle$, summed over all spin orientations of the neutrino and averaged over all spin orientations of the muon. A straightforward calculation gives

$$\langle|\text{M.E.}^{(\mu)}|^2\rangle = \frac{1}{(2\pi)^3}\left[\frac{1}{\pi}\left(\frac{Zm_\mu'}{137}\right)^3\right]\frac{1}{2} \cdot |\text{M.E.}_{\text{nucl}}^{(\mu)}(a\rightarrow b)|^2, \quad \text{(2a)}$$

where $m_\mu' = m_\mu/[1+(m_\mu/Am_p)]$ is the muon reduced mass in the parent mu-mesic atom and the nuclear matrix element is expressed as

$$\begin{aligned}|\text{M.E.}_{\text{nucl}}^{(\mu)}(a\rightarrow b)|^2 &= (G_V^{(\mu)})^2|\langle b|\textstyle\sum_i \tau_i^{(-)}\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)\varphi(r_i)|a\rangle|^2 \\ &+ (G_A^{(\mu)})^2|\langle b|\textstyle\sum_i \tau_i^{(-)}\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)\varphi(r_i)\boldsymbol{\sigma}_i|a\rangle|^2 \\ &+ [(G_P^{(\mu)})^2 - 2G_A^{(\mu)}G_P^{(\mu)}] \\ &\times|\langle b|\textstyle\sum_i \tau_i^{(-)}\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)\varphi(r_i)\boldsymbol{\sigma}_i\cdot\mathbf{v}_1|a\rangle|^2. \quad \text{(2b)}\end{aligned}$$

In Eqs. (2a) and (2b), $|a\rangle$, $|b\rangle$ represent wave functions of the two nuclear states involved in the capture process; the quantity $\varphi(r_i)$ is the muon space orbital wave function normalized in such a way that $\varphi(r_i)\rightarrow 1$ as $Z\rightarrow 0$, i.e., for small Z,

$$\varphi(r_i) \cong \exp(-Zm_\mu' r_i/137).$$

[5] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); M. Gell-Mann, Phys. Rev. **111**, 362 (1958); J. Bernstein and R. R. Lewis, Phys. Rev. **112**, 232 (1958).

The emitted neutrino momentum, ν_{ba}, is obtained from the energy and momentum conservation laws as

$$\nu_{ba} = m_\mu\left(1 - \frac{\epsilon_a}{m_\mu} - \frac{(E_b - E_a)}{m_\mu}\right)\left(1 - \frac{m_\mu}{2(m_\mu + Am_p)}\right), \quad \text{(2c)}$$

where ϵ_a is the binding energy of the muon in the lowest Bohr orbit of the mu-mesic atom and E_a, E_b are the energies of the nuclear states a, b. Thus, taking proper account of the density of final states available to the emitted neutrino, the total muon capture rate of the parent nucleus in the state $a|\rangle$, $\Lambda^{(\mu)}(a)$, is

$$\Lambda^{(\mu)}(a) = 2\pi\cdot 4\pi\sum_b \int \frac{d\mathbf{v}_1}{4\pi}\langle|\text{M.E.}^{(\mu)}|^2\rangle(\nu_{ba})^2 \times[1+\nu_{ba}/[(\nu_{ba})^2 + (Am_p)^2]^{\frac{1}{2}}]^{-1} \quad \text{(3a)}$$

so that using Eqs. (2a) to (2c)

$$\begin{aligned}\Lambda^{(\mu)}(a) &= Z^3\frac{1}{2\pi^2}\frac{m_\mu^5}{(137)^3}\sum_b(\eta_{ba})^2\int\frac{d\mathbf{v}_1}{4\pi} \\ &\times\{(G_V^{(\mu)})^2|\langle b|\textstyle\sum_i \tau_i^{(-)}\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)\varphi(r_i)|a\rangle|^2 \\ &+ (G_A^{(\mu)})^2|\langle b|\textstyle\sum_i \tau_i^{(-)}\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)\varphi(r_i)\boldsymbol{\sigma}_i|a\rangle|^2 \\ &+ [(G_P^{(\mu)})^2 - 2G_A^{(\mu)}G_P^{(\mu)}] \\ &\times|\langle b|\textstyle\sum_i \tau_i^{(-)}\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)\varphi(r_i)\boldsymbol{\sigma}_i\cdot\mathbf{v}_1|a\rangle|^2\} \quad \text{(3b)}\end{aligned}$$

where

$$(\eta_{ba})^2 \equiv \left(1 - \frac{\epsilon_a}{m_\mu} - \frac{(E_b - E_a)}{m_\mu}\right)^2\left(1 - \frac{m_\mu}{2(m_\mu + Am_p)}\right)^2 \times[1+\nu_{ba}/[(\nu_{ba})^2+(Am_p)^2]^{\frac{1}{2}}]^{-1}\left(1+\frac{m_\mu}{Am_p}\right)^{-3}, \quad \text{(3c)}$$

or, dropping terms in m_μ/Am_p,

$$(\eta_{ba})^2 \cong \left(1 - \frac{\epsilon_a}{m_\mu} - \frac{(E_b - E_a)}{m_\mu}\right)^2 \cong (\nu_{ba}/m_\mu)^2. \quad \text{(3d)}$$

The approximate expression for $(\eta_{ba})^2$ in Eq. (3d), involving neglect of the corrections for daughter nucleus recoil and parent mu-mesic atom reduced mass, holds to better than 5% for $A > 12$.

The sum over b in Eqs. (3a) and (3b) runs over all energetically accessible states of the daughter nucleus, i.e., over all states b for which $E_b \leqq E_a + (m_\mu - \epsilon_a)$. Such energetically accessible states are very numerous since $m_\mu - \epsilon_\mu \approx 100$ Mev and $(E_b)_{\text{ground state}} - E_a$ is normally ≈ 2–3 Mev and never exceeds 15 Mev; in addition, the matrix elements

$$\langle b|\textstyle\sum_i \tau_i^{(-)}\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)\varphi(r_i)|a\rangle,$$
$$\langle b|\textstyle\sum_i \tau_i^{(-)}\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)\varphi(r_i)\boldsymbol{\sigma}_i|a\rangle,$$
$$\langle b|\textstyle\sum_i \tau_i^{(-)}\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)\varphi(r_i)\boldsymbol{\sigma}_i\cdot\mathbf{v}_1|a\rangle$$

are largest for low lying states b. Thus one may have confidence in the accuracy of a closure approximation which extends the sum over all energetically accessible states b to a sum over all states b without restriction, and which replaces the explicitly E_b-dependent quantities, ν_{ba}, η_{ba}, by suitable averages $\langle\nu\rangle_a$, $\langle\eta\rangle_a$, which do not depend explicitly on E_b. Such a closure approximation applied to Eq. (3b) yields

$$\Lambda^{(\mu)}(a)=Z^3(\langle\eta\rangle_a)^2\frac{1}{2\pi^2}\frac{m_\mu{}^5}{(137)^3}\int\frac{d\mathbf{v}_1}{4\pi}\Big\{(G_V{}^{(\mu)})^2\langle a|\sum_{i,j}\tau_i{}^{(+)}\tau_j{}^{(-)}\exp(i\langle\nu\rangle_a\mathbf{v}_1\cdot\mathbf{r}_{ij})\varphi^*(r_i)\varphi(r_j)|a\rangle$$
$$+(G_A{}^{(\mu)})^2\langle a|\sum_{i,j}\tau_i{}^{(+)}\tau_j{}^{(-)}\exp(i\langle\nu\rangle_a\mathbf{v}_1\cdot\mathbf{r}_{ij})\varphi^*(r_i)\varphi(r_j)\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j|a\rangle$$
$$+((G_P{}^{(\mu)})^2-2G_P{}^{(\mu)}G_A{}^{(\mu)})\langle a|\sum_{i,j}\tau_i{}^{(+)}\tau_j{}^{(-)}\exp(i\langle\nu\rangle_a\mathbf{v}_1\cdot\mathbf{r}_{ij})\varphi^*(r_i)\varphi(r_j)\boldsymbol{\sigma}_i\cdot\mathbf{v}_1\boldsymbol{\sigma}_j\cdot\mathbf{v}_1|a\rangle\Big\}\tag{4a}$$

$$\cong Z^4(\langle\eta\rangle_a)^2\frac{1}{2\pi^2}\frac{m_\mu{}^5}{(137)^3}((G_V{}^{(\mu)})^2+3(\Gamma_A{}^{(\mu)})^2)\left\{\frac{\left\langle a\left|\sum_i\left(\frac{1+\tau_i{}^{(3)}}{2}\right)|\varphi(r_i)|^2\right|a\right\rangle}{\left\langle a\left|\sum_i\left(\frac{1+\tau_i{}^{(3)}}{2}\right)\right|a\right\rangle}\right.$$
$$\left.+\frac{\left\langle a\left|\sum_{i,j}{}'\left[\tfrac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i{}^{(3)}\tau_j{}^{(3)})((G_V{}^{(\mu)})^2+(\Gamma_A{}^{(\mu)})^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\right]\left[\frac{\sin(\langle\nu\rangle_ar_{ij})}{(\langle\nu\rangle_ar_{ij})}\varphi^*(r_i)\varphi(r_j)\right]\right|a\right\rangle}{Z((G_V{}^{(\mu)})^2+3(\Gamma_A{}^{(\mu)})^2)}\right\}$$

$$\equiv(Z_{\text{eff}})^4(\langle\eta\rangle_a)^2\frac{1}{2\pi^2}\frac{m_\mu{}^5}{(137)^3}((G_V{}^{(\mu)})^2+3(\Gamma_A{}^{(\mu)})^2)$$
$$\times\left\{1+\frac{\left\langle a\left|\sum_{i,j}{}'\left[\tfrac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i{}^{(3)}\tau_j{}^{(3)})((G_V{}^{(\mu)})^2+(\Gamma_A{}^{(\mu)})^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\right]\left[\frac{\sin(\langle\nu\rangle_ar_{ij})}{(\langle\nu\rangle_ar_{ij})}\varphi^*(r_i)\varphi(r_j)\right]\right|a\right\rangle}{Z((G_V{}^{(\mu)})^2+3(\Gamma_A{}^{(\mu)})^2)\left(\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}\right)}\right\},$$

where

$$(\Gamma_A{}^{(\mu)})^2\equiv(G_A{}^{(\mu)})^2+\tfrac{1}{3}((G_P{}^{(\mu)})^2-2G_A{}^{(\mu)}G_P{}^{(\mu)});\quad\sum_{i,j}{}'\cdots\equiv\sum_{i,j;\,(i\neq j)}\cdots;\tag{4b}$$

$$(Z_{\text{eff}})^4\equiv Z^4\frac{\left\langle a\left|\sum_i\frac{1+\tau_i{}^{(3)}}{2}|\varphi(r_i)|^2\right|a\right\rangle}{\left\langle a\left|\sum_i\frac{1+\tau_i{}^{(3)}}{2}\right|a\right\rangle}=Z^4\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}$$

with

$$\mathfrak{D}_a(r)\equiv\frac{1}{2J_a+1}\sum_{M_a}\frac{\left\langle E_a,J_a,M_a\left|\sum_i\left(\frac{1+\tau_i{}^{(3)}}{2}\right)\delta(\mathbf{r}-\mathbf{r}_i)\right|E_a,J_a,M_a\right\rangle}{\left\langle E_a,J_a,M_a\left|\sum_i\left(\frac{1+\tau_i{}^{(3)}}{2}\right)\right|E_a,J_a,M_a\right\rangle}.\tag{4c}$$

The approximate equality in Eq. (4a) refers to the replacement of

$$\int\frac{d\mathbf{v}_1}{4\pi}\left\langle a\left|\sum_{i,j}{}'\tau_i{}^{(+)}\tau_j{}^{(-)}\exp(i\langle\nu\rangle_a\mathbf{v}_1\mathbf{r}_{ij})\varphi^*(r_i)\varphi(r_j)\boldsymbol{\sigma}_i\cdot\mathbf{v}_1\boldsymbol{\sigma}_j\cdot\mathbf{v}_1\right|a\right\rangle\tag{4d}$$

by

$$\frac{1}{3}\int \frac{d\mathbf{v}_1}{4\pi}\left\langle a\left|\sum_{i,j}{}' \tau_i^{(+)}\tau_j^{(-)}\frac{\sin(\langle\nu\rangle_a r_{ij})}{(\langle\nu\rangle_a r_{ij})}\varphi^*(r_i)\varphi(r_j)\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right|a\right\rangle, \tag{4e}$$

a replacement which is justified since the d wave, g wave, $\cdots$ parts of $\exp(i\langle\nu\rangle_a\mathbf{v}_1\cdot r_{ij})$ make a relatively small contribution to

$$\int \frac{d\mathbf{v}_1}{4\pi}\left\langle a\left|\sum_{i,j}{}' \tau_i^{(+)}\tau_j^{(-)}\exp(i\langle\nu\rangle_a\mathbf{v}_1\cdot\mathbf{r}_{ij})\varphi^*(r_i)\varphi(r_j)\boldsymbol{\sigma}_i\cdot\mathbf{v}_1\boldsymbol{\sigma}_j\cdot\mathbf{v}_1\right|a\right\rangle.$$

The quantity $(Z_{\mathrm{eff}})^4$, introduced by Wheeler in his original estimates of muon capture rates,[6] describes the variation of the muon space orbital wave function over the extent of the nucleus the quantity $\mathfrak{D}_a(r)$ being the (directionally-averaged-over) density function of the protons in the parent nucleus—the electrostatic potential appropriate to $\mathfrak{D}_a(r)$ enters into the muon energy eigenfunction-eigenvalue Dirac equation which determines ϵ_a and $\varphi(r)$. It is clear that $((Z_{\mathrm{eff}})^4/Z^4)<1$, and $\rightarrow 1$ as $Z\rightarrow 0$.

Introducing the expression for the neutron decay rate,

$$\frac{\ln 2}{(\tau_{\frac{1}{2}})_{\mathrm{neu}}}=\frac{1}{(2\pi)^3}m_e^5 f_{\mathrm{neu}}[(g_V^{(\beta)})^2+3(g_A^{(\beta)})^2];\quad f_{\mathrm{neu}}(\tau_{\frac{1}{2}})_{\mathrm{neu}}=(1180\pm35)\ \mathrm{sec},^7 \tag{5}$$

we have, from Eqs. (4a) to (4c) and (5),

$$\begin{aligned}\Lambda^{(\mu)}(a)&=(Z_{\mathrm{eff}})^4(\langle\eta\rangle_a)^2\frac{(m_\mu/m_e)^5}{(137)^3}\frac{\pi(\ln 2)}{(f_{\mathrm{neu}}(\tau_{\frac{1}{2}})_{\mathrm{neu}})}\mathfrak{R}(1-\mathcal{g}_a)\\ &=(Z_{\mathrm{eff}})^4(\langle\eta\rangle_a)^2(272\ \mathrm{sec}^{-1})\mathfrak{R}(1-\mathcal{g}_a)\end{aligned} \tag{6a}$$

with

$$\mathfrak{R}\equiv[(G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2]/[(g_V^{(\beta)})^2+3(g_A^{(\beta)})^2], \tag{6b}$$

$$\mathcal{g}_a\equiv-\frac{\left\langle a\left|\sum_{i,j}{}'[\frac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i^{(3)}\tau_j^{(3)})((G_V^{(\mu)})^2+(\Gamma_A^{(\mu)})^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)]\left[\frac{\sin(\langle\nu\rangle_a r_{ij})}{(\langle\nu\rangle_a r_{ij})}\varphi^*(r_i)\varphi(r_j)\right]\right|a\right\rangle}{Z((G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2)\left(\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}\right)}. \tag{6c}$$

The quantity $\mathcal{g}_a(\mathcal{g}_a>0)$, which would vanish if the nucleus had $Z=A$ [in such a case: $(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)|a\rangle=|a\rangle$; $\tau_i^{(3)}\tau_j^{(3)}|a\rangle=|a\rangle$] and also if $\langle\nu\rangle_a$ were $\gg\{(r_{ij})_{\mathrm{Av}}\}^{-1}\approx$ mean nucleon momentum within nucleus, describes, within the context of the closure approximation, the inhibitory effect of the Pauli exclusion principle on the muon capture process. This inhibition may be visualized as arising from the fact that the neutron created in the $\mu^-+p\rightarrow n+\nu$ process cannot be produced in states already occupied by pre-existing neutrons of the parent nucleus, and the corresponding $\mathcal{g}_a$ may be expressed in terms of appropriate nucleon-nucleon correlation functions in the parent nucleus. We have

$$\begin{aligned}-\mathcal{g}_a=&\frac{\left\langle a\left|\sum_{i,j}{}'\{\frac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i^{(3)}\tau_j^{(3)})[(G_V^{(\mu)})^2+(\Gamma_A^{(\mu)})^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j]\}\left(\frac{1+P_{ij}}{2}\right)\right|a\right\rangle}{Z[(G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2]}\\ &\times\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}\varphi^*(r)\varphi(r')F_a^{(+)}(\mathbf{r},\mathbf{r}')d\mathbf{r}d\mathbf{r}'\Big/\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}\\ &+\frac{\left\langle a\left|\sum_{i,j}{}'\{\frac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i^{(3)}\tau_j^{(3)})[(G_V^{(\mu)})^2+(\Gamma_A^{(\mu)})^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j]\}\left(\frac{1-P_{ij}}{2}\right)\right|a\right\rangle}{Z[(G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2]}\\ &\times\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}\varphi^*(r)\varphi(r')F_a^{(-)}(\mathbf{r},\mathbf{r}')d\mathbf{r}d\mathbf{r}'\Big/\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}\end{aligned} \tag{7a}$$

