REVIEWS OF MODERN PHYSICS VOLUME 31, NUMBER 3 JULY, 1959

Mu Capture, Beta Decay, and Pi-Meson Decay

M. L. GOLDBERGER
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

In the original Yukawa formulation of meson theory, the π meson (as we now believe Yukawa's particle to be) was to provide a natural explanation for β decay. The process $\pi \rightarrow e+\bar{\nu}$ was regarded as an elementary interaction and nuclear β decay was imagined to proceed by the route $n \rightarrow p+\pi \rightarrow \mu+e+\bar{\nu}$. There are a variety of reasons why this scheme fails. Just the opposite point of view is now generally adopted, namely, that the nuclear β decay is fundamental and that the observed decay of the π meson is to be explained in terms of it. We do not exclude the possibility that β decay be described in terms of an as yet unknown heavy intermediate. Nevertheless, the nuclear β decay is to be regarded as essentially primary. In order to describe the actual dominant π-meson decay mode $\pi \rightarrow \mu+\bar{\nu}$ it is necessary to assume the existence of another β decay like process, μ-meson capture. The elementary process may be described as $\mu+p \rightarrow n+\nu$, or equally well as $n+\bar{p} \rightarrow \mu+\bar{\nu}$; the first is the experimentally observed μ-meson absorption reaction, whereas, the second, the annihilation of a neutron and an antiproton, plays an important role in π-meson decay.

Since this is a conference on weak interactions, I shall not be able to say that one of the π mesons, the neutral one, decays into two gamma rays; electromagnetic interactions are too strong to be mentioned! Furthermore, I will not be able to point out that a theory of the π^0 decay can be given which is very similar to what we describe for charged pions.

During the past year or two the field of weak interactions has become a surprisingly orderly one. The two-component theory of the neutrino, as well as the principle of lepton conservation now both seem to be well established. Both nuclear β decay and μ-meson decay seem to be describable in terms of a vector (V) and axial vector (A) coupling. This statement has to be qualified somewhat in the case of nuclear β decay. There is the additional fact that in both β decay and μ decay the vector coupling constants are almost identical. Rather less is known of the coupling types for the μ-meson capture reaction, but the dominant couplings seem to have about the same strength as in β decay. This is discussed by Primakoff. We tentatively assume that the apparently universal (V,A) interaction extends also to this Fermi process.

Precisely what do we mean by a *universal* interaction? This can mean only that the basic interaction Lagrangian contains only these two coupling types. Given this basic definition let us see whether there is anything surprising in the observed decays. First, in μ decay the V and A couplings are forced to be equal if we adopt the two component neutrino theory. In β decay, $g_A=1.25\ g_V$, which need not be disturbing. The amazing thing is, with β decay and μ capture involving strongly interacting particles and μ decay involving only weakly interacting ones, that there is any kind of universality whatsoever. One would expect the existence of pions and other strongly interacting particles to modify greatly the effective matrix element for transitions between physical nucleons as compared to the μ-decay process.

Insofar as the vector coupling is concerned, Gerstein and Zel'dorich and Feynman and Gell-Mann have made a very attractive suggestion: They propose that there may be a principle analogous to gauge invariance in electrodynamics which would insure that the vector coupling constant in β decay be the same even when the strong interactions are turned on. Recall that as a result of current conservation, or, if you prefer, gauge invariance, the charge of a bare and physical proton is the same. In order to achieve this goal the β decay "vector current density" $g_V\bar{\psi}\gamma_\mu\psi$ [ψ is a nucleon field operator] must be augmented by terms which ultimately couple leptons directly to pions, etc., and such that the total "current density" $j_\mu{}^V$ satisfies $\partial j_\mu{}^V/\partial x_\mu$. The difference between vector and axial vector couplings in β decay is attributed to renormalization of the axial vector interaction.

