degree of distortion before separation into fragments must take place. It is also of interest to note that R_0 has increased more for symmetric fission than for asymmetric fission.

The experimental data for the spontaneous fission of Cf^{252} show that an R_0 curve is required which peaks at a smaller mass ratio than in Fig. 1 and which falls more steeply from the peak in the direction of symmetry. The data for the fast neutron fission of Ra^{226} indicate that an R_0 curve is required which peaks at a greater mass ratio than in Fig. 1 and which probably falls less steeply from the peak in the direction of symmetry. These preliminary analyses suggest that there are systematic trends in the properties of the R_0 curve as one passes from one nucleus to another, but much more work remains to be done before such trends can be established in detail.

The general conclusions concerning nuclear sizes which can be drawn from this work are that there tends to be an expansion in the nuclear surface with excitation energy which is probably not closely correlated with a volume expansion coefficient, and that there is a smaller radius of separation into fragments in symmetric fission than in asymmetric fission.

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Interpretation of Scattering Cross Sections in Terms of Nuclear Size

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1. INTRODUCTION

As experimental measurements of scattering cross sections are made with greater and greater accuracy, so our ideas of interpreting them in terms of nuclear size have to be continuously refined. Thus while only a few years ago we referred to something vaguely called the nuclear radius R, which obeyed approximately the law,

$$R=r_0A^{\frac{1}{3}}, \tag{1}$$

we now distinguish a halfway radius c from a root-mean-square radius $[\langle r^2\rangle]^{\frac{1}{2}}$, we investigate a surface transition region and the relative density of neutrons and protons in it, we try to relate the density distribution to an optical model potential and we attempt to fit experimental proton-nucleus cross sections of such complexity that even the variation of an arbitrary function and four arbitrary constants is not enough to obtain a good fit.

Let us summarize the deductions that can be made from the experimental results.

A. The Stanford electron scattering experiments show that the proton distribution is characterized by a central region of constant density and a surface region in which the density drops to zero, which within quite narrow limits ($\pm 5\%$) is of constant thickness for all spherical nuclei. Because of the extreme insensitiveness of the electrostatic potential to small variations in the charge density, no information can be obtained about the functional form of the proton density distribution.[1]

B. Nucleon-nuclear scattering experiments in the 0–90-Mev region yield information about the parameters of the optical model potential. The complexity of the results is matched here by the complexity of the parameters, some of which are certainly energy dependent. All that can be said at present is that although there is no single set of parameters that fits the experimental results better than all others, the optical potential would appear to extend beyond the distribution of nuclear matter.[2,3]

C. The absorption cross sections for pions and nucleons by nuclei in the Bev region give the most direct information about nuclear size. They have shown that there is no significant difference between the neutron and proton distributions.[4]

2. ANALYSIS OF ASSUMPTIONS MADE

It is now necessary to point out two conclusions which have been drawn erroneously by other investiga-

[1] R. Hofstadter, Revs. Modern Phys. 28, 214 (1956).
[2] Bjorklund, Blandford, and Fernbach, Phys. Rev. 108, 795 (1957); A. E. Glassgold and P. J. Kellogg, Phys. Rev. 107, 1372 (1957), and private communication; R. Jastrow and I. Harris, O.N.R. Decennial Symposium 1957.
[3] L. R. B. Elton, Nuclear Phys. 5, 173 (1958).
[4] Abashian, Cool, and Cronin, Phys. Rev. 104, 855 (1956).

tors from the results of the Stanford group. The first is that the halfway radius, whether of the density distribution or of the potential, is proportional to $A^{\frac{1}{3}}$. This is caused partly by a hangover from the days when an $A^{\frac{1}{3}}$ proportionality was all we had to hold onto, and partly to a mistaken reading of a remark by the Stanford group[5] that the halfway radius of the charge distribution "varies as $A^{\frac{1}{3}}$ to within $\pm 2\%$." This statement was carefully qualified to apply only to the nuclei investigated, which ranged from Ca to Bi. Later results for C, Mg and Si clearly showed that there was no such simple proportionality over the whole range of A. Furthermore, there has never been any good reason why the halfway radius for the potential should be proportional to $A^{\frac{1}{3}}$, and such a proportionality should most certainly not be used as a criterion of goodness of fit when analyzing experimental results.