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[6] J. A. Wheeler, Revs. Modern Phys. **21**, 133 (1949); J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 153 (1949).
[7] Sosnovskii, Spivak, Prokofiev, Kutikov, and Dobrinin, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1059 (1958).

with

$$F_a^{(\pm)}(\mathbf{r},\mathbf{r}')\equiv\frac{\left\langle a\left|\sum_{i,j}'\{\frac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i^{(3)}\tau_j^{(3)})[(G_V^{(\mu)})^2+(\Gamma_A^{(\mu)})^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j]\}[\delta(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{r}'-\mathbf{r}_j)]\left(\frac{1\pm P_{ij}}{2}\right)\right|a\right\rangle}{\left\langle a\left|\sum_{i,j}'\{\frac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i^{(3)}\tau_j^{(3)})[(G_V^{(\mu)})^2+(\Gamma_A^{(\mu)})^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j]\}\left(\frac{1\pm P_{ij}}{2}\right)\right|a\right\rangle},\tag{7b}$$

$$P_{ij}\equiv-\left(\frac{1+\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j}{2}\right)\left(\frac{1+\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j}{2}\right).\tag{7c}$$

The $F_a^{(\pm)}(\mathbf{r},\mathbf{r}')$ are nucleon-nucleon correlation density functions associated with a space symmetric, space antisymmetric relative motion of the two nucleons since for $|a\rangle$ antisymmetric in $\mathbf{r}_i$, $\mathbf{r}_j$; $\sigma_i^{(3)}$, $\sigma_j^{(3)}$; $\tau_i^{(3)}$, $\tau_j^{(3)}$; $P_{ij}|a\rangle=$ (exchange operator for $\mathbf{r}_i$, $\mathbf{r}_j$) $|a\rangle$; it is to be noted that $F_a^{(-)}(\mathbf{r},\mathbf{r})=0$. One has further

$$\begin{aligned}&\langle a|\sum_{i,j}'\{\tfrac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i^{(3)}\tau_j^{(3)})[(G_V^{(\mu)})^2+(\Gamma_A^{(\mu)})^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j]\}|a\rangle\\&\quad=-[(G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2]\tfrac{1}{2}A+\langle a|(G_V^{(\mu)})^2[(\mathbf{T})^2-(T^{(3)})^2]+(\Gamma_A^{(\mu)})^2[(\mathbf{Y}^{(1)})^2+(\mathbf{Y}^{(2)})^2]|a\rangle,\\&\langle a|\sum_{i,j}'[\tfrac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i^{(3)}\tau_j^{(3)})[(G_V^{(\mu)})^2+(\Gamma_A^{(\mu)})^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j]P_{ij}|a\rangle\\&\quad=-[(G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2]\frac{Z(A-Z)}{2}+[(G_V^{(\mu)})^2-(\Gamma_A^{(\mu)})^2]\langle a|(\mathbf{S}_{\mathrm{pr}})^2+(\mathbf{S}_{\mathrm{neu}})^2-(\mathbf{S}_{\mathrm{pr}}+\mathbf{S}_{\mathrm{neu}})^2|a\rangle\end{aligned}\tag{7d}$$

with

$$\mathbf{T}\equiv\sum_i\tfrac{1}{2}\boldsymbol{\tau}_i;\quad \mathbf{Y}^{(1),(2),(3)}\equiv\sum_i\tfrac{1}{2}\tau_i^{(1),(2),(3)}\boldsymbol{\sigma}_i;\quad \mathbf{S}_{\mathrm{pr}}\equiv\sum_i\left(\frac{1+\tau_i^{(3)}}{2}\right)(\boldsymbol{\sigma}_i/2);\quad \mathbf{S}_{\mathrm{neu}}\equiv\sum_i\left(\frac{1-\tau_i^{(3)}}{2}\right)(\boldsymbol{\sigma}_i/2)\tag{7e}$$

as may be verified by using relations of the type:

$$\sum_{i,j}'\tfrac{1}{4}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j=\sum_{i,j}\tfrac{1}{4}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\sum_i\tfrac{1}{4}(\boldsymbol{\tau}_i)^2=(\mathbf{T})^2-\tfrac{3}{4}A.$$

Equations (7a) to (7e) and (6a) to (6c) yield

$$\begin{aligned}\Lambda^{(\mu)}(a)=(Z_{\mathrm{eff}})^4(\langle\eta\rangle_a)^2(272\text{ sec}^{-1})\mathcal{R}\Bigg\{1&-\left[\frac{A}{2Z}-\frac{\langle a|(G_V^{(\mu)})^2[(\mathbf{T})^2-(T^{(3)})^2]+(\Gamma_A^{(\mu)})^2[(\mathbf{Y}^{(1)})^2+(\mathbf{Y}^{(2)})^2]|a\rangle}{Z[(G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2]}\right]\\&\times\left[\frac{\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}\varphi^*(r)\varphi(r')\tfrac{1}{2}[F_a^{(+)}(\mathbf{r},\mathbf{r}')+F_a^{(-)}(\mathbf{r},\mathbf{r}')]d\mathbf{r}d\mathbf{r}'}{\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}}\right]\\&-\left[\frac{A-Z}{2}-\frac{[(G_V^{(\mu)})^2-(\Gamma_A^{(\mu)})^2]\langle a|(\mathbf{S}_{\mathrm{pr}})^2+(\mathbf{S}_{\mathrm{neu}})^2-(\mathbf{S}_{\mathrm{pr}}+\mathbf{S}_{\mathrm{neu}})^2|a\rangle}{Z[(G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2]}\right]\\&\times\left[\frac{\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}\varphi^*(r)\varphi(r')\tfrac{1}{2}[F_a^{(+)}(\mathbf{r},\mathbf{r}')-F_a^{(-)}(\mathbf{r},\mathbf{r}')]d\mathbf{r}d\mathbf{r}'}{\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}}\right]\Bigg\}.\end{aligned}\tag{8}$$

Equation (8) for $\Lambda_{(a)}^{(\mu)}$ is essentially exact within the limitations of the closure approximation.

We proceed to evaluate $\Lambda^{(\mu)}(a)$ for the heavier nuclei, $Z>6$, $A>12$. Dropping terms $\sim 1/Z$ in Eq. (8) yields

$$\Lambda^{(\mu)}(a)\cong(Z_{\rm eff})^4(\langle\eta\rangle_a)^2(272\ {\rm sec}^{-1})\mathfrak{R}\left\{1-\left(\frac{A}{2Z}\right)\left[\frac{\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}\varphi^*(r)\varphi(r')\tfrac{1}{2}[F_a^{(+)}(\mathbf{r},\mathbf{r}')+F_a^{(-)}(\mathbf{r},\mathbf{r}')]d\mathbf{r}d\mathbf{r}'}{\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}}\right]\right.$$
$$\left.-\left(\frac{A-Z}{2}\right)\left[\frac{\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}\varphi^*(r)\varphi(r')\tfrac{1}{2}[F_a^{(+)}(\mathbf{r},\mathbf{r}')-F_a^{(-)}(\mathbf{r},\mathbf{r}')]d\mathbf{r}d\mathbf{r}'}{\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}}\right]\right\}. \tag{9}$$

Then, adopting physically reasonable functional forms for $F_a^{(\pm)}(\mathbf{r},\mathbf{r}')$, $\mathfrak{D}_a(r)$ such as,

$$F_a^{(\pm)}(\mathbf{r},\mathbf{r}')=C_a^{(\pm)}\mathfrak{D}_a(r)\mathfrak{D}_a(r')(1\pm f_a(|\mathbf{r}-\mathbf{r}'|));\quad f_a(|\mathbf{r}-\mathbf{r}'|)=\begin{matrix}1:|\mathbf{r}-\mathbf{r}'|\leqq d\\ 0:|\mathbf{r}-\mathbf{r}'|>d\end{matrix};$$
$$(C_a^{\pm})^{-1}=\int\int\mathfrak{D}_a(r)\mathfrak{D}_a(r')(1\pm f_a(|\mathbf{r}-\mathbf{r}'|))d\mathbf{r}d\mathbf{r}';\quad \mathfrak{D}_a(r)=\begin{cases}1/[(4\pi/3)r_0^3A]:r\leqq r_0A^{\frac{1}{3}}\\ 0\qquad\qquad\qquad :r>r_0A^{\frac{1}{3}}\end{cases}; \tag{10}$$

remembering that $[\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)]/(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)$ and $f_a(|\mathbf{r}-\mathbf{r}'|)$ are both *short ranged* functions with comparable ranges $(\langle\nu\rangle_a)^{-1}\approx d\approx r_0\ll r_0A^{\frac{1}{3}}$, and again neglecting terms $\sim 1/Z$, we have

$$\Lambda^{(\mu)}(a)\cong(Z_{\rm eff})^4(\langle\eta\rangle_a)^2(272\ {\rm sec}^{-1})\mathfrak{R}$$
$$\times\left\{1-\left(\frac{A-Z}{2A}\right)\left[\frac{A\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}\varphi^*(r)\varphi(r')\mathfrak{D}_a(r)\mathfrak{D}_a(r')f_a(|\mathbf{r}-\mathbf{r}'|)d\mathbf{r}d\mathbf{r}'}{\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}}\right]\right\}$$
$$\equiv(Z_{\rm eff})^4(\langle\eta\rangle_a)^2(272\ {\rm sec}^{-1})\mathfrak{R}\left\{1-\left(\frac{A-Z}{2A}\right)\delta_a\right\}. \tag{11a}$$

Further, using once more the *short ranged* character of $f_a(|\mathbf{r}-\mathbf{r}'|)$, we obtain

$$\delta_a\cong A\left(\frac{1}{(4\pi/3)r_0^3A}\right)\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}\int\frac{\sin(\langle\nu\rangle_a\rho)}{(\langle\nu\rangle_a\rho)}f_a(\rho)d\boldsymbol{\varrho}\Big/\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}$$
$$=\left(\frac{d}{r_0}\right)^3\left[1-\frac{1}{10}(\langle\nu\rangle_ar_0)^2\left(\frac{d}{r_0}\right)^2+\cdots\right]\cong\left(\frac{d}{r_0}\right)^3\left[1-0.024\left(\frac{d}{r_0}\right)^2+\cdots\right] \tag{11b}$$

where we have taken $\langle\nu\rangle_a\cong0.75m_\mu$ (see below), $r_0\cong1.25\times10^{-13}$ cm$=0.67/m_\mu$. Thus, within the present approximation, the "nucleon-nucleon correlation" parameter δ_a and the exclusion principle inhibition factor $\mathcal{I}_a$, proportional to the fraction of nucleons which are neutrons and to δ_a: $\mathcal{I}_a\cong((A-Z)/2A)\delta_a$ [Eqs. (6a), (6c), (11a), and (11b)], are both essentially determined by the parameter d/r_0, i.e., by the ratio of the characteristic lengths entering into $f_a(|\mathbf{r}-\mathbf{r}'|)$ and $\mathfrak{D}_a(r)$. The characteristic length d is the radius of the Pauli-Fermi "correlation region of influence," e.g., the Pauli-Fermi "correlation hole" surrounding each nucleon, and is determined by the interplay of the nucleon-nucleon forces and the exclusion principle. It is to be noted that up to terms $\sim 1/Z$, $\mathcal{I}_a$ vanishes with vanishing d, i.e., with vanishing nucleon-nucleon correlation.

To obtain the numerical value of d/r_0 and so of δ_a we consider the expression for the Coulomb energy of the parent nucleus. We have

$$E_{\rm Coul}=\left\langle a\left|\sum_{i,j}{}'\frac{1}{2}\left(\frac{1+\tau_i^{(3)}}{2}\right)\left(\frac{1+\tau_j^{(3)}}{2}\right)\left(\frac{e^2}{r_{ij}}\right)\right|a\right\rangle, \tag{12}$$

and using a procedure analogous to that employed in the evaluation of $\mathcal{J}_a$ [Eqs. (6c) to (7e)] we obtain a formula first given by Feenberg and Goertzel,[8]

$$E_{\rm Coul}=\tfrac{1}{2}Z(Z-1)\int\int\frac{e^2}{|\mathbf{r}-\mathbf{r}'|}\tfrac{1}{4}(G_a^{(+)}(\mathbf{r},\mathbf{r}')+3G_a^{(-)}(\mathbf{r},\mathbf{r}'))d\mathbf{r}d\mathbf{r}'$$
$$+(\tfrac{3}{4}Z-\langle a|(\mathbf{S}_{\rm pr})^2|a\rangle)\int\int\frac{e^2}{|\mathbf{r}-\mathbf{r}'|}\tfrac{1}{2}(G_a^{(+)}(\mathbf{r},\mathbf{r}')-G_a^{(-)}(\mathbf{r},\mathbf{r}'))d\mathbf{r}d\mathbf{r}', \quad (13a)$$

where

$$G_a^{(\pm)}(\mathbf{r},\mathbf{r}')\equiv\frac{\left\langle a\left|\sum_{i,j}'\left[\left(\frac{1+\tau_i^{(3)}}{2}\right)\left(\frac{1+\tau_j^{(3)}}{2}\right)\right][\delta(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{r}'-\mathbf{r}_j)]\left(\frac{1+P_{ij}}{2}\right)\right|a\right\rangle}{\left\langle a\left|\sum_{i,j}'\left[\left(\frac{1+\tau_i^{(3)}}{2}\right)\left(\frac{1+\tau_j^{(3)}}{2}\right)\right]\left(\frac{1\pm P_{ij}}{2}\right)\right|a\right\rangle} \quad (13b)$$

is the proton-proton correlation density function associated with a space symmetric, space antisymmetric relative motion of the two protons. Comparing Eq. (13b) for $G_a{}^{\pm}(\mathbf{r},\mathbf{r}')$ with Eq. (7b) for $F_a^{(\pm)}(\mathbf{r},\mathbf{r}')$ we see that, in each case, the same spin and isobaric-spin operators occur in the integrals in the numerator and in the denominator so that the influence of these spin and isobaric-spin operators may be expected roughly to cancel. (This is almost rigorously true for nuclei in the 1s shell, e.g., $He_2{}^4$, where $|a\rangle$ factorizes into $\Phi_a(\mathbf{r}_1,\cdots,\mathbf{r}_4)$ $\times X_a(\sigma_1^{(3)},\cdots,\sigma_4^{(3)};\tau_1^{(3)},\cdots,\tau_4^{(3)})$ to a good approximation.) It thus appears reasonable to identity $G_a^{(\pm)}(\mathbf{r},\mathbf{r}')$ with $F_a^{(\pm)}(\mathbf{r},\mathbf{r}')$ so that from Eqs. (13a) and (10),

$$E_{\rm Coul}=(\tfrac{1}{2}Z(Z-1))\left(\frac{6}{5}\frac{e^2}{r_0A^{\frac{1}{3}}}\left(1-\frac{(d/r_0)^2}{A^{\frac{2}{3}}}\right)\right)+(\tfrac{3}{4}Z-\langle a|(\mathbf{S}_{\rm pr})^2|a\rangle)\left(\frac{6}{5}\frac{e^2}{r_0A^{\frac{1}{3}}}\frac{(d/r_0)^2}{A^{\frac{2}{3}}}\right). \quad (13c)$$

A "best fit" of Eq. (13c) to the experimentally determined Coulomb energy differences of various light and medium-heavy mirror nuclei yields

$$\frac{d}{r_0}=1.47;\quad \delta_a=3.0 \text{ [from Eq. (11b)]}. \quad (14)$$

In this "best fit" the value of r_0 used is consistent with the electron-nucleus elastic scattering data[9]—the d/r_0 values of the individual nuclei naturally fluctuate somewhat about the aforementioned "best fit" value so that an assignment of, say, 10% uncertainty to the resultant value of 3.0 for δ_a appears quite reasonable. Equations (11a) and (14) yield values of the total muon capture rate, $\Lambda^{(\mu)}(a)$, which can be compared with experiment—it is to be noted that the effect of the exclusion principle inhibition is very important numerically since

$$\{1-\mathcal{J}_a\}\cong\left\{1-\left(\frac{A-Z}{2A}\right)\delta_a\right\}\cong\left\{1-\left(\frac{A-Z}{2A}\right)3\right\}=\frac{1}{4}$$

for $Ca_{20}{}^{40}$; $=\frac{1}{8}$ for $Ca_{20}{}^{48}$; $=5/32$ for $Mo_{42}{}^{96}$; $=19/208$ for $Pb_{82}{}^{208}$, etc. The associated *isotope effect* in the total muon capture rate is thus expected to be quite large, e.g., $\Lambda^{(\mu)}(Ca_{20}{}^{48})/\Lambda^{(\mu)}(Ca_{20}{}^{40})\cong\frac{1}{2}$, and would be interesting to verify experimentally using separated isotope targets.