One troublesome point in connection with this proposal has been raised by Wightman, Telegdi, and Michel. When one computes the electromagnetic radiative corrections for μ decay and β decay, one finds that to lowest order in all couplings not a finite correction for μ decay, but a logarithmic divergence in β decay. One may argue that if the nucleons are "dressed" properly and the radiative corrections are then computed (something no one knows how to do exactly) the result will be convergent. Nevertheless, it is not clear why, even if the β-decay effect is made finite, the two radiatively-corrected vector coupling constants should continue to be equal.

Let us discuss in a systematic way the role of strong interactions in Fermi processes. The work to be reviewed was carried out by Treiman and me and has been, for the most part, published elsewhere. I apologize for this, but in order to talk about something new, I would have to make an obviously wrong new theory—the correct one already having been given.

We suppose that β decay and μ capture are described by the Lagrangian density,

$$\mathfrak{L}_I = Z_2 f_A \bar{\psi}_\nu (1-\gamma_5) i\gamma_\lambda\gamma_5\psi_1 (\psi_n i\gamma_\lambda\gamma_5\psi_p) + Z_2 f_V \bar{\psi}_\nu (1-\gamma_5)\gamma_\lambda\psi_1 (\bar{\psi}_n\gamma_\lambda\psi_p) + \text{Hermitian conjugate}, \quad (1)$$

where f_A and f_V are the unrenormalized coupling constants, and Z_2 is the nucleon wave-function renormalization constant. The ψ's are field operators associated with the particles indicated by the subscripts; l stands for either an electron or a μ meson. There may be other interactions of leptons. Among these are the direct pion couplings of Feynman and Gell-Mann, or perhaps couplings to baryons other than nucleons. For the time being we do not consider such possibilities. We consider the processes $(e,\mu)+p\rightarrow n+\nu$. To lowest order in the weak interaction, the matrix element computed from (1) is given by

$$S=i(2\pi)^4\delta(n+p_\nu-p-p_l)M, \tag{2}$$

where n, p_ν, p, p_l are the four-momenta of the neutron, neutrino proton, and electron (or μ meson), and

$$M=\bar{u}_\nu(1-\gamma_5)i\gamma_\lambda\gamma_5u_l\langle n|P_\lambda|p\rangle+\bar{u}_\nu(1-\gamma_5)\gamma_\lambda u_l\langle n|V_\lambda|p\rangle \tag{3}$$

$|n\rangle$ and $|p\rangle$ represent physical neutron and proton states and

$$P_\lambda=Z_2f_A\bar{\psi}_ni\gamma_\lambda\gamma_5\psi_p,\quad V_\lambda=Z_2f_V\bar{\psi}_n\gamma_\lambda\psi_p. \tag{4}$$

The lepton spinors have been normalized according to $\bar{u}_l\gamma_4u_l=\bar{u}_\nu\gamma_4u_\nu=1$.

In the Feynman-Gell-Mann theory, V_λ would have additional terms. We do not use the explicit forms of V_λ and P_λ.

The general forms of the matrix elements of P_λ, V_λ required for Eq. (3) may be deduced from invariance principles, and they are

$$\langle n|P_\lambda|p\rangle=\left(\frac{m^2}{n_0p_0}\right)^{\frac{1}{2}}\bar{u}(n)\{ai\gamma_\lambda\gamma_5-b(p-n)_\lambda\gamma_5\}u(p), \tag{5}$$

$$\langle n|V_\lambda|p\rangle=\left(\frac{m^2}{n_0p_0}\right)^{\frac{1}{2}}\bar{u}(n)\{c\gamma_\lambda-d\sigma_{\lambda\mu}(p-n)_\mu\}u(p). \tag{6}$$

In these formulas, m is the nucleon mass, and the spinors are normalized according to $\bar{u}u=1$. The fact that only the momentum combination $p-n$ appears above is a consequence of charge symmetry and time reversal invariance in the strong interactions. Finally, the coefficients a, b, c, and d are functions of $(n-p)^2$, the momentum transfer squared.