The second error is that the nuclear scattering results are independent of the functional form of the nuclear potential. The reason why electron scattering is so insensitive to variations in the charge density is of course that the electrostatic potential is insensitive to such changes. There is no evidence that changes in the nuclear potential shape will not affect nuclear scattering and every reason to suppose that they will.

Recently[3] an analysis of the electron scattering results has been based on the assumptions that

(a) neutrons and protons have the same density distribution,

(b) the central density in nuclei is independent of the mass number A,

(c) the thickness of the surface region is independent of A, i.e., that in the usual formula for the nuclear density,

$$\rho(r)=\rho(0)\frac{1+\exp(-c/a)}{1+\exp[(r-c)/a]}, \tag{2}$$

only c is A dependent. The condition

$$A=4\pi\int_0^\infty \rho(r)r^2dr \tag{3}$$

then leads to the formulas

$$c=pA^{\frac{1}{3}}-\frac{\pi^2a^2}{3p}A^{-\frac{1}{3}}+0(A^{-5/3}), \tag{4}$$

$$R=\surd(5/3)[\langle r^2\rangle]^{\frac{1}{2}}$$
$$=pA^{\frac{1}{3}}+\frac{5\pi^2a^2}{6p}A^{-\frac{1}{3}}-\frac{7\pi^4a^4}{8p^3}A^{-1}+0(A^{-5/3}), \tag{5}$$

where

$$\tfrac{4}{3}\pi\rho(0)p^3=1,$$

and (5) gives the radius R of the equivalent uniform distribution. A very good fit to the experimental results is then obtained with

$$p=1.121\pm0.005f,\quad a=0.575\pm0.005f, \tag{6}$$

which corresponds to a 90%–10% transition thickness of

$$t=2.53\pm0.02f. \tag{7}$$

Here, $1f=1$ fermi $=10^{-13}$ cm.

The halfway radius c_v for the nuclear potential may differ from this for any or all of the following reasons:

(a) The neutron distribution in heavy nuclei extends beyond the proton distribution, owing to the effect of the exclusion principle.

(b) The proton distribution extends beyond the neutron distribution, owing to the effect of the electrostatic repulsion.

(c) The potential extends beyond the matter distribution, due to the finite range of nuclear forces.

Although the effect due to (c) will be independent of A, that due to (a) increases with A and is negligible for $A<40$, since then the numbers of protons and neutrons in the nucleus are equal. The effect due to (b), which is in the opposite direction to that due to (a), also increases with A. An analysis of the available optical model data indicates that $c-c_v$ is approximately constant, so that the combined effect of (a) and (b) must be very small. In what follows we assume it to be negligible.

3. ABSORPTION CROSS SECTION AT HIGH ENERGIES

Unfortunately the optical model data still have rather large error brackets, and so a new analysis has been undertaken of the high-energy data for the scattering of neutrons and pions by nuclei in the Bev region.[6,7] Williams[8] has pointed out that it is possible to obtain the absorption cross section at these energies merely from a knowledge of the density distribution and the elementary two-body total cross section $\bar{\sigma}$. Following his method we obtain for the absorption cross section

$$\sigma_a=2\pi\int_0^\infty[1-e^{-KS(b)}]bdb, \tag{8}$$

where

$$K=2\bar{\sigma}\rho(0),\ S(b)=\frac{1}{\rho(0)}\int_0^\infty\rho[\surd(b^2+s^2)]ds. \tag{9}$$

For the functional form of the density we take

$$\rho(r)=\rho(0)\frac{1+\exp(-C^2/a^2)}{1+\exp[(r^2-C^2)/a^2]}, \tag{10}$$

since this form is more convenient algebraically, and

[5] Hahn, Ravenhall, and Hofstadter, Phys. Rev. **101**, 1131 (1956).

[6] Coor, Hill, Hornyak, Smith, and Snow, Phys. Rev. **98**, 1369 (1955).

[7] Cronin, Cool, and Abashian, Phys. Rev. **107**, 1121 (1957).