Alternatively, one need not appeal to any relation between $F_a^{(\pm)}(\mathbf{r},\mathbf{r}')$ and $G_a^{(\pm)}(\mathbf{r},\mathbf{r}')$ with $G_a^{(\pm)}(\mathbf{r},\mathbf{r}')$ found from $E_{\rm Coul}$ in order to determine δ_a, but adopt instead the less ambitious procedure of fitting Eq. (11a) for $\Lambda^{(\mu)}(a)$ to the available data for the total muon capture rates, with use of a single Z, A-independent adjustable parameter δ_a. A successful fit of this type would also determine $(\langle\eta\rangle_a)^2\mathcal{R}$, i.e., with a reliable estimate of $\langle\eta\rangle_a\cong\langle\nu\rangle_a/m_\mu$ [Eq. (3d)] would determine $\mathcal{R}\equiv((G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2)/((g_V^{(\beta)})^2+3(g_A^{(\beta)})^2)$ [Eq. (6b)]. A more rigorous treatment would involve use of Eq. (8) for $\Lambda^{(\mu)}(a)$, without neglect of terms $\sim 1/Z$, with $\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}\equiv(Z_{\rm eff})^4/Z^4$ appropriately expressed via a proton density function $\mathfrak{D}_a(r)$ obtained from electron-nucleus elastic scattering data,[9] and with the nucleon-nucleon correlation density functions $F_a^{(\pm)}(\mathbf{r},\mathbf{r}')$, again identified with $G_a{}^{\pm}(\mathbf{r},\mathbf{r}')$ as deduced from an analysis of *inelastic* electron-nucleus scattering.[9] Such a more rigorous treatment must await the performance and the interpretation of the appropriate inelastic electron-nucleus scattering experiments.[10]

[8] E. Feenberg and G. Goertzel, Phys. Rev. **70**, 597 (1946).
[9] R. Hofstadter, Ann. Rev. Nuclear Sci. **7**, 231 (1957).
[10] H. A. Tolhoek, in a preprint received after this manuscript was completed, has applied the closure approximation to the calculation of $\Lambda^{(\mu)}(a)$ in a manner which, in effect, involves the expansion of $\exp(i\langle\nu\rangle_a\mathbf{v}_1\cdot\mathbf{r}_{ij})$ or of $\sin(\langle\nu\rangle_a r_{ij})/(\langle\nu\rangle_a r_{ij})$ [as in Eq. (4a) or (6c) or (8)] in powers of $\langle\nu\rangle_a r_{ij}\approx\langle\nu\rangle_a r_0 A^{\frac{1}{3}}$. Tolhoek keeps terms only up to $(\langle\nu\rangle_a r_{ij})^2$ in this expansion. It is however to be emphasized that such an expansion converges rather slowly for the heavier nuclei ($Z>6$, $A>12$) and the results obtained by means of it are quite unreliable. It is to be noted that, contrary to the impression given by Tolhoek, no such expansion is used in the present procedure in the crucial passage from Eq. (8) to Eq. (11a).

TABLE I.

	$\langle\nu\rangle_a/m_\mu$	$(\langle\eta\rangle_a)^2$	$\langle a\|(\mathbf{T})^2-(T^{(3)})^2\|a\rangle$	$\langle a\|(\mathbf{Y}^{(1)})^2+(\mathbf{Y}^{(2)})^2\|a\rangle$	$\langle a\|(\mathbf{S}_{pr})^2+(\mathbf{S}_{neu})^2-(\mathbf{S}_{pr}+\mathbf{S}_{neu})^2\|a\rangle$
H_1^1	0.94	0.58	1/2	3/2	0
H_1^2	0.90	0.65	0	2	−1/2
He_2^3	0.95	0.78	1/2	3/2	0
He_2^4	0.75	0.50	0	0	0

In concluding this section we apply Eq. (8) for $\Lambda^{(\mu)}(a)$ to the light nuclei H_1^1, H_1^2, He_2^3, He_2^4. We have from Table I and Eq. (8),

$$\Lambda^{(\mu)}(H_1^1)=1^4\times 0.58\times 272\ \text{sec}^{-1}\times\mathcal{R}\times 1,$$

$$\Lambda^{(\mu)}(H_1^2)\cong 1^4\times 0.65\times 272\ \text{sec}^{-1}\times\mathcal{R}\times\left\{1-\left[\frac{(G_V^{(\mu)})^2+(\Gamma_A^{(\mu)})^2}{(G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2}\right]\left[\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}F_a^{(+)}(\mathbf{r},\mathbf{r}')d\mathbf{r}d\mathbf{r}'\right]\right\},\qquad(15)$$

$$\Lambda^{(\mu)}(He_2^3)\cong 2^4\times 0.78\times 272\ \text{sec}^{-1}\times\mathcal{R}\times\left\{1-\frac{1}{2}\left[\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}F_a^{(+)}(\mathbf{r},\mathbf{r}')d\mathbf{r}d\mathbf{r}'\right]\right\},$$

$$\Lambda^{(\mu)}(He_2^4)\cong 2^4\times 0.50\times 272\ \text{sec}^{-1}\times\mathcal{R}\times\left\{1-\left[\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}F_a^{(+)}(\mathbf{r},\mathbf{r}')d\mathbf{r}d\mathbf{r}'\right]\right\},$$

which exhibits forms for the corresponding exclusion principle inhibition factors, $\mathcal{I}_a$ [Eqs. (15) and (6a) to (6c)]. With regard to the first two columns of Table I for $\langle\nu\rangle_a$ [Eq. (2c)] and $\langle\eta\rangle_a$ [Eq. (3c)] reasonable estimates have been made for $\langle E_b-E_a\rangle=\langle$excitation energy of daughter nucleus $b\rangle+((E_b)_{\text{ground state}}-E_a)$; also $\varphi(r)\approx\exp(-Zm_\mu'r/137)$ has been approximated by 1 within the integrals. Further, from Eq. (7b), the $\mathbf{r}_i$ being here coordinates relative to the nucleus' center of mass,

$$F_a^{(+)}(\mathbf{r},\mathbf{r}')=\int\int|\Phi_a(\mathbf{r}_1,\mathbf{r}_2)|^2\delta(\mathbf{r}_1+\mathbf{r}_2)\delta(\mathbf{r}-\mathbf{r}_1)\delta(\mathbf{r}'-\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2=|\Phi_a(\mathbf{r},\mathbf{r}')|^2\delta(\mathbf{r}+\mathbf{r}'):H_1^2$$

$$F_a^{(+)}(\mathbf{r},\mathbf{r}')=\int\int\int|\Phi_a(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)|^2\delta(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)\delta(\mathbf{r}-\mathbf{r}_1)\delta(\mathbf{r}'-\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3=|\Phi_a[\mathbf{r},\mathbf{r}',-(\mathbf{r}+\mathbf{r}')]|^2:He_2^3\qquad(16)$$

$$F_a^{(+)}(\mathbf{r},\mathbf{r}')=\int\int\int\int|\Phi_a(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)|^2\delta(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4)\delta(\mathbf{r}-\mathbf{r}_1)\delta(\mathbf{r}'-\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3d\mathbf{r}_4$$

$$=\int|\Phi_a[\mathbf{r},\mathbf{r}',\mathbf{r}_3,-(\mathbf{r}+\mathbf{r}'+\mathbf{r}_3)]|^2d\mathbf{r}_3:He_2^4,$$

where the Φ_a are space wave functions in the corresponding H_1^2, He_2^3, He_2^4 space, spin, isobaric-spin wave functions $|a\rangle\cong\Phi_a(\mathbf{r}_1,\mathbf{r}_2,\cdots)X_a(\sigma_1^{(3)},\sigma_2^{(3)},\cdots;\tau_1^{(3)},\tau_2^{(3)},\cdots)$ with $\Phi_a(\mathbf{r}_1,\mathbf{r}_2,\cdots)=\Phi_a(r_{12},\cdots)=P_{ij}\Phi_a(\mathbf{r}_1,\mathbf{r}_2,\cdots)$. The appropriate spin, isobaric-spin wave functions, X_a, have been used together with Eq. (7e) in order to obtain the entries in the last three columns of Table I. Use of simple variational trial forms for Φ_a enables evaluation of the $F_a^{(+)}(\mathbf{r},\mathbf{r}')$ in Eq. (16) which, together with the $\langle\nu\rangle_a$ estimates of the first column, give the integrals in Eq. (15). Employing also numerical values of $\mathcal{R}$ quoted below [Eq. (19)] and a value of $(\Gamma_A^{(\mu)}/G_V^{(\mu)})^2=1.53$ calculated from Eqs. (1b), (1c), and (4b) with $(g_A^{(\beta)}/g_V^{(\beta)})=-1.21$,[11] we obtain

$$\Lambda^{(\mu)}(H_1^1)=1^4\times 0.58\times 272\ \text{sec}^{-1}\times\mathcal{R}\times 1=158\ \text{sec}^{-1}\times\mathcal{R}=169\ \text{sec}^{-1},$$

$$\Lambda^{(\mu)}(H_1^2)\cong 1^4\times 0.65\times 272\ \text{sec}^{-1}\times\mathcal{R}\times\left\{1-\left[\frac{(G_V^{(\mu)})^2+(\Gamma_A^{(\mu)})^2}{(G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2}\right]\times 0.64\right\}$$

$$=177\ \text{sec}^{-1}\times\mathcal{R}\times\left\{1-\left[\frac{1+(\Gamma_A^{(\mu)}/G_V^{(\mu)})^2}{1+3(\Gamma_A^{(\mu)}/G_V^{(\mu)})^2}\right]\times 0.64\right\}=135\ \text{sec}^{-1}\qquad(17)$$

$$\Lambda^{(\mu)}(He_2^{(3)})\cong 2^4\times 0.78\times 272\ \text{sec}^{-1}\times\mathcal{R}\times(1-\tfrac{1}{2}\times 0.66)=23\times 10^2\ \text{sec}^{-1}\times\mathcal{R}=25\times 10^2\ \text{sec}^{-1}$$

$$\Lambda^{(\mu)}(He_2^{(4)})\cong 2^4\times 0.50\times 272\ \text{sec}^{-1}\times\mathcal{R}\times(1-0.80)=4.4\times 10^2\ \text{sec}^{-1}\times\mathcal{R}=4.7\times 10^2\ \text{sec}^{-1}.$$

[11] C. S. Wu, Revs. Modern Phys. **30**, 783 (1959), this issue; V. L. Telegdi, Conference on Weak Interactions, Gatlinburg, Tennessee (1958).

The dependence of $\Lambda^{(\mu)}(H_1^2)$ on the ratio $(\Gamma_A^{(\mu)}/G_V^{(\mu)})^2$ is especially to be noted[12] as is the enormous isotope effect—factor $\cong 5$—between $\Lambda^{(\mu)}(He_2^3)$ and $\Lambda^{(\mu)}(He_2^4)$. Both of these are essentially manifestations of the inhibitions of the exclusion principle on the total muon capture rate. Thus in $\mu^- + H_1^2 \rightarrow \nu + n + n$ the dineutron must be produced in the $^3P_1$ state if $\Gamma_A^{(\mu)}=0$ while for $|\Gamma_A^{(\mu)}| \gtrsim G_V^{(\mu)}$ production in the dineutron $^1S_0$ state is possible. Since the dineutron $^1S_0$ state spatially overlaps far better with the deuteron ground $^3S_1$ state than does the dineutron $^3P_1$ state, dineutron production in the $^1S_0$ state when $|\Gamma_A^{(\mu)}| \gtrsim G_V^{(\mu)}$ is indeed predominant, and one expects $\Lambda^{(\mu)}(H_1^2)$ to be greater when $(\Gamma_A^{(\mu)}/G_V^{(\mu)})^2 \gg 1$ than when $(\Gamma_A^{(\mu)}/G_V^{(\mu)})^2 \ll 1$ just as predicted by Eq. (17). In a similar way the factor of 5 difference between the rates of $\mu^- + He_2^3 \rightarrow \nu + H_1^3$ and $\mu^- + He_2^4 \rightarrow \nu + H_1^4$ may be viewed as largely arising from the fact that the H_1^3 may be formed in $^2S_{\frac{1}{2}}$ states (bound or unbound) which spatially overlap well with the $^2S_{\frac{1}{2}}$ ground state of He_2^3 while the H_1^4 is formed in (unbound) $^3P_{2,1,0}$, $^1P_0$ states which all have a poor spatial overlap with the $^1S_0$ ground state of He_2^4.

3. COMPARISON OF CLOSURE APPROXIMATION EXPRESSION FOR THE TOTAL MUON CAPTURE RATE WITH EXPERIMENT

Equation (11a) for $\Lambda^{(\mu)}(a)$ has been compared by Telegdi, Sens, Swanson, and Yovanovitch[13] with their definitive measurements of the total muon capture rates in 29 elements from C_6^{12} to U_{92}^{238}. These investigators first calculated values of $(Z_{\text{eff}})^4$ [according to Eq. (4c)] for the various elements studied and then found that a plot of their $[\Lambda^{(\mu)}(a)]_{\text{exper}}/(Z_{\text{eff}})^4$ values *versus* the corresponding $(A-Z)/2A$ values gave a nice straight line in agreement with Eq. (11a); the values of δ_a and $[(\langle\eta_a\rangle_a)^2(272\ \text{sec}^{-1})\mathcal{R}]$, determined by a weighted least squares fit of their individual experimental points to this straight line, were

$$\delta_a = 3.15 \quad [vs\ \delta_a = 3.0 \text{ in Eq. (14)}] \tag{18a}$$

and

$$(\langle\eta\rangle_a)^2(272\ \text{sec}^{-1})\mathcal{R} = 188\ \text{sec}^{-1}\ \text{(experimental)}. \tag{18b}$$

On the other hand, Eqs. (1b), (1c), (4b), (6b), and (3d), and $(g_A^{(\beta)}/g_V^{(\beta)}) = -1.21$ [11] yield:

$$\mathcal{R} = \begin{matrix} 1.06: \langle\eta\rangle_a \cong \langle\nu\rangle_a/m_\mu \cong 0.75 \\ 1.07: \langle\eta\rangle_a \cong \langle\nu\rangle_a/m_\mu \cong 0.85-0.95, \end{matrix} \tag{19}$$

while the best *a priori* estimate for

$$\langle\eta\rangle_a \cong \langle\nu\rangle_a/m_\mu \cong \{1 - (\epsilon_a/m_\mu - \langle\text{excitation energy of daughter nucleus } b\rangle/m_\mu - [(E_b)_{\text{ground state}} - E_a]/m_\mu\},$$

[Eq. (3d)] supposed valid in the mean for all the various pairs of nuclei (b,a) involved, is $\langle\eta\rangle_a \cong \langle\nu\rangle_a/m_\mu \cong 0.75 = 80\ \text{Mev}/m_\mu$ with an uncertainty of, say, 10%. This value of $\langle\nu\rangle_a/m_\mu$ corresponds to an $\langle$excitation energy of daughter nucleus $b\rangle \cong 15$ Mev which is of the order of those empirically observed. The value of $(\langle\eta\rangle_a)^2(272\ \text{sec}^{-1})\mathcal{R}$ for $\langle\eta\rangle_a \cong \langle\nu\rangle_a/m_\mu \cong 0.75 = 80\ \text{Mev}/m_\mu$ is, using Eq. (19);

$$(\langle\eta\rangle_a)^2(272\ \text{sec}^{-1})\mathcal{R} = 161\ \text{sec}^{-1}\ \text{(theoretical)} \tag{20}$$

which, in view of the uncertainty in the value of $(\langle\eta\rangle_a)^2$, must be considered in essential agreement with the experimental value of 188 sec^{-1} [Eq. (18b)]—thus, for example, the not unreasonable choice of $\langle\nu\rangle_a/m_\mu \cong 0.80 = 85\ \text{Mev}/m_\mu$ yields $(\langle\eta\rangle_a)^2(272\ \text{sec}^{-1})\mathcal{R} = 185$ sec^{-1} (theoretical). In this way one finds support for the combination of basic assumptions on which our effective Hamiltonian $H_{\text{eff}}^{(\mu)}$ [Eqs. (1a) to (1c)] rests, *viz.*: (a) "universality" between muon-bare nucleon and electron-bare nucleon coupling constants which implies the numerical relations of Eq. (1c) between the effective muon-dressed nucleon and electron-dressed nucleon coupling constants; (b) the presence of an "induced" pseudoscalar interaction[3,4] with an effective muon-dressed nucleon coupling constant $g_P^{(\mu)} = 8g_A^{(\beta)}$ [Eq. (1c)]; and (c) the presence of anomalous nucleon magnetic moment contributions in the effective muon-dressed nucleon interaction associated with the assumption of a "conserved vector current." [5] In particular, if this assumption of the conserved vector current is abandoned and the anomalous nucleon magnetic moment contributions $\sim(\mu_p - \mu_n)$ to the effective muon-dressed coupling constants $G_A^{(\mu)}$, $G_p^{(\mu)}$ omitted from Eqs. (1b), (4b), and (6b), the quantity $\mathcal{R}$ of Eq. (6b) is 0.90 rather than 1.06 and

$$(\langle\eta\rangle_a)^2(272\ \text{sec}^{-1})\mathcal{R} = 137\ \text{sec}^{-1}\ \text{(theoretical)}. \tag{20}$$

It thus appears that somewhat better agreement is reached between theoretical and experimental values of the total muon capture rates if the assumption of a conserved vector current is retained.