Substituting these matrix elements in Eq. (3) and using the Dirac equation for the leptons we find for M the result

$$M=\left(\frac{m^2}{p_0n_0}\right)^{\frac{1}{2}}\{a\bar{u}_\nu(1-\gamma_5)i\gamma_\nu\gamma_5u_l\bar{u}_ni\gamma_\lambda\gamma_5u_p+m_lb\bar{u}_\nu(1-\gamma_5)\gamma_5u_l\bar{u}_n\gamma_5u_p+c\bar{u}_\nu(1-\gamma_5)\gamma_\lambda u_l\bar{u}_n\gamma_\lambda u_p+d\bar{u}_\nu(1-\gamma_5)\gamma_\lambda(p_l-p_\nu)_\mu u_l\bar{u}_n\sigma_{\lambda\mu}u_p\}.$$

The first and third terms are in the form of the usual axial vector and vector interactions. The functions $a[(n-p)^2]$ and $c[(n-p)^2]$ are, for zero value of the momentum transfer, simply the coupling constants g_A and g_V of β decay. In μ capture $(n-p)^2\sim m_\mu^2$ but a and c do not deviate much from g_A and g_V over such an interval. The second term has the form of a conventional pseudoscalar interaction with an effective coupling constant m_lb. Barring strong dependence of b on momentum transfer, this term is relatively much less important in β decay than in μ capture. The last term is identical with what has been called weak magnetism by Gell-Mann and is present whether or not the conserved vector current of Feynman and Gell-Mann is assumed. The magnitude of d depends critically, however, on this assumption.

This is as far as one can go on more or less general grounds. What we have done is to study the four functions a, b, c, and d by dispersion techniques. It is not practical to discuss this investigation in detail, so we outline the elements that go into such a treatment and quote the relevant results. We wish to represent the functions in the following form:

$$a(\xi)=g_A-\frac{\xi}{\pi}\int_{(3m_\pi)^2}^{\infty}d\xi'\frac{\mathrm{Im}a(-\xi')}{\xi'(\xi'+\xi-i\epsilon)},$$

$$b(\xi)=\frac{1}{\pi}\int_{m_\pi^2}^{\infty}d\xi'\frac{\mathrm{Im}b(-\xi')}{\xi'+\xi-i\epsilon},$$

$$c(\xi)=g-\frac{\xi}{\pi}\int_{(2m_\pi)^2}^{\infty}d\xi'\frac{\mathrm{Im}c(-\xi')}{\xi'(\xi'+\xi-i\epsilon)},$$

$$d(\xi)=\frac{1}{\pi}\int_{(2m_\pi)^2}^{\infty}d\xi'\frac{\mathrm{Im}d(-\xi')}{\xi'+\xi-i\epsilon}.$$

We have written the dispersion relations explicitly in such a way that $a(0)=g_A$, $c(0)=g_V$ and make essentially no effort to relate the renormalized coupling constants g_A, g_V to the unrenormalized ones appearing in the original Lagrangian. The quantities Ima, etc., represent the imaginary parts of the various amplitudes (which are real for positive arguments) and these may be expressed in terms of the amplitudes for certain real physical processes.

Consider first the vertex $\langle n|P_\lambda|p\rangle$; it is slightly more convenient to study $\langle 0|P_\lambda|\bar{n}p,in\rangle$ which is related to our other amplitude according to

$$\left(\frac{p_0\bar{n}_0}{m^2}\right)^{\frac{1}{2}}\langle 0|P_\lambda|n,\bar{p},in\rangle=\bar{v}_{\bar{n}}[ai\gamma_\lambda\gamma_5-b(p+\bar{n})_\lambda\gamma_5]u(p),$$

where $v_{\bar{n}}$ is a negative energy spinor and a is now a function of $(p+\bar{n})^2$. This matrix element may be imagined as describing the annihilation of a proton-antineutron pair to produce leptons via the interaction P_λ. The dependence on the lepton variables may be factored out so that they no longer appear explicitly. The sort of things that can contribute to this matrix element are shown in Fig. 1.