[8] R. W. Williams, Phys. Rev. **98**, 1387 (1955).

yields the same results for electron scattering provided the transition thickness T, which in this case is given by

$$T=\sqrt{(c^2+2a^2\ln 3)}-\sqrt{(c^2-2a^2\ln 3)} \qquad (11)$$

is taken about 10% larger than for the form (2).[5,9] Then

$$S(b)=\frac{a}{2}\int_0^\infty \frac{x^{-\frac{1}{2}}}{1+\exp(x-k)}dx, \quad k=\frac{C^2-b^2}{a^2}, \qquad (12)$$

and this integral is tabulated.[10] The density distribution (10) is somewhat more realistic than that employed by Williams and by Cool *et al.* which cuts off at a definite point. Because the distribution employed here is more spread out, it will give slightly larger cross sections than that of either Williams or Cool for the same values of C and T. However the difference, even for the lightest nuclei, is at present within the experimental error.

It is clear from (8) that σ_a is a function of three parameters, C, T, and K. We must now look more closely at the meaning of the density distribution. The incident particle is affected by a target nucleon only if it can scatter from it, and, compared with scattering from a free nucleon, this scattering is modified by the exclusion principle. This has two effects working in opposite directions. On the one hand, very small momentum transfers to the target nucleon are inhibited, since momentum states near that of the target nucleon are occupied. On the other hand, scattering as a whole is enhanced by a correlation between the target nucleons, which is due to the antisymmetrization of the target wave function. At energies in the Bev region neither effect is large, and we assume the effective two-body cross section $\bar{\sigma}$ to be the same as for free particles. However, a modification of the surface may well occur,

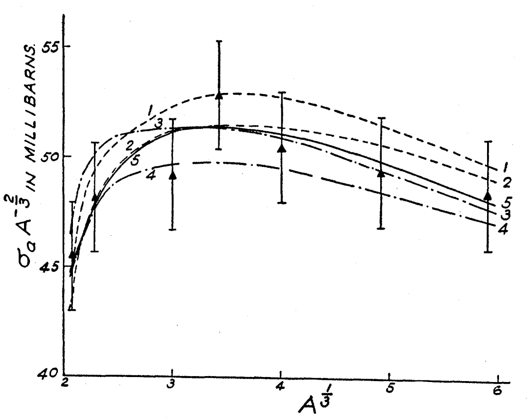

FIG. 1. Absorption cross sections for 1.4 Bev pions: (1) $C-c=0.35f$, $T=2.61f$, (2) $C-c=0.30f$, $T=2.61f$, (3) $C-c=0.40f$, $T=2.30f$, (4) $C-c=0.35f$, $T=2.30f$, (5) $C-c=0.35f$, $T=2.45f$. In all cases $K=1.15f^{-1}$, corresponding to $\rho(0)=0.175f^{-3}$, $\bar{\sigma}=3.3f^2$.

[9] G. E. Brown and L. R. B. Elton, Phil. Mag. **46**, 164 (1955).
[10] J. McDougall and E. C. Stoner, Trans. Roy. Soc. (London) **A237**, 67 (1939).

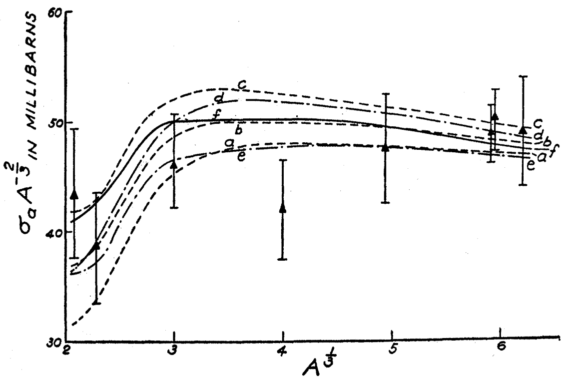

FIG. 2. Absorption cross sections for 1.4 Bev neutrons: (a) $C-c=0$, $T=2.61f$, (b) $C-c=0.1f$, $T=2.61f$, (c) $C-c=0.2f$, $T=2.61f$, (d) $C-c=0.1f$, $T=2.80f$, (e) $C-c=0.1f$, $T=2.45f$, (f) $C-c=0.2f$, $T=2.45f$. In all cases $K=1.50f^{-1}$, corresponding to $\rho(0)=0.175f^{-3}$, $\bar{\sigma}=4.3f^2$.

and we therefore analyze the high-energy results on the following assumptions:

(a) the central density of nuclear matter is given by the electron scattering results (6),

(b) the halfway radius C differs from the electron scattering radius c by a constant amount,

(c) the surface thickness T is constant, but not necessarily the same as for electron scattering,

(d) the two-body cross section $\bar{\sigma}$ is given by the free cross section, within a few percent.