4. MUON CAPTURE TO PARTICULAR FINAL STATES OF DAUGHTER NUCLEUS

The first investigation in which a *partial* muon capture rate was determined is due to Godfrey.[14] Godfrey studied experimentally the rate of that muon capture reaction

$$\mu^- + C_6^{12} \rightarrow \nu + B_5^{12}$$

[12] H. Primakoff, Phys. Rev. **91**, 480 (A) (1953), and *Proceedings of the Fifth Annual Rochester Conference on High Energy Physics*, New York (1955), p. 174; A. Rudik, Doklady Akad. Nauk. **92**, 739 (1953); H. Überall and L. Wolfenstein, Nuovo cimento **10**, 136 (1958).

[13] V. Telegdi (private communication); Sens, Swanson, Telegdi, and Yovanovitch, Phys. Rev. **107**, 1464 (1957); J. Sens, Ph.D. thesis, University of Chicago (1958) and Phys. Rev. **113**, 679 (1959).

[14] T. N. K. Godfrey, Ph.D. thesis, Princeton University (1954), and Phys. Rev. **92**, 512 (1953).

which was followed by the beta decay of B_5^{12} and accordingly obtained the partial rate of muon capture to all the bound states of B_5^{12}. He further gave a qualitative argument in favor of the view that most of the muon capture transitions to the bound states of B_5^{12} actually go to the ground state of B_5^{12}. As a result he identified his *observed* partial muon capture rate with the rate from C_6^{12} to the B_5^{12} ground state. Godfrey then established an approximate theoretical relation between the nuclear matrix elements for muon capture and for beta decay between the ground states of C_6^{12} and B_5^{12}. By using this relation and comparing his observed muon capture rate with the known B_5^{12} decay rate he concluded that the Gamow-Teller coupling constants in muon capture and in beta decay are approximately equal.

Fujii and Primakoff[1] recently re-examined and refined the relation between the nuclear matrix elements for muon capture and for beta decay between the ground states of C_6^{12} and B_5^{12}, and also extended the argument to the calculation of the ground state to ground-state partial muon capture rates in the reactions,

$$\mu^- + Li_3^6 \to \nu + He_2^6,$$
$$\mu^- + He_2^3 \to \nu + H_1^3.$$

Using the expressions for the muon capture transition matrix elements, M.E.$^{(\mu)}$, given in Eqs. (2a) and (2b) and the analogous expressions for the corresponding beta-decay transition matrix elements, M.E.$^{(\beta)}$,

$$|\mathrm{M.E.}^{(\beta)}|^2 = \frac{1}{(2\pi)^6} F_{ba}(Z,E_e)|\mathrm{M.E.}_{\mathrm{nucl}}^{(\beta)}(b\to a)|^2, \qquad (21a)$$

$$|\mathrm{M.E.}_{\mathrm{nucl}}^{(\beta)}(b\to a)|^2 = (g_V^{(\beta)})^2|\langle a|\textstyle\sum_i \tau_i^{(+)}|b\rangle|^2 + (g_A^{(\beta)})^2|\langle a|\textstyle\sum_i \tau_i^{(+)}\boldsymbol{\sigma}_i|b\rangle|^2. \qquad (21b)$$

Fujii and Primakoff obtained an expression for the ratio of transition rates of muon capture from $|a\rangle$ to $|b\rangle$ and beta decay from $|b\rangle$ to $|a\rangle$, $\Lambda^{(\mu)}(a\to b)/\Lambda^{\beta}(b\to a)$,

$$\frac{\Lambda^{(\mu)}(a\to b)}{\Lambda^{(\beta)}(b\to a)} = \left[\pi(\eta_{ba})^2\frac{Z^3 m_\mu^5}{(137)^3}\frac{1}{f_{ba}}\right]\times\left(\frac{2J_b+1}{2J_a+1}\right) \times\left[\frac{\displaystyle\int\frac{d\mathbf{v}_1}{4\pi}\sum_{M_b,M_a}|\mathrm{M.E.}_{\mathrm{nucl}}^{(\mu)}(a\to b)|^2}{\displaystyle\sum_{M_b,M_a}|\mathrm{M.E.}_{\mathrm{nucl}}^{(\beta)}(b\to a)|^2}\right] \equiv k\times\left(\frac{2J_b+1}{2J_a+1}\right)\times X, \qquad (22a)$$

where

$$f_{ba} = \int_1^{(E_e)_{\max}} = F_{ba}(Z,E_e)((E_e)_{\max}-E_e)^2 \times E_e((E_e)^2-1)^{\frac{1}{2}}dE_e \qquad (22b)$$

$F_{ba}(Z,E)$ = Fermi function.

Fujii and Primakoff then calculated, employing appropriate approximations for $|a\rangle$, $|b\rangle$, the ratio of the nuclear matrix elements for muon capture and for beta decay—X in Eq. (22a)—as:

$$X(He_2^3 \leftrightarrows H_1^3) = 0.791$$
$$X(Li_3^6 \leftrightarrows He_2^6) = 0.619 \qquad (23)$$
$$X(C_6^{12} \leftrightarrows B_5^{12}) = 0.612$$

and, using also known values for η_{ba} [Eq. (3c)] and $\Lambda^{(\beta)}(b\to a)/f_{ba} = \ln 2/[f_{ba}\tau_{\frac{1}{2}}(b\to a)]$, found, from Eqs. (23) and (22a),

$$\Lambda^{(\mu)}(He_2^3\to H_1^3) = 1.46\times10^3\ \mathrm{sec}^{-1},$$
$$\Lambda^{(\mu)}(Li_3^6\to He_2^6) = 1.79\times10^3\ \mathrm{sec}^{-1}, \qquad (24)$$
$$\Lambda^{(\mu)}(C_6^{12}\to B_5^{12}) = 7.86\times10^3\ \mathrm{sec}^{-1},$$

with an over-all uncertainty of some 10 to 15%. A calculation by Wolfenstein,[15] based on general assumptions entirely similar to those of Fujii and Primakoff, yields $\Lambda^{(\mu)}(C_6^{12}\to B_5^{12}) = 7.4\times10^3\ \mathrm{sec}^{-1}$.

As regards experimental values of $\Lambda^{(\mu)}(a\to b)$, data are at present available only in the $C_6^{12}\to B_5^{12}$ case and are:

$$[\Lambda^{(\mu)}(C_6^{12}\to B_5^{12})]_{\mathrm{exper}} = \begin{cases}(9.05\pm0.95)\times10^3\ \mathrm{sec}^{-1},^{16}\\ (9.18\pm0.5)\times10^3\ \mathrm{sec}^{-1},^{17}\\ (6.6\pm1.1)\times10^3\ \mathrm{sec}^{-1},^{18}\\ (6.8\pm1.5)\times10^3\ \mathrm{sec}^{-1},^{19}\\ (5.9\pm1.5)\times10^3\ \mathrm{sec}^{-1}.^{14}\end{cases} \qquad (25)$$

[15] L. Wolfenstein, Conference on Weak Interactions, Gatlinburg, Tennessee (1958, to be published).
[16] Argo, Harrison, Kruse, and McGuire, Conference on Weak Interactions, Gatlinburg, Tennessee (1958), and preprint (to be published).
[17] Burgman, Fischer, Leontic, Lundby, Meunier, Stroot, and Teja, Phys. Rev. Letters 1, 469 (1958).
[18] Fetkovich, Fields, and McIlwain, Conference on Weak Interactions, Gatlinburg, Tennessee (1958).
[19] Love, Marder, Nadelhaft, Siegel, and Taylor, Conference on Weak Interactions, Gatlinburg, Tennessee (1958, to be published), R. Siegel (private communication).

TABLE II.

$\Lambda^{(\mu)}(C_6^{12}\to B_5^{12})$ in $sec^{-1}\times 10^{-3}$	Terms $\sim(\mu_p-\mu_n)$	$g_P^{(\mu)}/g_A^{(\beta)}$
7.86	included	+8
6.34	omitted	+8
11.80	included	−8
10.25	omitted	−8

It is thus clear, from Eqs. (24) and (25), that the theoretical value of $\Lambda^{(\mu)}(C_6^{12}\to B_5^{12})$ agrees, within the over-all theoretical and experimental uncertainties, with the corresponding experimental value. This agreement offers further support for the validity of our effective Hamiltonian $H_{\text{eff}}^{(\mu)}$ [Eqs. (1a) to (1c)].

It may be of interest in concluding this section to append a table—Table II—giving the theoretical values of $\Lambda^{(\mu)}(C_6^{12}\to B_5^{12})$ with the assumption of the conserved vector current abandoned, i.e., with anomalous magnetic moment contributions $[\sim(\mu_p-\mu_n)]$ to $G_A^{(\mu)}$, $G_P^{(\mu)}$ [Eq. (1b)] omitted and with $g_P^{(\mu)}$ {within $G_P^{(\mu)}$ [Eq. (1c)]} taken as $-8g_A^{(\beta)}$ as well as $+8g_A^{(\beta)}$. Such an ambiguity in the sign of $g_P^{(\mu)}/g_A^{(\beta)}$ may be contemplated since, without a sufficiently detailed theory of the $\pi^+\to\mu^++\nu$ process,[3] one can fix, on the basis of the known π^+ lifetime, only the square of the effective coupling constant for the $\mu^-+\pi^+\to\nu$ "step"[3,4] in the $\mu^-+p\to\mu^-+\pi^++n\to\nu+n$ "two-step" process. We find from Eqs. (2a), (2b), (21a) to (22b), (1b), and (1c), the numerical results given in Table II. These results, together with the values of $[\Lambda^{(\mu)}(C_6^{12}\to B_5^{12})]_{\text{exper}}$ in Eq. (25), indicate that (1) assumption of $g_P^{(\mu)}/g_A^{(\beta)}=-8$, which is inconsistent with the usually accepted detailed theory of the $\pi^+\to\mu^++\nu$ process which involves the dominance of the $p+\bar{n}$ "intermediate state",[3] yields values of $\Lambda^{(\mu)}(C_6^{12}\to B_5^{12})$ which fit experiment less well than values of $\Lambda^{(\mu)}(C_6^{12}\to B_5^{12})$ calculated with the assumption of $g_P^{(\mu)}/g_A^{(\beta)}=+8$; (2) if future observations uphold the first pair of experimental values in Eq. (25), use of $g_P^{(\mu)}/g_A^{(\beta)}=+8$ will, for agreement between experiment and theory of $\Lambda^{(\mu)}(C_6^{12}\to B_6^{12})$, require inclusion of the anomalous magnetic moment contributions to $G_A^{(\mu)}$, $G_P^{(\mu)}$ and so support the assumption of a conserved vector current. This last conclusion agrees with that reached at the end of Sec. 3 on the basis of a comparison of experiment and theory for the total muon capture rate, $\Lambda^{(\mu)}(a)$.

5. "HYPERFINE" EFFECT IN MUON CAPTURE RATE—MUON CAPTURE IN HYDROGEN

The total muon capture rate, $\Lambda^{(\mu)}(a)$, calculated in Sec. 2 or the partial muon capture rate, $\Lambda^{(\mu)}(a\to b)$, calculated in Sec. 4 are actually appropriate averages of the total or partial muon capture rates from the two individual hyperfine states of the parent mu-mesic atom, $\Lambda^{(\mu)}(J_a\pm\frac{1}{2};a)$ or $\Lambda^{(\mu)}(J_a\pm\frac{1}{2};a\to b)$, e.g.,

$$\Lambda^{(\mu)}(a)=\frac{2J_a+2}{4J_a+2}\Lambda^{(\mu)}(J_a+\tfrac{1}{2};a)+\frac{2J_a}{4J_a+2}\Lambda^{(\mu)}(J_a-\tfrac{1}{2};a). \tag{26}$$

Equation (26) represents such an appropriate average for $\Lambda^{(\mu)}(a)$ in terms of $\Lambda^{(\mu)}(J_a\pm\frac{1}{2};a)$; this average is "incoherent" and is weighted only according to the degeneracies of the two hyperfine states involved since (1) the energy difference, $\delta\epsilon_a$, between these two hyperfine states is much greater than their width, i.e.,

$$\delta\epsilon_a\gg\hbar[\Lambda_{\text{decay}}^{(\mu)}+\Lambda^{(\mu)}(J_a+\tfrac{1}{2};a)];$$
$$\delta\epsilon_a\gg\hbar[\Lambda_{\text{decay}}^{(\mu)}+\Lambda^{(\mu)}(J_a-\tfrac{1}{2};a)];$$

and (2) the rate of conversion from one of these states to the other is (with the exception of hydrogen and deuterium) sufficiently smaller than $\Lambda_{\text{decay}}^{(\mu)}+\Lambda^{(\mu)}(J_a\pm\frac{1}{2};a)$ for the various mu-mesic atoms of interest.[20] The physical reason for the difference between $\Lambda^{(\mu)}(J_a+\frac{1}{2};a)$ and $\Lambda^{(\mu)}(J_a-\frac{1}{2};a)$ arises, as discussed by Bernstein, Lee, Yang, and Primakoff[21] (B.L.Y. and P.), from the combined action of the following three effects (1) the correlation between the spin of the muon, $\frac{1}{2}\boldsymbol{\sigma}$, and the spin of the parent nucleus $\mathbf{J}_a$ is different in the two hyperfine states $F_a^{(\pm)}=J_a\pm\frac{1}{2}$; (2) there is, in general, a correlation between the spin $\frac{1}{2}\boldsymbol{\sigma}_i$ of the proton that captures the muon and $\mathbf{J}_a$; (3) the capture rate of the muon by the proton depends on their relative spin orientation via the terms $G_A^{(\mu)}\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_i$ and $G_P^{(\mu)}\boldsymbol{\sigma}\cdot\mathbf{v}_1\boldsymbol{\sigma}_i\cdot\mathbf{v}_1$ in the effective Hamiltonian $H_{\text{eff}}^{(\mu)}$ of Eqs. (1a) to (1c).

The total muon capture rates from the two individual hyperfine states $F_a^{(\pm)}=J_a\pm\frac{1}{2}$, $\Lambda^{(\mu)}(J_a\pm\frac{1}{2};a)$, can be calculated on the basis of the $H_{\text{eff}}^{(\mu)}$ of Eqs. (1a) to (1c) by the closure approximation method described in Sec. 2. By using a procedure similar to that involved in the derivation of Eqs. (3a) to (8) we find that

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[20] Conversion from the energetically higher to the energetically lower of the two hyperfine mu-mesic atom states occurs (1) via collisions with atoms of the muon moderating medium—this is important only for the electrically neutral and hence mobile hydrogen and deuterium mu-mesic atoms—see below; (2) via spontaneous magnetic dipole radiation; (3) via Auger electron ejection. The rates of (2) and (3) may be readily estimated and are, respectively:

$$R_{(2)}\approx 10\left(\frac{1}{137}\right)^{13}(Z_{\text{eff}})^9\left(\frac{m_\mu}{m_p}\right)^3\frac{m_\mu c^2}{\hbar}$$

and

$$R_{(3)}\approx 10\left(\frac{1}{137}\right)^{6}Z\left(\frac{m_e}{m_\mu}\right)^3\frac{m_\mu c^2}{\hbar},$$

so that for example, $R_{(2)}\approx 1\times 10^3$ sec^{-1}, $R_{(3)}\approx 4\times 10^5$ sec^{-1} for Al_{13}^{27}; on the other hand, the value of

$$\{\Lambda_{\text{decay}}^{(\mu)}+\Lambda^{(\mu)}(J_a\pm\tfrac{1}{2};a)\}$$

for Al_{13}^{27} is $\approx 1.2\times 10^6$ sec^{-1}.