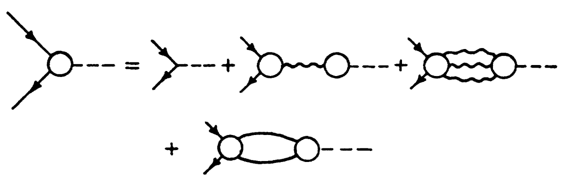

FIG. 1. The structure of the pseudoscalar matrix element $\langle 0|P_\lambda|n\bar{p}\rangle$ is pictured: The first "term" is effectively the bare interaction, the second shows a virtual transition to a π meson which decays into leptons, the third shows the reaction passing through a three pion state, and finally we have nucleon-antinucleon scattering followed by lepton emission.

The first element is essentially the bare interaction; in the second the pair annihilates to form a pion which then undergoes $\pi-\mu$ decay; the third diagram shows the pair annihilating into three pions, which ultimately combine to yield the lepton pair; the fourth diagram shows the pair undergoing a scattering interaction before annihilating to produce the leptons by the very matrix element we are studying. We are, thus, generating an integral equation. Needless to say, there are an infinite number of diagrams which we have not shown; we cannot even compute all of the ones we have. The intermediate state involving three pions is too hard for us to handle, but the remaining three are manageable and the integral equation for a and b can be easily solved.

The resulting solutions involve the ${}^3P_1$ and ${}^1S_0$ complex phase shifts for proton-antineutron scattering, the renormalized strong pion-nucleon coupling constant (G), the renormalized β-constant $[g_A=a(0)]$ and the experimental $\pi\rightarrow\mu+\nu$ lifetime. The latter enters via the one pion intermediate state which contributes only to the effective pseudoscalar interaction b. We find

$$a(\xi)=g_A\exp\left\{-\frac{\xi}{\pi}\int_{4m^2}^{\infty}dy\frac{\phi_1(y)}{y(y+\xi-i\epsilon)}\right\},$$

$$a-\frac{\xi}{2m}b=\left[g_A+\frac{\xi}{\xi+m_\pi^2}\frac{\sqrt{2}GF(-m_\pi^2)}{2m}\times\exp\left\{-\frac{m_\pi^2}{\pi}\int_{4m^2}^{\infty}dy\frac{\phi_0(y)}{y(y-m_\pi^2)}\right\}\right]\times\exp\left\{-\frac{\xi}{\pi}\int_{4m^2}^{\infty}dy\frac{\phi_0(y)}{y(y+\xi-i\epsilon)}\right\},$$

where $F(-m_\pi^2)$ may be related to the $\pi-\mu$ lifetime; ϕ_0 and ϕ_1 are related to the ${}^1S_0$ and ${}^3P_1$ phase shifts δ_0, δ_1 according to

$$\tan\phi=\frac{\mathrm{Re}e^{i\delta}\sin\delta}{1-\mathrm{Im}e^{i\delta}\sin\delta}.$$

We find, using the experimental value of the $\pi-\mu$ lifetime, $F=-0.115(\sqrt{2}Gmg_A/2\pi^2)$. Using this value and neglecting the contributions from the proton-antineutron scattering (i.e., set $\phi_0=\phi_1=0$) we find "g_p"$=m_\mu b(m_\mu^2)\approx 8g_A$ as the effective pseudoscalar coupling constant that would be effective in μ capture. The deviations of a from the value at $\xi=0$ are of order m_μ^2/m^2; of course, the three pion intermediate state could cause slightly larger corrections but one would expect that for β decay or μ capture the leading terms

$$a\approx g_A,$$
$$b\approx-\frac{\sqrt{2}GF(-m_\pi^2)}{\xi+m_\pi^2},$$

are certainly adequate. In π decay, one needs a and b for values of $-\xi>4m^2$ in which case the neglect of the many less massive states (such as 3, 5$\cdots$pions) could be much more serious. It is our feeling that since the leptons are coupled directly only to the nucleon pairs (or perhaps more generally to other baryon pairs) that such pair states are more important than the lighter pion states.