From this discussion it is clear that the density thus obtained is not necessarily the actual mass density and that in particular there is no good reason why it should satisfy (3).

In making a detailed analysis of the variation of $y=\sigma_a A^{-\frac{2}{3}}$ with C, T, and K, it was found (see Figs. 1 and 2) that an increase in C resulted in an increase in y for all A, that an increase in T had almost no effect on y for small A but a large effect for large A, and that a 10% decrease in K [which could be due to a change of either $\rho(0)$ or $\bar{\sigma}$] could be compensated for by an increase of C by $0.05f$. Since our $\rho(r)$ does not satisfy (3), arguments based on the effect of a redistribution of nucleons do not apply.

The pion scatterings results would appear to be more accurate—they certainly show a greater internal consistency as regards variation with A—and these were analyzed first. We took[7] $\bar{\sigma}=33$ mb, which gave $K=1.15f^{-1}$. Figure 1 shows that a very good fit could be obtained with $C-c=0.35f$, $T=2.45f$. More detailed study shows that the results for small A are very sensitive to quite small variations in $C-c$, while those for large A are more sensitive to variations in T, so that we can put fairly conservatively

$$C-c=(0.35\pm0.05)f, \quad T=(2.45\pm0.15)f. \qquad (13)$$

The most obvious difference between the pion and neutron scattering results is that for neutrons $\sigma_a A^{-\frac{2}{3}}$

does not appear to have a maximum for medium A. Further, the cross sections generally are slightly smaller, which is most surprising, since the free cross section[8] is $\bar{\sigma}=43$ mb. To reduce the cross sections in general we must reduce C as compared with (13), and, to enhance relatively those for large A, we must increase T. This has been done in the analysis, as shown in Fig. 2, where the error brackets on the experimental results have been doubled as compared with those quoted in reference 8, in the belief that the results should show a much smoother variation with A than they do. The error brackets for $C-c$ and T must clearly be much larger in this case, but it has been impossible to obtain even moderately good fits outside

$$C-c=(0.1\pm0.1)f, \quad T=(2.60\pm0.30)f. \tag{14}$$

4. CONCLUSION

The most important conclusion is that $C-c$ is very small and that there is no evidence for any variation of it with A. This shows that any difference between the proton and neutron distributions must be very small. In fact $C-c$ is of the order of the reduced wavelength of the incident particles ($0.09f$ for neutrons, $0.13f$ for pions) and may well be due at least partially to the fact that the incident particles have a finite wavelength. That the surface should be different in its effect on pions and neutrons is reasonable; what is difficult to understand is why the effect on the pions should be so much larger. That the radii are no larger than they are, and certainly much smaller than those required by the optical model analysis of elastic scattering,[3] shows that Williams's basic assumption is sound—the particles that are absorbed interact with the mass distribution and not the potential. The transition region is about the same for neutron scattering as for electron scattering, for pion scattering it is somewhat, but not significantly, smaller. The density distributions of Pb for electron, neutron, and pion scattering are plotted in Fig. 3. Lastly, very little can be said about any significant effect of the exclusion principle on $\bar{\sigma}$, since a quite large change in $\bar{\sigma}$ can be compensated for by a small change in C.

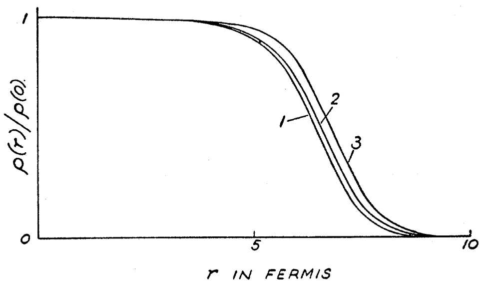

FIG. 3. The density distribution in the nucleus of Pb, as obtained from (1) elastic electron scattering, (2) neutron absorption, and (3) pion absorption.

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