[21] Bernstein, Lee, Yang, and Primakoff, Phys. Rev. 111, 313 (1958).

$$\frac{\Lambda^{(\mu)}(J_a\pm\frac{1}{2};a)}{\Lambda^{(\mu)}(a)}-1\cong\pm(J_a+\tfrac{1}{2}\pm\tfrac{1}{2})^{-1}\frac{b^{(\mu)}}{a^{(\mu)}}\frac{\langle a|2\mathbf{J}\cdot\mathbf{S}_{\mathrm{pr}}|a\rangle}{Z}$$
$$\times\left\{\frac{\xi_a-\left[\frac{1}{2}+\frac{1}{2}\frac{d^{(\mu)}}{b^{(\mu)}}\frac{\langle a|2\mathbf{J}\cdot\mathbf{S}_{\mathrm{neu}}|a\rangle}{\langle a|2\mathbf{J}\cdot\mathbf{S}_{\mathrm{pr}}|a\rangle}-\frac{1}{2}\frac{\langle a|2\mathbf{J}\cdot\mathbf{K}|a\rangle}{\langle a|2\mathbf{J}\cdot\mathbf{S}_{\mathrm{pr}}|a\rangle}\right]\alpha_a'^{(+)}-\left[\frac{A-Z}{2}+\frac{Z}{2}\frac{d^{(\mu)}}{b^{(\mu)}}\frac{\langle a|2\mathbf{J}\cdot\mathbf{S}_{\mathrm{neu}}|a\rangle}{\langle a|2\mathbf{J}\cdot\mathbf{S}_{\mathrm{pr}}|a\rangle}\right]\alpha_a'^{(-)}}{1-\left[\frac{A}{2Z}-\frac{\gamma_a^{(+)}}{Za^{(\mu)}}\right]\alpha_a^{(+)}-\left[\frac{A-Z}{2}-\frac{\gamma_a^{(-)}}{Za^{(\mu)}}\right]\alpha_a^{(-)}}\right\},\qquad(27a)$$

where

$$b^{(\mu)}\equiv2(G_V^{(\mu)}G_A^{(\mu)}-\tfrac{1}{3}G_V^{(\mu)}G_P^{(\mu)})-2[(G_A^{(\mu)})^2-\tfrac{2}{3}G_A^{(\mu)}G_P^{(\mu)}],$$

$$d^{(\mu)}\equiv2(G_V^{(\mu)}G_A^{(\mu)}-\tfrac{1}{3}G_V^{(\mu)}G_P^{(\mu)})+2[(G_A^{(\mu)})^2-\tfrac{2}{3}G_A^{(\mu)}G_P^{(\mu)}],$$

$$a^{(\mu)}\equiv(G_V^{(\mu)})^2+3(\Gamma_A^{(\mu)})^2=(G_V^{(\mu)})^2+3(G_A^{\mu})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)},$$

$$\mathbf{K}\equiv(\mathbf{Y}^{(1)}T^{(1)}+\mathbf{Y}^{(2)}T^{(2)})-\left(\frac{b^{(\mu)}-d^{(\mu)}}{2b^{(\mu)}}\right)(\mathbf{Y}^{(1)}T^{(1)}+\mathbf{Y}^{(2)}T^{(2)}+\mathbf{Y}^{(1)}\times\mathbf{Y}^{(2)}),$$

$$\xi_a\equiv\left(\int|\varphi(r)|^2\mathfrak{D}_a'(r)d\mathbf{r}\right)\bigg/\left(\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}\right),$$

$$\alpha_a'^{(\pm)}\equiv\left[\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}\varphi^*(r)\varphi(r')\tfrac{1}{2}(F_a'^{(+)}(\mathbf{r},\mathbf{r}')\pm F_a'^{(-)}(\mathbf{r},\mathbf{r}'))d\mathbf{r}d\mathbf{r}'\right]\bigg/\left[\int|\varphi(r)|^2\mathfrak{D}_a'(r)d\mathbf{r}\right],\qquad(27b)$$

$$\mathfrak{D}_a'(r)\equiv\frac{1}{2J_a+1}\sum_{M_a}\frac{\langle E_a,J_a,M_a|\mathbf{J}\cdot\sum_i[(1+\tau_i^{(3)})/2]\boldsymbol{\sigma}_i\delta(\mathbf{r}-\mathbf{r}_i)|E_a,J_a,M_a\rangle}{\langle E_a,J_a,M_a|\mathbf{J}\cdot\sum_i[(1+\tau_i^{(3)})/2]\boldsymbol{\sigma}_i|E_a,J_a,M_a\rangle},$$

$$F_a'^{(\pm)}(\mathbf{r},\mathbf{r}')\equiv\frac{\langle a|\mathbf{J}\cdot\sum'_{i,j}[\frac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i^{(3)}\tau_j^{(3)})(b^{(\mu)}+d^{(\mu)})(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)-\frac{1}{4}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{(3)}(b^{(\mu)}-d^{(\mu)})(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)]\times[\delta(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{r}'-\mathbf{r}_j)](1+P_{ij})/2|a\rangle}{\langle a|\mathbf{J}\cdot\sum'_{i,j}[\frac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i^{(3)}\tau_j^{(3)})(b^{(\mu)}+d^{(\mu)})(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)-\frac{1}{4}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{(3)}(b^{(\mu)}-d^{(\mu)}(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)]\times(1+P_{ij})/2|a},$$

$$\alpha_a^{(\pm)}\equiv\left[\int\int\frac{\sin(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}{(\langle\nu\rangle_a|\mathbf{r}-\mathbf{r}'|)}\varphi^*(r)\varphi(r')\tfrac{1}{2}(F_a^{(+)}(\mathbf{r},\mathbf{r}')\pm F_a^{(-)}(\mathbf{r},\mathbf{r}'))d\mathbf{r}d\mathbf{r}'\right]\bigg/\left[\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}\right],$$

$$\gamma_a^{(+)}\equiv\langle a|(G_V^{(\mu)})^2((\mathbf{T})^2-(T^{(3)})^2)+(\Gamma_A^{(\mu)})^2((\mathbf{Y}^{(1)})^2+(\mathbf{Y}^{(2)})^2)|a\rangle,$$

$$\gamma_a^{(-)}\equiv((G_V^{(\mu)})^2-(\Gamma_A^{(\mu)})^2)\langle a|(\mathbf{S}_{\mathrm{pr}})^2+(\mathbf{S}_{\mathrm{neu}})^2-(\mathbf{S}_{\mathrm{pr}}+\mathbf{S}_{\mathrm{neu}})^2|a\rangle.$$

We now apply Eq. (27a) to the case of an odd Z, odd A nucleus so that $\langle a|2\mathbf{J}\cdot\mathbf{S}_{\mathrm{neu}}|a\rangle=0$. Considering also the heavier nuclei, $Z>6$, $A>12$, and hence dropping terms $\sim1/Z$ within the curly bracket of Eq. (27a), we have:

$$\frac{\Lambda^{(\mu)}(J_a\pm\frac{1}{2};a)}{\Lambda^{(\mu)}(a)}-1\cong\pm(J_a+\tfrac{1}{2}\pm\tfrac{1}{2})^{-1}\frac{b^{(\mu)}}{a^{(\mu)}}\frac{\langle a|2\mathbf{J}\cdot\mathbf{S}_{\mathrm{pr}}|a\rangle}{Z}\left\{\frac{\xi_a-\left[\frac{1}{2}-\frac{1}{2}\frac{\langle a|2\mathbf{J}\cdot\mathbf{K}|a\rangle}{\langle a|2\mathbf{J}\cdot\mathbf{S}_{\mathrm{pr}}|a\rangle}\right]\alpha_a'^{(+)}-\left[\frac{A-Z}{2}\right]\alpha_a'^{(-)}}{1-[A/2Z]\alpha_a^{(+)}-[(A-Z)/2]\alpha_a^{(-)}}\right\}\qquad(28a)$$

so that

$$\frac{\Lambda^{(\mu)}(J_a+\frac{1}{2};a)-\Lambda^{(\mu)}(J_a-\frac{1}{2};a)}{\Lambda^{(\mu)}(a)}\cong\frac{2J_a+1}{J_a(J_a+1)}\frac{b^{(\mu)}}{a^{(\mu)}}\frac{\langle a|2\mathbf{J}\cdot\mathbf{S}_{\mathrm{pr}}|a\rangle}{Z}\left\{\frac{\xi_a-\left[\frac{1}{2}-\frac{1}{2}\frac{\langle a|2\mathbf{J}\cdot\mathbf{K}|a\rangle}{\langle a|2\mathbf{J}\cdot\mathbf{S}_{\mathrm{pr}}|a\rangle}\right]\alpha_a'^{(+)}-\left[\frac{A-Z}{2}\right]\alpha_a'^{(-)}}{1-[A/2Z]\alpha_a^{(+)}-[(A-Z)/2]\alpha_a^{(-)}}\right\}.\qquad(28b)$$

The quantity in the curly brackets in Eq. (28a) or (28b) is the ratio of the exclusion principle inhibition factors for $\Lambda^{(\mu)}(J_a+\frac{1}{2};a)-\Lambda^{(\mu)}(J_a-\frac{1}{2};a)$ and for $\Lambda^{(\mu)}(a)$ and should be close to unity. Assuming in addition that for the purpose of calculating $\langle a|2\mathbf{J}\cdot\mathbf{S}_{\rm pr}|a\rangle$ one can visualize the odd Z, odd A nucleus as consisting of an "outside" proton with orbital angular momentum $\mathbf{L}_a$ moving about a spinless "core," one has

$$\langle a|2\mathbf{J}\cdot\mathbf{S}_{\rm pr}|a\rangle=J_a(J_a+1)-L_a(L_a+1)+\tfrac{3}{4} = \begin{matrix} J_a+1: J_a=L_a+\frac{1}{2} \\ -J_a \;: J_a=L_a-\frac{1}{2} \end{matrix} \qquad (29)$$

so that, substituting into Eq. (28b),

$$\frac{\Lambda^{(\mu)}(J_a+\frac{1}{2};a)-\Lambda^{(\mu)}(J_a-\frac{1}{2};a)}{\Lambda^{(\mu)}(a)} \cong \frac{b^{(\mu)}}{a^{(\mu)}}\frac{1}{Z}\times\begin{bmatrix}(2J_a+1)/J_a & : J_a=L_a+\frac{1}{2} \\ -(2J_a+1)/(J_a+1) & : J_a=L_a-\frac{1}{2}\end{bmatrix} \qquad (30)$$

in essential agreement with the result for

$$(\Lambda^{(\mu)}(J_a+\tfrac{1}{2};a)-\Lambda^{(\mu)}(J_a-\tfrac{1}{2};a))/\Lambda^{(\mu)}(a)$$

obtained in another way by B.L.Y. and P.[21] The combination of coupling constants in Eq. (30), $b^{(\mu)}/a^{(\mu)}$, has the numerical value, using Eqs. (27b), (1b), (1c), and (3d), and $(g_A^{(\beta)}/g_V^{(\beta)})=-1.21$,[11]

$$\frac{b^{(\mu)}}{a^{(\mu)}}=-0.945. \qquad (31)$$

As an example of the magnitude of the hyperfine effect we calculate the quantity

$(\Lambda^{(\mu)}(J_a+\frac{1}{2};a)-\Lambda^{(\mu)}(J_a-\frac{1}{2};a))/\Lambda^{(\mu)}(a)$ from Eqs. (30) and (31) for Al_{13}^{27}. Here, $Z=13$, $J_a=\frac{5}{2}$, $L_a=2$, so that we have

$$\frac{\Lambda^{(\mu)}(J_a+\frac{1}{2};a)-\Lambda^{(\mu)}(J_a-\frac{1}{2};a)}{\Lambda^{(\mu)}(a)}\cong-0.17, \qquad (32)$$

which appears sufficiently large to be observed with available experimental techniques.

We now apply Eqs. (27a) and (27b) to the case of muon capture in hydrogen. Here

$$\langle a|2\mathbf{J}\cdot\mathbf{S}_{\rm neu}|a\rangle=0;\quad \langle a|2\mathbf{J}\cdot\mathbf{S}_{\rm pr}|a\rangle=2\cdot\tfrac{3}{4};$$
$$\langle a|2\mathbf{J}\cdot\mathbf{K}|a\rangle=2\cdot\tfrac{3}{4};\quad \langle a|(\mathbf{T})^2-(T^{(3)})^2|a\rangle=\tfrac{1}{2};$$
$$\langle a|(\mathbf{Y}^{(1)})^2+(\mathbf{Y}^{(2)})^2|a\rangle=\tfrac{3}{2};$$
$$\langle a|(\mathbf{S}_{\rm pr})^2+(\mathbf{S}_{\rm neu})^2-(\mathbf{S}_{\rm pr}+\mathbf{S}_{\rm neu})^2|a\rangle=0;$$
$$b^{(\mu)}/a^{(\mu)}=-0.922$$

[from Eqs. (27b), (1b), (1c), (2c), and $(g_A^{(\beta)}/g_V^{(\beta)})=-1.21$ [11]], $\Lambda^{(\mu)}(H_1^1)=169$ sec^{-1} [Eq. (17)], so that

$$\Lambda^{(\mu)}(\tfrac{1}{2}+\tfrac{1}{2};H_1^1)=\Lambda^{(\mu)}(H_1^1)\left(1+\frac{b^{(\mu)}}{a^{(\mu)}}\right)=13\ \text{sec}^{-1}, \qquad (33a)$$

$$\Lambda^{(\mu)}(\tfrac{1}{2}-\tfrac{1}{2};H_1^1)=\Lambda^{(\mu)}(H_1^1)\left(1-\frac{3b^{(\mu)}}{a^{(\mu)}}\right)=636\ \text{sec}^{-1}. \qquad (33b)$$

Thus the hyperfine effect is enormous—

$$(\Lambda^{(\mu)}(\tfrac{1}{2}-\tfrac{1}{2};H_1^1)/\Lambda^{(\mu)}(\tfrac{1}{2}+\tfrac{1}{2};H_1^1))\cong 50.$$

Muon capture in hydrogen is also unique in that within a time $\approx[p\mu^-]$ mu-mesic atom mean life $\tau^{\pm}$

$$(\tau^{\pm}=\{\Lambda_{\rm decay}^{(\mu)}+\Lambda^{(\mu)}(\tfrac{1}{2}\pm\tfrac{1}{2};H_1^1)\}^{-1}\approx 1/\Lambda_{\rm decay}^{(\mu)}=2.21\times10^{-6}\ \text{sec})$$

there is, under certain circumstances, a high probability of conversion from the energetically higher triplet state with $F_a^{(+)}=\frac{1}{2}+\frac{1}{2}$ to the energetically lower singlet state with $F_a^{(-)}=\frac{1}{2}-\frac{1}{2}$. This is a consequence of the fact that, as pointed out by Gershtein and Zeldovich,[22] the $[p\mu^-]$ mu-mesic is electrically neutral and so wanders through the hydrogen gas or liquid making collisions with the $(H_1^1)_2$ molecules—these collisions occasionally result in "exchange" between the proton in the incident $[p\mu^-]$ mu-mesic atom with one direction of spin and a proton in the target $(H_1^1)_2$ molecule with the opposite direction of spin, with the net result that the $[p\mu^-]$ is converted from the triplet state to the singlet state. The rate of such a collisional conversion process is estimated by Gershtein[22] to be $\approx 5\times10^9$ sec^{-1} at an $(H_1^1)_2$ molecule number density, $\mathfrak{N}/V$, of $2\times10^{22}/\text{cm}^3$—thus at $\mathfrak{N}/V$ less than, say, $2\times10^{17}/\text{cm}^3$ there is no appreciable collisional triplet to singlet conversion and the muon capture rate is, from Eqs. (33a), (33b), and (26),

$$\tfrac{3}{4}\Lambda^{(\mu)}(\tfrac{1}{2}+\tfrac{1}{2};H_1^1)+\tfrac{1}{4}\Lambda^{(\mu)}(\tfrac{1}{2}-\tfrac{1}{2};H_1^1)=\Lambda^{(\mu)}(H_1^1)=169\ \text{sec}^{-1}.$$

With increasing $\mathfrak{N}/V$ the collisional triplet to singlet conversion becomes progressively more important and at $\mathfrak{N}/V\approx10^{20}/\text{cm}^3$ practically all the $[p\mu^-]$ mu-mesic atoms are in the singlet state at the instant of muon decay or capture—the corresponding muon capture rate is, from Eq. (33b), $\Lambda^{(\mu)}(\frac{1}{2}-\frac{1}{2};H_1^1)=636$ sec^{-1}. At still higher $\mathfrak{N}/V$, e.g., at $\mathfrak{N}/V$ of the order of those in liquid hydrogen, formation of mu-mesic hydrogen molecule ions, $[p\mu^-p]$, becomes dominant[23] and it is expected that most of the muons will be found at the instant of their decay or capture in the lowest Bohr orbit of a

[22] S. S. Gershtein, J. Exptl. Theoret. Phys. U.S.S.R.) **34**, 463 (1958); *ibid.* **34**, 993 (1958).
[23] Y. B. Zeldovich, Doklady Akad. Nauk (S.S.S.R.) **95**, 493 (1954), and J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 310 (1957); J. D. Jackson, Phys. Rev. **106**, 330 (1957); T. H. R. Skyrme, Phil. Mag. **2**, 910 (1957); Y. B. Zeldovich and S. S. Gershtein, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 649 (1958); Cohen, Judd, and Riddell, Phys. Rev. **110**, 1471 (1958).