The effective vector interaction matrix element may be analyzed in a manner quite similar to our treatment of $\langle 0|P_\lambda|\bar{n}p,in\rangle$. We do not go into the analysis in much detail since the vector interaction plays no role in π decay. The matrix element $\langle 0|V_\lambda|\bar{n}p,in\rangle$ is identical in form to that encountered in the study of the electromagnetic structure of nucleons. If we follow Feynman and Gell-Mann we see that this parallel is essentially exact. As in the electromagnetic problem, the two-pion intermediate state is expected to play a dominant role for the relatively small values of $(\bar{n}+p)^2$, namely, about m_μ^2 encountered in μ capture. In order to evaluate the two-pion contribution, we must know the matrix element for pair annihilation into two pions (even when the total energy extends into the unphysical region of total energy W, $4m^2>W^2>4\mu^2$) and also that for the pions to annihilate into a lepton pair. The latter process may also be analyzed by dispersion methods and we have done so in a rather crude fashion. For its evaluation one requires the matrix element for production of a proton-antineutron pair by two pions; the pair then annihilates via the original matrix element $\langle 0|V_\lambda|\bar{n}p,in\rangle$ (strictly speaking, there is a two pion intermediate state also which we neglect). We approximated all the matrix elements encountered in the problem by lowest order perturbation theory and found

$$c(\xi)=g_V\left[1+\frac{4}{9\pi}\frac{f^2}{4\pi}(0.24)\frac{\xi}{m_\pi^2}\right],$$

$$d(\xi)=1.7\frac{g_V}{2m}\times\frac{16}{3\pi}\frac{f^2}{4\pi}\left[1-\frac{0.12}{6}\frac{\xi}{m_\pi^2}\right],$$

where $f^2/4\pi=0.08$ is the effective pseudovector coupling constant of pion physics.

Adopting the Feynman—Gell-Mann theory, then, at least as far as the static terms (i.e., $\xi=0$) are concerned,

FIG. 2. Dispersion theoretic diagrams for π decay. The first shows a transition to an intermediate state with three pions, which we neglect; the second shows the usually contemplated transition through a nucleon-antinucleon pair.

the calculation can be carried out exactly. In their theory there is a complete analogy with the electromagnetic problem (except for slight numerical isotopic spin factors) so that we have without calculation,

$$c(0)\equiv g_V,$$
$$d(0)=(\mu_p-\mu_N)g_V/2m,$$

where μ_p, μ_N are the anomalous moments of proton and neutron in units of nucleon magnetons. Thus there is a clear-cut difference between the prediction of $d(0)$ made by the Feynman—Gell-Mann theory and the conventional theory: The value of d is about fifteen times larger in their case, and raises it out of the undetectable range.

An experiment to test the correctness of the Feynman—Gell-Mann proposal for conserved vector currents has been proposed by Gell-Mann. It is possible that the magnetic moment term, d, as well as the induced pseudoscalar interaction, b, may be detectable in certain μ-capture effects as discussed by Primakoff.

We turn now to a discussion of the decay of the π meson. It is some relief to be able to say that only the mode $\pi^-\rightarrow\mu^-+\bar{\nu}$ need be discussed. The correct branching ratio for the channel $\pi^-\rightarrow e^-+\bar{\nu}$ presumably then follows from our basic Lagrangian containing only vector and axial vector couplings. It is easy to see that the axial vector coupling alone plays any role in π decay. The leptons are assumed to emerge from a point (in the sense of an arbitrary Feynman diagram); hence there is only one momentum vector in the problem, say that of the pion, p_π. The pion is presumably a pseudoscalar and thus it is impossible to construct anything other than a pseudoscalar or a pseudovector to be coupled to the leptons. Formally, the S-matrix element for the transition is proportional to Λ where

$$\Lambda=\bar{u}(p_\mu)i\gamma_\lambda\gamma_5(1+\gamma_5)u(p_\nu)\langle 0|P_\lambda|\pi\rangle + \bar{u}(p_\mu)\gamma_\lambda(1+\gamma_5)u(p_\nu)\langle 0|V_\lambda|\pi\rangle$$

and the second term vanishes if one assumes parity is conserved in the strong interactions.

We concentrate attention, therefore, on $\langle 0|P_\lambda|\pi\rangle$ which we write as

$$\langle 0|P_\lambda|\pi\rangle=-i(p_\pi)_\lambda F(p_\pi^2)/(2p_{\pi 0})^{\frac{1}{2}}.$$

We can easily show that $F(p_\pi^2)$ satisfies a dispersion relation of the form

$$F(\xi)=\frac{1}{\pi}\int d\xi'\frac{\mathrm{Im}F(-\xi')}{\xi'+\xi-i\epsilon},$$

and our task is to express $\mathrm{Im}F$ in terms of calculable quantities. Analyzing the structure of the intermediate states which can contribute, we find the first few (judging them in terms of increasing rest mass) are as shown in Fig. 2.

The first diagram shows the uncomputable transition from one to three pions which ultimately combine to yield the leptons. There should then come states with 5, 7, $\cdots$ pions, perhaps followed by zero strangeness states involving K mesons and pions. Finally, one comes to the neutron-antiproton state. There are three reasons for concentrating attention on this state: (1) it is the one conventionally envisaged in a qualitative discussion of π-decay; (2) the leptons are directly coupled to nucleons, hence, such states might be expected to be of great importance; and (3) we can do quite a reasonable job of evaluating its contribution (and cannot calculate any of the others).

The individual pieces of our diagram are also treated by dispersion methods. We have already discussed the weak vertex in detail and so we now concentrate on the strong one, describing the virtual dissociation of the pion into a neutron-antiproton pair. In Fig. 3 the first diagram shows the "bare" interaction, the second, a three-pion state which by this time we neglect quite automatically, and, finally, the one we retain, namely, that involving a neutron-antiproton pair. This pair (in the rest system of the pion) is in a $^1S_0$ state (isotopic triplet) and to the indicated approximation can be characterized by a complex phase shift, δ_0.

For this vertex function, $K(\xi)$, say, one finds

$$K(\xi)=\sqrt{2}G\exp\left\{-\frac{\xi+m_\pi^2}{\pi}\int_{4m^2}^{\infty}d\xi'\frac{\phi_0(\xi')}{(\xi'-m_\pi^2)(\xi'+\xi-i\epsilon)}\right\},$$

where ϕ_0 is the same function introduced in connection with our previous discussion of $a-\xi b/2m$, the quantity arising from the weak vertex. Putting our dispersion pictures together, we find for $\mathrm{Im}F(\xi)$, neglecting small terms $\sim m_\pi^2/m^2$,

$$\mathrm{Im}F(\xi)=-\frac{\sqrt{2}G}{4\pi}\left[mg_A+\frac{\sqrt{2}GF(-m_\pi^2)\xi}{\xi+m_\pi^2}\right]\times\left[\frac{\xi+4m^2}{\xi}\right]^{\frac{1}{2}}H(\xi)$$
$$(\text{for } -\xi>4m^2,\ =0 \text{ otherwise}),$$

where $H(\xi)$ is given by

$$H(\xi)=\exp\left\{-\frac{2(\xi+m_\pi^2)}{\pi}P\int_{4m^2}^{\infty}d\xi'\frac{\phi_0(\xi')}{(\xi'-m_\pi^2)(\xi'+\xi)}\right\},$$

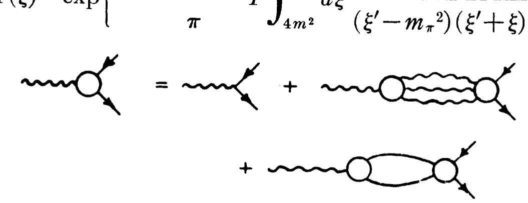

FIG. 3. Dispersion theoretic diagrams for the strong pion-nucleon vertex. The first one is the direct interaction, the second the uncomputable three pion state and finally the important state, the one involving a nucleon-antinucleon pair.