$[p\mu^- p]$. Any such $[p\mu^- p]$ must have a total spin angular momentum of $\frac{1}{2}$ (since one of its parents is a singlet $[p\mu^-]$) and so possesses a muon spin $\boldsymbol{\sigma}$—proton spin $\boldsymbol{\sigma}_1$, $\boldsymbol{\sigma}_2$ configuration intermediate between that in a triplet and that in a singlet $[p\mu_-]$—in fact

$$\langle(\mathbf{F})^2\rangle_a=\tfrac{3}{4}=\langle[\tfrac{1}{2}\boldsymbol{\sigma}+(\tfrac{1}{2}\boldsymbol{\sigma}_1+\tfrac{1}{2}\boldsymbol{\sigma}_2)]^2\rangle_a$$
$$=\tfrac{3}{4}+\langle(\tfrac{1}{2}\boldsymbol{\sigma}_1+\tfrac{1}{2}\boldsymbol{\sigma}_2)^2\rangle_a+2\langle(\tfrac{1}{2}\boldsymbol{\sigma})\cdot(\tfrac{1}{2}\boldsymbol{\sigma}_1+\tfrac{1}{2}\boldsymbol{\sigma}_2)\rangle_a$$

so that

$$\langle\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_1\rangle_a=\langle\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_2\rangle_a=-\langle(\tfrac{1}{2}\boldsymbol{\sigma}_1+\tfrac{1}{2}\boldsymbol{\sigma}_2)^2\rangle_a=0,\ -2$$

in *para*, *ortho* $[p\mu^- p]$ while $\langle\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_1\rangle_a=1,\ -3$ in triplet, singlet $[p\mu^-]$. Further, since from Eqs. (33a) and (33b),

$$\Lambda^{(\mu)}(\tfrac{1}{2}\pm\tfrac{1}{2};\,\mathrm{H}_1{}^1)\sim\left(1+\langle\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_1\rangle_{a;\pm}\frac{b^{(\mu)}}{a^{(\mu)}}\right)$$
$$=\left(1+\begin{pmatrix}1\\-3\end{pmatrix}\times\frac{b^{(\mu)}}{a^{(\mu)}}\right)$$

the muon capture rate in *para* $[p\mu^- p]$, *ortho* $[p\mu^- p]$ will, in an analogous fashion, be

$$\sim\left(1+\langle\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_1\rangle_{a;\,para,\,ortho}\frac{b^{(\mu)}}{a^{(\mu)}}\right)=\left(1+\begin{pmatrix}0\\-2\end{pmatrix}\times\frac{b^{(\mu)}}{a^{(\mu)}}\right).$$

It follows that at high $\mathfrak{N}/V$ the muon capture rate has the form

$$x_1\left(\frac{\mathfrak{N}}{V}\right)\left[\Lambda^{(\mu)}(\mathrm{H}_1{}^1)\left(1-\frac{3b^{(\mu)}}{a^{(\mu)}}\right)\right]$$
$$+x_2\left(\frac{\mathfrak{N}}{V}\right)[(2\gamma)\Lambda^{(\mu)}(\mathrm{H}_1{}^1)]$$
$$+x_3\left(\frac{\mathfrak{N}}{V}\right)\left[(2\gamma)\Lambda^{(\mu)}(\mathrm{H}_1{}^1)\left(1-\frac{2b^{(\mu)}}{a^{(\mu)}}\right)\right]$$
$$=\left\{x_1\left(\frac{\mathfrak{N}}{V}\right)[636]+x_2\left(\frac{\mathfrak{N}}{V}\right)[(2\gamma)(169)]\right.$$
$$\left.+x_3\left(\frac{\mathfrak{N}}{V}\right)[(2\gamma)(480)]\right\}\ \mathrm{sec}^{-1},\quad(34)$$

where $x_1(\mathfrak{N}/V)$, $x_2(\mathfrak{N}/V)$, $x_3(\mathfrak{N}/V)$, are the fractions of muons found in singlet $[p\mu^-]$, *para* $[p\mu^- p]$ and *ortho* $[p\mu^- p]$ at the $\mathfrak{N}/V$ in question—$x_1(\mathfrak{N}/V)+x_2(N/V)+x_3(\mathfrak{N}/V)=1$—and γ is the ratio of the absolute square of the muon orbital wave function at the proton position in $[p\mu^- p]$ and in $[p\mu^-]$. Equations (33a), (33b), and (34) show that the muon capture rate in hydrogen, considered as a function of $\mathfrak{N}/V$, exhibits a maximum, $\cong 636$ sec^{-1}, at an intermediate value of $\mathfrak{N}/V$; if in addition $2\gamma\cong 1$ and $(x_2(\mathfrak{N}/V)/x_3(\mathfrak{N}/V))\gg 1$ this rate falls to values $\cong 169$ sec^{-1} at high $\mathfrak{N}/V$ as well as at very low $\mathfrak{N}/V$.

It is necessary for the physical relevance of the whole above discussion that the isotopically natural hydrogen in which the muons stop be purified of deuterium to an extent—factor ≈ 25–50 in relative abundance—sufficient to prevent any frequent occurrence of the "exchange" reaction $[p\mu^-]+d\rightarrow[d\mu^-]+p$. Any appreciable rate for such an exchange reaction with the subsequent formation of $[d\mu^- p]$ and the resultant "muon catalysis" of $d+p\rightarrow \mathrm{He}_2{}^3+\gamma$ complicates the situation very considerably. On the other hand, a similar analysis may be given for muons stopping in pure deuterium with the major difference that at high $\mathfrak{N}/V$ the sequence of reactions:

$$[d\mu^-]+d\rightarrow[d\mu^- d];\quad [d\mu^- d]\begin{array}{l}\nearrow \mathrm{He}_2{}^3+n+\mu^-\\ \searrow \mathrm{H}_1{}^3+p+\mu^-\end{array}$$

is anticipated with most of the muons eventually being trapped in Bohr orbits about the $\mathrm{He}_2{}^3$ nuclei.[24] Thus most muons stopping in pure liquid deuterium which are eventually captured by nuclei, are captured by nuclei of $\mathrm{He}_2{}^3$, a circumstance which permits study of muon capture in $\mathrm{He}_2{}^3$ without any anterior possession of a $\mathrm{He}_2{}^3$ target.

In concluding this section we remark again that the study of muon capture in hydrogen will not only ultimately yield the most reliable values of the effective muon—dressed nucleon coupling constants, $G_V{}^{(\mu)}$, $G_A{}^{(\mu)}$, $G_P{}^{(\mu)}$, i.e., values free of "nuclear physics" uncertainties, but will also shed light on several other interesting effects such as the collisional conversion in $[p\mu^-]$ and the formation of *para* $[p\mu^- p]$ and *ortho* $[p\mu^- p]$.

6. RADIATIVE MUON CAPTURE—TOTAL RATE AND PHOTON-NEUTRINO ANGULAR CORRELATION

We now discuss the process of radiative muon capture. Here calculations of the total rate and of the shape of the corresponding internal bremsstrahlung (I.B.) momentum spectrum have been made by Cantwell[25] for capture by light and medium-heavy nuclei ($Z/137 \ll 1$). Cantwell uses the effective Hamiltonian of Eqs. (1a) and (1b) but with all nucleon recoil corrections, i.e. all terms $\sim\nu/m_p$ omitted so that $G_V{}^{(\mu)}=g_V{}^{(\mu)}$; $G_A{}^{(\mu)}=g_A{}^{(\mu)}$; $G_P{}^{(\mu)}=0$. The radiative capture is visualized as predominantly due to the "two-step process"

$$\mu^-+p\rightarrow\gamma+(\mu^-)'+p\rightarrow\gamma+\nu+n\qquad(35)$$

and a second-order perturbation calculation, using an appropriate (free particle) Green's function to describe the virtual intermediate μ^- states, is carried out. In this way Cantwell obtains a relation for the relative

[24] J. D. Jackson, reference 23.
[25] R. M. Cantwell, Ph.D. thesis, Washington University (1956).

probability of radiative to nonradiative muon capture

$$\frac{\sum_b \Lambda_{\text{rad}}^{(\mu)}(a\rightarrow b;\,\boldsymbol{\gamma}_{ba})d(\gamma_{ba}/\gamma_{ba}^{\max})d\boldsymbol{\gamma}_1}{\sum_b \Lambda^{(\mu)}(a\rightarrow b)}$$

$$=\frac{\displaystyle\sum_b\left[\left(\frac{1}{4\pi^2}\frac{1}{137}\right)\left(\frac{\gamma_{ba}^{\max}}{m_\mu}\right)^2\left(1-\frac{\gamma_{ba}}{\gamma_{ba}^{\max}}\right)^2\left(\frac{\gamma_{ba}}{\gamma_{ba}^{\max}}\right)d\left(\frac{\gamma_{ba}}{\gamma_{ba}^{\max}}\right)d\boldsymbol{\gamma}_1\right]\times\left[\int\frac{d\mathbf{v}_1}{4\pi}\{(g_V^{(\mu)})^2(1+\mathbf{v}_1\cdot\boldsymbol{\gamma}_1)|\langle b|\exp'|a\rangle|^2+(g_A^{(\mu)})^2(1-\mathbf{v}_1\cdot\boldsymbol{\gamma}_1)|\langle b|(\exp')\boldsymbol{\sigma}|a\rangle|^2+(g_A^{(\mu)})^2 2\,\mathrm{Re}(\mathbf{v}_1\cdot\langle b|(\exp')\boldsymbol{\sigma}|a\rangle^*\boldsymbol{\gamma}_1\cdot\langle b|(\exp')\boldsymbol{\sigma}|a\rangle)\right]}{\displaystyle\sum_b\left[\int\frac{d\mathbf{v}_1}{4\pi}\{(g_V^{(\mu)})^2|\langle b|\exp|a\rangle|^2+(g_A^{(\mu)})^2|\langle b|(\exp)\boldsymbol{\sigma}|a\rangle|^2\}\right]},\quad\text{(36a)}$$

where

$$\exp'\equiv\sum_i\tau_i^{(-)}\exp[-i(\mathbf{v}_{ba}'+\boldsymbol{\gamma}_{ba})\cdot\mathbf{r}_i]\varphi(r_i);\quad(\exp')\boldsymbol{\sigma}\equiv\sum_i\tau_i^{(-)}\exp[-i(\mathbf{v}_{ba}'+\boldsymbol{\gamma}_{ba})\cdot\mathbf{r}_i]\varphi(r_i)\boldsymbol{\sigma}_i,$$
$$\exp\equiv\sum_i\tau_i^{(-)}\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)\varphi(r_i);\quad(\exp)\boldsymbol{\sigma}\equiv\sum_i\tau_i^{(-)}\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)\varphi(r_i)\boldsymbol{\sigma}_i.\quad\text{(36b)}$$

In Eq. (36a) the sum over b runs over all the energetically accessible states of the daughter nucleus; $\mathbf{v}_{ba}'=\nu_{ba}'\mathbf{v}_1$, $\boldsymbol{\gamma}_{ba}=\gamma_{ba}\boldsymbol{\gamma}_1$ are the momenta of the neutrino and I.B. photon emitted in the radiative capture; the energy, momentum conservation laws yield, analogously to Eqs. (2c) and (3d),

$$\nu_{ba}'+\gamma_{ba}\cong m_\mu\left[1-\frac{\epsilon_a}{m_\mu}-\frac{(E_b-E_a)}{m_\mu}\right]\cong\nu_{ba}.\quad\text{(36c)}$$

The first square bracket in the numerator represents an I.B. momentum spectrum of the same shape as that found by Morrison and Schiff in allowed K-electron orbital capture[26]; the second square bracket describes the dependence of the I.B. momentum spectrum on the wave functions $|a\rangle$, $|b\rangle$ of the parent and daughter nuclei—this second square bracket is however such for $\gamma_{ba}/\gamma_{ba}^{\max}=\gamma_{ba}/\nu_{ba}\cong0$ and $\cong1$ that the I.B. momentum spectrum has the allowed K-capture shape both near its low-energy end and its high-energy end. Thus a careful study of the shape of I.B. momentum spectrum near its high-energy end will yield $\langle\gamma\rangle_a^{\max}$, the average value of $\gamma_{ba}^{\max}$ over the states b; since $\langle\gamma\rangle_a^{\max}=\langle\nu\rangle_a$ one will in this way find empirically the average value of the neutrino energy involved in nonradiative muon capture by a particular parent nucleus and so remove the major uncertainty associated with the evaluation of the coupling constant ratio $\mathfrak{R}$ [Eq. (6b)] from the experimental data [see Sec. 3, Eqs. (19), (20), *et seq.*].

We now apply the closure approximation for the evaluation of the sums over b in Eq. (36a). By using the techniques of Eqs. (3b) to (6c), we obtain

$$\frac{\Lambda_{\text{rad}}^{(\mu)}(a;\langle\boldsymbol{\gamma}\rangle_a)dx_ad\boldsymbol{\gamma}_1}{\Lambda^{(\mu)}(a)}=\left[\left(\frac{1}{4\pi^2}\frac{1}{137}\right)\left(\frac{\langle\gamma\rangle_a^{\max}}{m_\mu}\right)^2(1-x_a)^2x_adx_ad\boldsymbol{\gamma}_1\right]$$
$$\times\left[\frac{((g_V^{(\mu)})^2+3(g_A^{(\mu)})^2)\displaystyle\int\frac{d\mathbf{v}_1}{4\pi}(1-\mathfrak{g}_a'(\mathbf{v}_1,\boldsymbol{\gamma}_1))+((g_V^{(\mu)})^2-(g_A^{(\mu)})^2)\int\frac{d\mathbf{v}_1}{4\pi}\mathbf{v}_1\cdot\boldsymbol{\gamma}_1(1-\mathfrak{g}_a''(\mathbf{v}_1,\boldsymbol{\gamma}_1))}{((g_V^{(\mu)})^2+3(g_A^{(\mu)})^2)(1-\mathfrak{g}_a)}\right],\quad\text{(37a)}$$

where $\mathfrak{g}_a$ is the nonradiative capture exclusion principle inhibition factor given in Eq. (6c) and $\mathfrak{g}_a'(\mathbf{v}_1,\boldsymbol{\gamma}_1)$, $\mathfrak{g}_a''(\mathbf{v}_1,\boldsymbol{\gamma}_1)$ are the corresponding radiative capture inhibition factors

$$\mathfrak{g}_a'(\mathbf{v}_1,\boldsymbol{\gamma}_1)=-\frac{\langle a|\sum_{i,j}'[\frac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i^{(3)}\tau_j^{(3)})((g_V^{(\mu)})^2+(g_A^{(\mu)})^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)][\exp\{i(\langle\nu'\rangle_a\mathbf{v}_1+\langle\gamma\rangle_a\boldsymbol{\gamma}_1)\cdot\mathbf{r}_{ij}\}\varphi^*(r_i)\varphi(r_j)]|a\rangle}{Z[(g_V^{(\mu)})^2+3(g_A^{(\mu)})^2]\left[\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}\right]},\quad\text{(37b)}$$

$$\mathfrak{g}_a''(\mathbf{v}_1,\boldsymbol{\gamma}_1)=-\frac{\langle a|\sum_{i,j}'[\frac{1}{4}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j-\tau_i^{(3)}\tau_j^{(3)})((g_V^{(\mu)})^2-\frac{1}{3}(g_A^{(\mu)})^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)][\exp\{i(\langle\nu'\rangle_a\mathbf{v}_1+\langle\gamma\rangle_a\boldsymbol{\gamma}_1)\cdot\mathbf{r}_{ij}\}\varphi^*(r_i)\varphi(r_j)]|a\rangle}{Z[(g_V^{(\mu)})^2-(g_A^{(\mu)})^2]\left[\int|\varphi(r)|^2\mathfrak{D}_a(r)d\mathbf{r}\right]}\quad\text{(37c)}$$

[26] P. Morrison and L. I. Schiff, Phys. Rev. 58, 24 (1940).

with

$$\langle\nu'\rangle_a+\langle\gamma\rangle_a\cong m_\mu\left\{1-\frac{\epsilon_a}{m_\mu}-\frac{\langle\text{excitation energy of daughter nucleus } b\rangle}{m_\mu}-\frac{[(E_b)_{\text{ground state}}-E_a]}{m_\mu}\right\}$$

$$x_a\equiv\frac{\langle\gamma\rangle_a}{\langle\gamma_a\rangle^{\max}}=\frac{\langle\gamma\rangle_a}{\langle\nu\rangle_a}. \tag{37d}$$

As a rough first approximation to evaluate Eq. (37a), we take $\mathcal{G}_a'(\mathbf{v}_1,\boldsymbol{\gamma}_1)\cong\mathcal{G}_a''(\mathbf{v}_1,\boldsymbol{\gamma}_1)\cong\mathcal{G}_a$; in this case the I.B. momentum spectrum has the allowed K-capture shape $\sim(1-x_a)^2x_a$ and the I.B. photon-neutrino angular correlation function is

$$1+\left(\frac{(g_V^{(\mu)})^2-(g_A^{(\mu)})^2}{(g_V^{(\mu)})^2+3(g_A^{(\mu)})^2}\right)\mathbf{v}_1\cdot\boldsymbol{\gamma}_1. \tag{38}$$

Equation (38) is rigorous for radiative muon capture by protons and has been derived directly for this case by Huang, Yang, and Lee[27] (H.Y. and L.). As a second approximation, we can use the techniques of Eqs. (7a) to (11b) and obtain for the heavier nuclei, $Z>6$, $A>12$

$$\mathcal{G}_a'(\mathbf{v}_1,\boldsymbol{\gamma}_1)\cong\mathcal{G}_a''(\mathbf{v}_1,\boldsymbol{\gamma}_1)\cong\left(\frac{A-Z}{2A}\right)\left(\frac{d}{r_0}\right)^3\times\left[1-\frac{1}{10}(\langle\nu'\rangle_a r_0\mathbf{v}_1+\langle\gamma\rangle_a r_0\boldsymbol{\gamma}_1)^2\left(\frac{d}{r_0}\right)^2+\cdots\right] \tag{39a}$$

entirely analogous to the result [Eqs. (6a), (11a), and (11b)]

$$\mathcal{G}_a\cong\left(\frac{A-Z}{2A}\right)\delta_a\cong\left(\frac{A-Z}{2A}\right)\left(\frac{d}{r_0}\right)^3\times\left[1-\frac{1}{10}(\nu_a r_0)^2\left(\frac{d}{r_0}\right)^2+\cdots\right]. \tag{39b}$$