and P means the principal value of the integral is to be taken at the singularity $\xi'=-\xi$. H is thus, aside from a factor, $|K(\xi)|^2$. On substituting $\mathrm{Im}F(\xi)$ into the dispersion relation for F, we find (neglecting m_π^2/m^2)

$$F(-m_\pi^2)\approx F(0)=-\frac{m\sqrt{2}}{4\pi}Gg_A\frac{J}{1+(G^2/4\pi)\cdot J},$$

where J is given by

$$J=\frac{1}{\pi}\int_{4m^2}^{\infty}d\xi\frac{(\xi-4m^2)^{\frac{1}{2}}}{\xi^{\frac{3}{2}}}H(-\xi).$$

If there were couplings to other baryon pairs (beside $\bar{p}n$) we would have

$$F(0)=-\frac{m}{4\pi}\sqrt{2}g_A\frac{\sum_i G_iJ_i}{1+(1/4\pi)\sum G_i^2J_i},$$

where the G_i are the strong coupling constants and the J_i are computed from H_i derived from the various ϕ_{0i}'s.

If we disregard the denominator and set the function H equal to unity, we get the familiar logarithmically-divergent result of perturbation theory. It has been conventional to say that if one puts the logarithm thus obtained equal to unity and computes the lifetime, the value of about sixty times the experimental one, which is found, is qualitative support for the correctness of the basic picture of π decay. The computation of the π-decay lifetime is really impossible for a person who believes seriously in the renormalization program based on perturbation theory. The π lifetime is a primitively divergent quantity whose presence must be accounted for by the existence of a so-called counter term which evidently then serves to remove the divergence and put in the observed decay rate by hand. One must prescribe the renormalized value of this divergent quantity. We obviously do not subscribe to this philosophy. Our feeling is that the function $H(\xi)$ plays a critical role and that the perturbation theoretical indications are irrelevant.

We obviously do not know enough about the complex $^1S_0$ phase shift for neutron-antiproton scattering to make a real quantitative study of J. What we have done, therefore, is to make a few simple models which have reasonable low energy behavior, and hope that they are not too insane at high energies. The reason that this may not be too unrealistic a procedure is that, provided only $H(\xi)\to 0$ however weakly for large ξ, the integral J exists. Furthermore, since it occurs in the denominator, multiplied by $G^2/4\pi\approx 15$, we see that if $J\gtrsim 1/15$ the J term dominates the denominator; neglecting the unity, then, J cancels out. This is a kind of strong coupling limit leading to $F(0)$ inversely proportional to G, instead of proportional to it, as would be given in weak coupling. In the case of several types of baryon loops there would evidently be a kind of mean value of $1/G$ defined by $\sum G_iJ_i/\sum G_i^2J_i$. In the global symmetry strong coupling limit ($G_i=G$) the earlier result continues to hold.

The models treated take for δ_0 the representation

$$\tan\delta_0=k(a+ib),\quad k=[(\xi/4)-m^2]^{\frac{1}{2}},$$

which leads to

$$\tan\phi_0=ka/(1+kb),$$

and we define $\delta_0(k=0)=0$. Various limiting cases of this expression have been studied ($a\gg b$, $b\gg a$) and in every case we find $J\gtrsim 0.7$. In a very unphysical case, namely, that of no absorption, $b=0$, all integrals may be evaluated analytically, and we find

$$J=\frac{2}{\pi}\frac{ma+1}{ma-1}\{1-(m^2a^2-1)^{-\frac{1}{2}}\tan^{-1}(m^2a^2-1)^{\frac{1}{2}}\},$$

for the not unreasonable value of $ma\approx 3$ we find $J=0.7$.

Finally, making our strong coupling approximation, we obtain

$$F(0)=-(\sqrt{2}Gmg_A/\pi^2)[0.11],$$

using $G^2/4\pi=15$. This is to be compared with the experimental value given earlier, namely,

$$F(0)=-(\sqrt{2}Gmg_A/\pi^2)[0.115].$$

The agreement is rather impressive. It would be nice to hope that the neglect of all of the millions of states which we have made is contained in the 5% discrepancy. We are not quite so optimistic, but it is our feeling that the most important elements of the π-decay problem have been taken into account, and that a reasonable quantitative understanding of the process has been obtained.