Equations (37a), (39a), and (39b) yield

$$\frac{\Lambda_{\text{rad}}^{(\mu)}(a;\langle\gamma\rangle_a)dx_a d\boldsymbol{\gamma}_1}{\Lambda^{(\mu)}(a)}\cong\left[\left(\frac{1}{4\pi^2}\frac{1}{137}\right)\left(\frac{\langle\gamma\rangle_a^{\max}}{m_\mu}\right)^2(1-x_a)^2x_a dx_a d\boldsymbol{\gamma}_1\right]\times\left\{1-(1-x_a)x_a\left\{\frac{1}{5}+\frac{1}{15}\left[\frac{(g_A^{(\mu)})^2-(g_V^{(\mu)})^2}{(g_V^{(\mu)})^2+3(g_A^{(\mu)})^2}\right]\right\}\right\}\times\left\{\frac{\left(\frac{A-Z}{2A}\right)\delta_a}{1-\left(\frac{A-Z}{2A}\right)\delta_a}\right\}\left[\left(\frac{d}{r_0}\right)^2(\langle\gamma\rangle_a^{\max}r_0)^2\right] \tag{40}$$

which shows that the correction factor to the allowed K-capture shape for the I.B. momentum spectrum involves [as does the I.B. photon-neutrino angular correlation, Eq. (38)] the quantity $(g_A^{(\mu)}/g_V^{(\mu)})^2$. Taking

$$(g_A^{(\mu)}/g_V^{(\mu)})^2\cong(g_A^{(\beta)}/g_V^{(\beta)})^2=(1.21)^{2\,11};$$

$$(d/r_0)^2=(1.47)^2\cong(\delta_a)^{\frac{2}{3}},$$

$$\delta_a=3.0 \text{ [Eqs. (11b) and (14)]};$$

$$\langle\gamma\rangle_a^{\max}=\langle\nu\rangle_a\cong\tfrac{3}{4}m_\mu,$$

$$r_0\cong\frac{0.67}{m_\mu} \text{ [Eq. (11b) } et\ seq.\text{]}$$

and $(A-Z)/2A\cong\frac{1}{4}$, Eq. (40) gives

$$\frac{\Lambda_{\text{rad}}^{(\mu)}(a;\langle\gamma\rangle_a)dx_a}{\Lambda^{(\mu)}(a)}\cong\left[\left(\frac{1}{\pi}\frac{1}{137}\right)\left(\frac{3}{4}\right)^2(1-x_a)^2x_a dx_a\right]\times[1-0.35(1-x_a)x_a] \tag{41}$$

so that the ratio of the total radiative capture rate,

$$\Lambda_{\text{rad}}^{(\mu)}(a)=\int_0^1\Lambda_{\text{rad}}^{(\mu)}(a,\langle\gamma\rangle_a)dx_a,$$

to the total nonradiative capture rate is

$$\frac{\Lambda_{\text{rad}}^{(\mu)}(a)}{\Lambda^{(\mu)}(a)}\cong\left[\left(\frac{1}{12\pi}\frac{1}{137}\right)\left(\frac{3}{4}\right)^2\right]\left[1-\frac{0.35}{5}\right]=1\times10^{-4}. \tag{42}$$

Equations (40), (41), and (42) hold also for radiative muon capture by a proton upon omission of the term $(\langle\gamma\rangle_a^{\max}/m_\mu)^2=(\frac{3}{4})^2$ and of the term $\sim(1-x_a)x_a$, $-0.35/5$—this has been shown directly in the case of Eq. (42) by H.Y. and L.[27] and may be seen from Eq. (37a) since for a proton $\langle\gamma\rangle_a^{\max}/m_\mu=0.94\cong1$ and $\mathcal{G}_a'(\mathbf{v}_1,\boldsymbol{\gamma}_1)=0$, $\mathcal{G}_a''(\mathbf{v}_1,\boldsymbol{\gamma}_1)=0$ (as $Z=A$).

Cantwell[25] has given, on the basis of the evaluation of Eqs. (37a) to (37d) with a simple variational trial form for

$$a|\rangle\cong\Phi_a(\mathbf{r}_1,\mathbf{r}_2,\cdots)X_a(\sigma_1^{(3)},\sigma_2^{(3)},\cdots;\tau_1^{(3)},\tau_2^{(3)},\cdots)$$

[Eqs. (16) *et seq.*], an expression for

$$\frac{\Lambda_{\text{rad}}^{(\mu)}(a,\langle\gamma\rangle_a)dx_a}{\Lambda^{(\mu)}(a)}$$

[27] Huang, Yang, and Lee, Phys. Rev. **108**, 1348 (1957).

in the case $\mu^- + He_2^4 \rightarrow \gamma + \nu + H_1^4$. This expression corresponds to a correction factor to the allowed K-capture shape $\cong[1-1.8(1-x_a)x_a]$ (for $(g_A^{(\mu)}/g_V^{(\mu)})^2 \cong (g_A^{(\beta)}/g_V^{(\beta)})^2 = (1.21)^2$ [11]) rather than

$$\cong[1-0.35(1-x_a)x_a]$$

as predicted by Eq. (41) for the heavier nuclei. We should also mention that all of the results in Eqs. (36a) to (42) refer, in the case $J_a \neq 0$, to the appropriate averages [as in Eq. (26)] of the various radiative and nonradiative muon capture rates over the two $(J_a+\frac{1}{2}, J_a-\frac{1}{2})$ hyperfine states of the parent mu-mesic atom.

The I.B. momentum spectrum of Eqs. (40) or (41), is susceptible to observational test—a plot of experimental values of $\Lambda_{\text{rad}}^{(\mu)}(a;\langle\gamma\rangle_a)/[(1-x_a)^2x_a]$ *vs* the corresponding $(1-x_a)x_a$ should, if our approximations are sufficiently accurate, lie on a straight line with a slope whose value yields $(g_A^{(\mu)}/g_V^{(\mu)})^2$ (subject to the uncertainty in the numerical value of δ_a). This I.B. spectrum peaks at $x_a \cong \frac{1}{3}$ or $\langle\gamma\rangle_a \cong 27$ Mev and still has very appreciable values at $\langle\gamma\rangle_a \geqq 55$ Mev, i.e., at I.B. photon energies high compared to the energies of any numerous background photons. This fact should permit detection of the muon radiative capture in spite of the relative rarity of the phenomenon.

In concluding the present section it is important to emphasize the interest of a calculation of radiative muon capture using an effective Hamiltonian which includes all nucleon recoil corrections (terms $\sim\nu/m_p$) and in particular contains the "induced" pseudoscalar and the "conserved vector current" anomalous magnetic moment contributions. Such a calculation is now being carried out by Bernstein[28] and is expected to exhibit additional terms in the correction factor to the allowed K-capture shape for the I.B. momentum spectrum.

7. PARITY NONCONSERVATION EFFECTS

The anticipated presence of parity nonconservation effects in maximum amounts in the muon capture process is incorporated into the effective Hamiltonian of Eq. (1a) through the assumption that the emitted neutrino carries unit negative helicity—this is expressed mathematically by the (two-component neutrino coupling type) spin projection operator $(1-\boldsymbol{\sigma}\cdot\mathbf{v}_l)/\sqrt{2}$ in the $H_{\text{eff}}^{(\mu)}$ of Eq. (1a). No direct experimental test of this assumption is as yet available for muon capture but it has now been established that neutrinos emitted in the analogous processes of electron orbital capture and positron beta decay do possess a helicity $=-1$.[11] It would clearly be of great interest to observe parity nonconservation effects in muon capture and the present section is devoted to a discussion of four phenomena in which pseudoscalar quantities are to be measured—cf. Eqs. (44), (53), (57), and (59).

(a) Angular Distribution of Recoil Nuclei in Capture of Polarized Muons

Experimental evidence is now available that negative muons still retain an appreciable fraction of their spin polarization at the instant of decay or capture from the lowest Bohr orbit of the parent mu-mesic atom. This evidence is based on the observation of an anisotropic angular distribution of the decay electrons relative to a unit vector, $\mathbf{s}_{\mu;1}$, in the direction of the muon spin,

$$1-P_\mu\tfrac{1}{3}(\mathbf{s}_{\mu;1}\cdot\mathbf{p}_{\text{el};1}), \tag{43a}$$

$$(\mathbf{s}_{\mu;1}\cdot\mathbf{p}_{\text{neg. muon};1})=+1,^{29} \tag{43b}$$

and corresponds to a residual muon polarization at the instant of decay or capture, P_μ, of about 15 to 20% for the case of various spin zero parent nuclei.[30] The angular distribution of recoil daughter nuclei in a particular state, the recoils being formed in muon capture by spin zero parent nuclei, is then also expected to exhibit an anisotropy relative to $\mathbf{s}_{\mu;1}$, *viz.*,

$$1+P_\mu\alpha(a\rightarrow b)(\mathbf{s}_{\mu;1}\cdot\mathbf{p}_{\text{rec};1}), \tag{44}$$

where $\mathbf{p}_{\text{rec};1}$ is a unit vector in the direction of the recoiling nucleus and the anisotropy coefficient, $\alpha(a\rightarrow b)$, is a quantity involving the muon capture nuclear matrix elements of Eq. (2b). To avoid complications associated with the "hyperfine" effect (Sec. 5) we confine our discussion until further notice to the case of zero spin parent nuclei—this case is in addition distinguished by a lack of hyperfine-structure induced muon depolarization which, for example if $J_a=\frac{1}{2}$, cuts down the otherwise effective value of P_μ by a factor of at least 2 [see Eqs. (52b) to (52d)].

Calculation of $\alpha(a\rightarrow b)$ on the basis of the $H_{\text{eff}}^{(\mu)}$ of Eq. (1a) yields

$$\alpha(a\rightarrow b)\cong\frac{\sum_{M_b,M_a}\{(G_V^{(\mu)})^2|\langle b|\exp|a\rangle|^2+\frac{1}{3}(-(G_A^{(\mu)})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)})|\langle b|(\exp)\boldsymbol{\sigma}|a\rangle|^2\}}{\sum_{M_b,M_a}\{(G_V^{(\mu)})^2|\langle b|\exp|a\rangle|^2+\frac{1}{3}(3(G_A^{(\mu)})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)})|\langle b|(\exp)\boldsymbol{\sigma}|a\rangle|^2\}} \tag{45}$$

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[28] J. Bernstein (to be published).

[29] Culligan, Frank, and Holt, Conference on Weak Interactions, Gatlinburg, Tennessee (1958, to be published); Macq, Crowe, and Haddock, Phys. Rev. **112**, 2061 (1958).

[30] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); Ignatenko, Yegorov, Khalupa, and Chulthem, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 5 (1958).

subject to the assumption that the wave functions $|a; J_a=0^+\rangle$, $|b; J_i\rangle$ are such that

$$\sum_{M_b, M_a} |\langle b|(\exp)\boldsymbol{\sigma}\cdot\mathbf{v}_1|a\rangle|^2 \cong \tfrac{1}{3} \sum_{M_b, M_a} |\langle b|(\exp)\boldsymbol{\sigma}|a\rangle|^2.^{1}$$

Formulas equivalent to that in Eq. (45), but without inclusion of the anomalous magnetic moment terms in the corresponding $H_{\text{eff}}^{(\mu)}$ and sometimes without the term in $G_P^{(\mu)}$, have been given by Ioffe,[31] H. Y. and L.,[27] Shapiro, Dolinsky, and Blokhintsev,[32] Wolfenstein,[33] Überall,[34] Treiman,[35] and Fulton.[36]

In a case such as the ground state ($J_a=0^+$) to ground state ($J_b=1^+$) transition in $\mu^-+C_6^{12}\rightarrow\nu+B_5^{12}$ (Sec. 4) the spin independent nuclear matrix element $\langle b|\exp|a\rangle$ [Eq. (36b)] vanishes[1] and Eq. (45) becomes [using also Eqs. (1b) and (1c) with $\nu_{ba}=0.86\, m_\mu$ [Eq. (2c)], and $(g_A^{(\beta)}/g_V^{(\beta)})=-1.21$ [11])

$$\alpha(a\rightarrow b)=\frac{-(G_A^{(\mu)})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)}}{3(G_A^{(\mu)})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)}}=-0.73. \tag{46}$$

Thus an accurate measurement of the angular distribution of the ground state B_5^{12} recoils, P_μ being determined by a parallel measurement of the muon decay electron angular distribution [Eq. (43a)], would yield information about the ratio $G_P^{(\mu)}/G_A^{(\mu)}$ and hence about the ratio $g_P^{(\mu)}/g_A^{(\beta)}$ [Eq. (1c) *et seq.*]. The corresponding $\alpha(a\rightarrow b)$ [Eq. (46)] is, fortunately, quite sensitive to the exact value of $g_P^{(\mu)}/g_A^{\beta}$ and to the omission or inclusion of the terms $\sim(\mu_p-\mu_n)$ being, for example, -0.33 if $g_P^{(\mu)}/g_A^{(\beta)}=0$ and if the terms $\sim(\mu_p-\mu_n)$ are absent.

We now apply the closure approximation to find the anisotropy coefficient, $\alpha(a)$,

$$\alpha(a)\equiv\frac{\sum_b\{(G_V^{(\mu)})^2|\langle b|\exp|a\rangle|^2+\frac{1}{3}(-(G_A^{(\mu)})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)})|\langle b|(\exp)\boldsymbol{\sigma}|a\rangle|^2\}}{\sum_b\{(G_V^{(\mu)})^2|\langle b|\exp|a\rangle|^2+\frac{1}{3}(3(G_A^{(\mu)})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)})|\langle b|(\exp)\boldsymbol{\sigma}|a\rangle|^2\}}$$

$$=\frac{(G_V^{(\mu)})^2\langle a|[\exp]_a^+[\exp]_a|a\rangle+\frac{1}{3}(-(G_A^{(\mu)})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)})\langle a|[(\exp)\boldsymbol{\sigma}]_a^+\cdot[(\exp)\boldsymbol{\sigma}]_a|a\rangle}{(G_V^{(\mu)})^2\langle a|[\exp]_a^+[\exp]_a|a\rangle+\frac{1}{3}(3(G_A^{(\mu)})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)})\langle a|[\exp)\boldsymbol{\sigma}]_a^+\cdot[(\exp)\boldsymbol{\sigma}]_a|a\rangle}, \tag{47a}$$

where

$$[\exp]_a\equiv\sum_i \tau_i^{(-)}\exp(-i\langle\nu\rangle_a\mathbf{v}_1\cdot\mathbf{r}_i)\varphi(r_i);\quad [(\exp)\boldsymbol{\sigma}]_a\equiv\sum_i \tau_i^{(-)}\exp(-i\langle\nu\rangle_a\mathbf{v}_1\cdot\mathbf{r}_i)\varphi(r_i)\boldsymbol{\sigma}_i \tag{47b}$$

which enters into the recoil nucleus angular distribution, $1+P_\mu\alpha(a)(\mathbf{s}_{\mu;1}\cdot\mathbf{p}_{\text{rec};1})$, appropriate to the total muon capture rate by the parent nucleus. Employing the techniques of Eqs. (4a) to (11b) then gives, for the heavier nuclei, $Z>6$, $A>12$ {using also Eqs. (1b) and (1c) with $\langle\nu\rangle_a=0.75\, m_\mu$ [Eq. (3d)] and $g_A^{(\beta)}/g_V^{(\beta)}=-1.21$ [11]},

$$\alpha(a)\cong\frac{(G_V^{(\mu)})^2-(G_A^{(\mu)})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)}}{(G_V^{(\mu)})^2+3(G_A^{(\mu)})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)}}=-0.39. \tag{48}$$

Further, if the daughter nucleus is unbound even in its ground state, e.g.,

$$\mu^-+Mg_{12}^{26}\rightarrow\nu+\{Na_{11}^{26}\rightarrow Na_{11}^{25}+n_0^1\}, \tag{49}$$

it is not unreasonable to suppose that in the great majority of energetically accessible final states a single neutron carries off most of the available recoil momentum. Under such circumstances the angular distribution of the recoiling neutrons is, approximately,

$$1+P_\mu\alpha(a)(\mathbf{s}_{\mu;1}\cdot\mathbf{p}_{\text{neu};1}), \tag{50}$$

where $\alpha(a)$ is as in Eq. (48) and $\mathbf{p}_{\text{neu};1}$ is a unit vector in the recoil neutron direction. Überall[34] has considered, on the basis of a Fermi gas model, the effective interaction of the recoiling neutron with the remaining nucleons and has concluded that this effective interaction is probably not sufficiently strong to distort appreciably the angular distribution in Eq. (50).

We now consider recoil anisotropy for the case of muon capture by parent nuclei with nonzero spin. Here the anisotropy coefficients are different in the two different hyperfine states of the parent mu-mesic atom, i.e., one must distinguish between $\alpha(J_a+\frac{1}{2};a)$ and $\alpha(J_a-\frac{1}{2};a)$. As a general rule it is obvious that $\alpha(J_a-\frac{1}{2};a)=0$ for $J_a=\frac{1}{2}$ since the corresponding parent atom hyperfine state is then a singlet and so is spatially spherical. As a particular illustration we may give the formulas for the case of the hydrogen mu-mesic atom $[p\mu^-]$. Here, by the remark just made,

$$\alpha(\tfrac{1}{2}-\tfrac{1}{2};H_1^1)=0, \tag{51a}$$

[31] B. L. Ioffe, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 308 (1957).
[32] Shapiro, Dolinsky, and Blokhintsev, Nuclear Phys. **4**, 273 (1957).
[33] L. Wolfenstein, Nuovo cimento **7**, 706 (1958).
[34] H. Überall, Nuovo cimento **6**, 533 (1957).
[35] S. B. Treiman, Phys. Rev. **110**, 448 (1958).
[36] T. Fulton, Nuclear Phys. **6**, 319 (1958).

and, as shown in a calculation by Bernstein and Primakoff,[37]

$$\alpha(\tfrac{1}{2}+\tfrac{1}{2};\,\mathrm{H_1{}^1})=\frac{\tfrac{1}{2}[(G_V{}^{(\mu)}+G_A{}^{(\mu)})^2+(G_P{}^{(\mu)})^2-2(G_V{}^{(\mu)}+G_A{}^{(\mu)})G_P{}^{(\mu)}]}{\tfrac{3}{4}[(G_V{}^{(\mu)}+G_A{}^{(\mu)})^2+(G_P{}^{(\mu)})^2-\tfrac{2}{3}(G_V{}^{(\mu)}+G_A{}^{(\mu)})G_P{}^{(\mu)}]}. \qquad (51b)$$

Thus at low $(\mathrm{H_1{}^1})_2$ molecule number densities, where there is no appreciable collisional hyperfine triplet to singlet conversion (Sec. 5), the angular distribution of the recoil neutrons is:

$$\frac{\tfrac{3}{4}\Lambda^{(\mu)}(\tfrac{1}{2}+\tfrac{1}{2};\,\mathrm{H_1{}^1})[1+P_\mu\alpha(\tfrac{1}{2}+\tfrac{1}{2};\,\mathrm{H_1{}^1})(\mathbf{s}_{\mu;1}\cdot\mathbf{p}_{\mathrm{neu};1})]+\tfrac{1}{4}\Lambda^{(\mu)}(\tfrac{1}{2}-\tfrac{1}{2};\,\mathrm{H_1{}^1})}{\Lambda^{(\mu)}(\mathrm{H_1{}^1})} \qquad (52a)$$

or {using also Eqs. (33a), (33b), (27b), (1b), and (1c) (with $\langle\nu\rangle_a=0.94\,m_\mu$ [Eq. (2c)], and $(g_A{}^{(\beta)}/g_V{}^{(\beta)})=-1.21$ [11])},

$$1+\frac{P_\mu}{2}\frac{[(G_V{}^{(\mu)}+G_A{}^{(\mu)})^2+(G_P{}^{(\mu)})^2-2(G_V{}^{(\mu)}+G_A{}^{(\mu)})G_P{}^{(\mu)}]}{[(G_V{}^{(\mu)})^2+3(G_A{}^{(\mu)})^2+(G_P{}^{(\mu)})^2-2G_A{}^{(\mu)}G_P{}^{(\mu)}]}(\mathbf{s}_{\mu;1}\cdot\mathbf{p}_{\mathrm{neu};1}) \qquad (52b)$$

$$=1+\frac{P_\mu}{2}[0.01](\mathbf{s}_{\mu;1}\mathbf{p}_{\mathrm{neu};1}), \qquad (52c)$$

while the corresponding muon decay electron angular distribution becomes

$$1-\frac{P_\mu}{2}\tfrac{1}{3}(\mathbf{s}_{\mu;1}\cdot\mathbf{p}_{\mathrm{el};1}). \qquad (52d)$$

P_μ must be interpreted in Eqs. (52a) to (52d) as what the residual muon polarization would be, at the instant of decay or capture, if the parent proton did not create any hyperfine-structure induced depolarization in the muon $1s_{\frac{1}{2}}$ orbit—it is seen that the numerical values of $G_V{}^{(\mu)}$, $G_A{}^{(\mu)}$, $G_P{}^{(\mu)}$ are such that the recoil neutron distribution is practically isotropic; this is expected since $[\Lambda^{(\mu)}(\tfrac{1}{2}+\tfrac{1}{2};\,\mathrm{H_1{}^1})/\Lambda^{(\mu)}(\tfrac{1}{2}-\tfrac{1}{2};\,\mathrm{H_1{}^1})]\ll 1$ [Eqs. (33a) and (33b)]. At higher $(\mathrm{H_1{}^1})_2$ molecule densities all the $[p\mu^-]$ are collisionally converted into the hyperfine singlet state (Sec. 5), so that as already noted above and as first pointed out by Gershtein and Zeldovich,[22] the recoil neutron distribution is here certainly expected to be isotropic.

(b) Angular Distribution of Photons in Radiative Capture of Polarized Muons

The internal bremsstrahlung (I.B.) photons emitted in the radiative capture of polarized muons are also expected to be characterized by an anisotropic angular distribution relative to $\mathbf{s}_{\mu;1}$; this distribution is, for the case of zero spin parent nuclei with the daughter nuclei formed in a particular state:

$$1+P_\mu\beta(a\to b;\,\gamma_{ba})(\mathbf{s}_{\mu;1}\cdot\boldsymbol{\gamma}_1), \qquad (53)$$

where $\boldsymbol{\gamma}_{ba}=\gamma_{ba}\boldsymbol{\gamma}_1$ is the I.B. photon momentum vector, $\beta(a\to b;\,\gamma_{ba})$ the corresponding anisotropy coefficient, and, for the reasons mentioned in Sec. 7(a), the discussion is again confined to the case of zero spin parent nuclei.

The anisotropy coefficient $\beta(a\to b;\,\gamma_{ba})$ of Eq. (53) may be directly calculated or may be obtained on the basis of a theorem of Cutkosky[38] which shows that this $\beta(a\to b;\,\gamma_{ba})$ is numerically equal to the helicity of a *massless* positron emitted in the beta-decay process: $|a;Z,A\rangle\to|b;Z-1,A\rangle+e^++\nu$ ($Z/137\ll 1$; beta-decay process allowed or forbidden). Now in a beta-decay theory described by an effective Hamiltonian of the type of Eq. (1a) with $G_V{}^{(\mu)}\to g_V{}^{(\beta)}$, $G_A{}^{(\mu)}\to g_A{}^{(\beta)}$, $G_P{}^{(\mu)}\to 0$ the helicity of such a massless positron is $+1$—thus in a muon radiative capture theory with an $H_{\mathrm{eff}}{}^{(\mu)}$ characterized by $G_V{}^{(\mu)}=g_V{}^{(\mu)}$, $G_A{}^{(\mu)}=g_A{}^{(\mu)}$, $G_P{}^{(\mu)}=0$, i.e., with an $H_{\mathrm{eff}}{}^{(\mu)}$ in which all nucleon recoil effects (terms $\sim\nu/m_p$) are omitted, one has

$$\beta(a\to b;\,\gamma_{ba})=1. \qquad (54)$$

Thus, summing over all the energetically accessible states of the daughter nucleus, we obtain

$$\beta(a;\,\langle\gamma\rangle_a)=\frac{\sum_b\beta(a\to b;\,\gamma_{ba})\Lambda_{\mathrm{rad}}{}^{(\mu)}(a\to b;\,\gamma_{ba}}{\sum_b\Lambda_{\mathrm{rad}}{}^{(\mu)}(a\to b;\,\gamma_{ba})}=1, \qquad (55)$$

which last pair of equations have also been explicitly derived by H.Y. and L.[27] and by Bernstein.[28]

There is now very considerable interest in a calculation of $\beta(a\to b;\,\gamma_{ba})$, $\beta(a;\,\langle\gamma\rangle_a)$, where one includes nucleon recoil terms $\sim\nu/m_p$ into the appropriate effective Hamiltonian, i.e., includes in particular the induced pseudoscalar and the conserved vector current anomalous magnetic moment contributions—such a calculation is being carried out by Bernstein.[28] The effect of the pseudoscalar term can be foreseen quali-

[37] J. Bernstein and H. Primakoff (to be published).

[38] R. E. Cutkosky, Phys. Rev. 107, 330 (1957).

tatively on the basis of Cutkosky's theorem[38] since it is known that with an effective V, A, P beta-decay interaction the helicity of a massless positron is less than unity; hence in a theory with $g_P^{(\mu)}\neq 0$ the calculated values of $\beta(a\rightarrow b;\gamma_{ba})$, $\beta(a;\langle\gamma\rangle_a)$ will also be less than unity. Thus any measurement of the I.B. photon angular distribution sufficiently accurate to fix a reliable value for $(1-\beta(a;\langle\gamma\rangle_a))$ will, upon comparison with Bernstein's theoretical expression[28] for $\beta(a;\langle\gamma\rangle_a)$, yield an "experimental" value of $g_P^{(\mu)}$ which can be compared with the Goldberger-Treiman[3]-Wolfenstein[4] theoretical value of $8g_A^{(\beta)}$ [Eq. (1c)].

(c) Polarization of Recoil Nuclei in Muon Capture

The recoil daughter nuclei formed in muon capture are in general polarized even when the polarization of the muon itself at the instant of capture is negligible. This effect has been discussed in a fairly general way by Treiman[35] and by Fulton,[36] and for the particular case of the hydrogen mu-mesic atom by Gershtein and Zeldovich[22] and by Bernstein and Primakoff.[37] For $[p\mu^-]$ in the hyperfine singlet state Gershtein and Zeldovich[22] point out that conservation of angular momentum and the assumption that the neutrino has unit negative helicity ensures, independent of the magnitudes of the various coupling constants in $H_{\text{eff}}^{(\mu)}$, that the helicity of the recil neutron is $=-1$. For $[p\mu^-]$ in the hyperfine triplet state an explicit calculation by Bernstein and Primakoff[37] shows that the recoil neutron helicity is

$$\frac{1}{3}\frac{(G_V^{(\mu)}+G_A^{(\mu)})^2+(G_P^{(\mu)})^2-6(G_V^{(\mu)}+G_A^{(\mu)})G_P^{(\mu)}}{(G_V^{(\mu)}+G_A^{(\mu)})^2+(G_P^{(\mu)})^2-\frac{2}{3}(G_V^{(\mu)}+G_A^{(\mu)})G_P^{(\mu)}} \tag{56a}$$

so that at low $(H_1{}^1)_2$ molecule number densities the over-all recoil neutron helicity is {using also Eqs. (33a), (33b), (27b), (1b), and (1c) (with $\langle\nu\rangle_a=0.94\,m_\mu$ [Eq. (2c)] and $(g_A^{(\beta)}/g_V^{(\beta)})=-1.21$ [11])}

$$\frac{3}{4}\frac{\Lambda^{(\mu)}(\frac{1}{2}+\frac{1}{2};H_1{}^1)}{\Lambda^{(\mu)}(H_1{}^1)}\left[\frac{1}{3}\frac{(G_V^{(\mu)}+G_A^{(\mu)})^2+(G_P^{(\mu)})^2-6(G_V^{(\mu)}+G_A^{(\mu)})G_P^{(\mu)}}{(G_V^{(\mu)}+G_A^{(\mu)})^2+(G_P^{(\mu)})^2-\frac{2}{3}(G_V^{(\mu)}+G_A^{(\mu)})G_P^{(\mu)}}\right]+\frac{1}{4}\frac{\Lambda^{(\mu)}(\frac{1}{2}-\frac{1}{2};H_1{}^1)}{\Lambda^{(\mu)}(H_1{}^1)}[-1]$$
$$=2\frac{G_V^{(\mu)}G_A^{(\mu)}-(G_A^{(\mu)})^2-G_V^{(\mu)}G_P^{(\mu)}}{(G_V^{(\mu)})^2+3(G_A^{(\mu)})^2+(G_P^{(\mu)})^2-2G_A^{(\mu)}G_P^{(\mu)}}=-0.99. \tag{56b}$$

This is very close to -1 as is indeed expected from the fact that $[\Lambda^{(\mu)}(\frac{1}{2}+\frac{1}{2};H_1{}^1)/\Lambda^{(\mu)}(\frac{1}{2}-\frac{1}{2};H_1{}^1)]\ll 1$ [Eqs. (33a) and (33b)].

In the calculation given by Treiman[35] and by Fulton[36] the $H_{\text{eff}}^{(\mu)}$ of Eq. (1a) with $G_V^{(\mu)}=g_V^{(\mu)}$, $G_A^{(\mu)}=g_A^{(\mu)}$, $G_P^{(\mu)}=0$, has been used; also, for the reasons mentioned in Sec. 7a, the discussion should be confined to the case of zero spin parent nuclei. Making the additional assumption that the wave functions $|a;J_a=0^+\rangle$, $|b;J_b\rangle$ of the parent and daughter nuclei are such that the s-wave part of $\exp(-i\mathbf{v}_{ba}\cdot\mathbf{r}_i)$ predominates over the d wave, g wave, $\cdots$ parts in the nuclear matrix elements,[1] so that $J_b=1^+$, one may calculate the polarization of the recoil daughter nuclei formed in a particular state with $J_b=1^+$. Since in a $J_a=0^+\rightarrow J_b=1^+$ transition, as for example $\mu^-+C_6{}^{12}=\nu+(B_5{}^{12})_{\text{ground state}}$, the spin independent nuclear matrix element $\langle b|\exp|a\rangle$ [Eq. (36b)] vanishes,[1] the spin dependent nuclear matrix element $\langle b|(\exp)\boldsymbol{\sigma}|a\rangle$ [Eq. (36b)] necessarily cancels out of the expression for the recoil polarization and this becomes

$$\langle\mathbf{J}_{b;1}(\mathbf{p}_{\text{rec};1})\rangle=\frac{2}{3}\frac{-\mathbf{p}_{\text{rec};1}+P_\mu\mathbf{s}_{\mu;1}}{1-\frac{1}{3}P_\mu\mathbf{s}_{\mu;1}\mathbf{p}_{\text{rec};1}}. \tag{57}$$

Thus the recoil polarization is (anti) parallel to $\mathbf{p}_{\text{rec};1}$ only if the muon polarization at the instant of capture, P_μ, vanishes. One can also obtain the recoil polarization averaged over all possible directions of recoil: $\{\langle\mathbf{J}_{b;1}\rangle\}_{\text{Av}}$—this turns out to be (Jackson, Treiman, and Wyld[39])

$$\{\langle\mathbf{J}_{b;1}\rangle\}_{\text{Av}}=\tfrac{2}{3}P_\mu\mathbf{s}_{\mu;1} \tag{58}$$

and is just what $\langle\mathbf{J}_{b;1}(\mathbf{p}_{\text{rec};1})\rangle$ would be in a theory with a parity conserving $H_{\text{eff}}^{(\mu)}$. The quantity $\{\langle\mathbf{J}_{b;1}\rangle\}_{\text{Av}}$ has recently been measured by Love, Marder, Nadelhaft, Siegel, and Taylor[40] on the basis of the observation of the angular anisotropy, relative to $\mathbf{s}_{\mu;1}$, of the decay electrons of the daughter nucleus: $(B_5{}^{12})_{\text{ground state}}$. $\{\langle\mathbf{J}_{b;1}\rangle\}_{\text{Av}}$ was found to be positive if one identified the directions $\mathbf{s}_{\mu;1}$ and $\mathbf{p}_{\text{neg. muon};1}$ [Eq. (43b)] while its magnitude was appropriate to a reasonable amount of depolarization of the $B_5{}^{12}$ by hyperfine interaction with its atomic electrons.

(d) Polarization of Photons in Radiative Muon Capture

The internal bremsstrahlung (I.B.) photons emitted in radiative muon capture are circularly polarized independent of any residual polarization of the muon itself. Cutkosky,[38] H.Y. and L.[27] and Bernstein[28] have shown that the degree of circular polarization, $\beta'(a\rightarrow b;\gamma_{ba})$ of any I.B. photon with momentum $\boldsymbol{\gamma}_{ba}$—$\beta'(a\rightarrow b;\gamma_{ba})=\pm 1$ for complete right-hand, left-hand

[39] Jackson, Treiman, and Wyld, Phys. Rev. **107**, 327 (1957).
[40] Love, Marder, Nadelhaft, Siegel, and Taylor, Phys. Rev. Letters **2**, 107 (1959).

circular polarization—is numerically equal to the parameter $\beta(a \rightarrow b; \gamma_{ba})$ which is the corresponding anisotropy coefficient of the I.B. photon momentum directional distribution [Eq. (53)]. The conclusions drawn in Sec. (7b) about $\beta(a \rightarrow b; \gamma_{ba})$, $\beta(a; \langle\gamma\rangle_a)$ therefore can be applied to the corresponding quantities $\beta'(a \rightarrow b; \gamma_{ba})$,

$$\beta'(a; \langle\gamma\rangle_a) = \frac{\sum_b \beta'(a \rightarrow b; \gamma_{ba}) \Lambda_{\mathrm{rad}}{}^{(\mu)}(a \rightarrow b; \gamma_{ba})}{\sum_b \Lambda_{\mathrm{rad}}{}^{(\mu)}(a \rightarrow b; \gamma_{ba})} \quad (59)$$

so that in particular one expects deviations from complete right circular polarization of the I.B. photons only to the extent that the induced pseudoscalar interaction is present.

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