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Isotope Shift Studies of Nuclei

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1. SUMMARY OF EXPERIMENTAL RESULTS

THE information about size and angular shape of atomic nuclei that is available from isotope shifts in the optical spectra of heavy elements can be condensed into one experimental number for each pair of isotopes. This is due to the fact that the shifts attributable to the field effect of valence *s* electrons are—within the theoretical framework and experimental accuracy available at present—proportional to the charge densities of these electrons at the nucleus. An isotope shift constant[1,2] C_{exp} can be defined, therefore, as

$$C_{\mathrm{exp}}=\Delta T_s/(\psi_s^2(0)\pi a_H^3/Z), \tag{1}$$

where ΔT_s is the level shift due to an *s* electron, $\psi_s(0)$ is the unrelativistic wave function of that electron at the origin, a_H is the Bohr radius, and Z the atomic number. The quantity $\psi_s^2(0)$ can be evaluated either from the magnetic hyperfine structure due to the *s* electron, or from the level scheme of the respective atom or ion. For elements with more than two isotopes it is usually convenient to measure C_{exp} for one selected pair, and relative isotope shifts for the others.

Isotope shift constants have so far been evaluated for the region $36<Z<83$ only. Figure 1 shows the data for isotopic pairs with an even number of neutrons. Apart from a factor $\beta\approx 1$, considered later, the ordinate is the ratio $C_{\mathrm{exp}}/C_{\mathrm{th}}$. C_{th} is the isotope shift constant calculated for a standard model of spherical, homogeneously charged, incompressible nuclei with radius r_0 proportional to $A^{\frac{1}{3}}$, used only for the purpose of comparison. The ratio $C_{\mathrm{exp}}/C_{\mathrm{th}}$ shows how the quantity

$$\langle r^{2\sigma}\rangle=Z^{-1}\int r^{2\sigma}\rho dV, \tag{2}$$

with

$$\sigma^2=1-\alpha^2Z^2 \quad (\alpha=e^2/\hbar c), \tag{3}$$

increases in going from mass number A_1 to $A_2=A_1+\delta A$, compared with the change

$$\delta\langle r^{2\sigma}\rangle_{\mathrm{std}}=(2\sigma/2\sigma+3)(\delta A/A)r_0^{2\sigma} \tag{4}$$

for the standard model. For nuclei which are not spherically symmetric, the nuclear charge density $e\rho$ will be a function of angle as well as radius.

The strong variations with neutron number of the quantity plotted in Fig. 1 can be accounted for in terms of changes in nuclear deformations. The isotope shifts, therefore, provide a sensitive measure of the change of intrinsic quadrupole moments of nuclei. The data show furthermore that addition of neutrons to a nucleus changes the charge radius less than given by the $A^{\frac{1}{3}}$ law. This can be understood by assuming a nuclear compressibility.

Several recent reviews[3–8] exist of the theory and interpretation of isotope shifts, to which the reader is referred for a detailed discussion. A complete literature survey up to July, 1956, is given by Mack and Arroe.[5] The published compilation of experimental isotope shift constants, however, is out of date now. Therefore, we present in Table I the numerical values and the literature used in the preparation of Fig. 1.

With the gross features of nuclear electric radii now reliably known from other sources, the isotope shifts should provide the most sensitive measure available of the change of nuclear charge distribution from one isotope to another. There are, unfortunately, several difficulties involved in obtaining the values of $\delta\langle r^{2\sigma}\rangle$ from the spectroscopic measurements.

(a) Uncertainties Attributable to C_{th}

Calculations should include effects of the intrinsic magnetic moment of the electron and of a possible polarization of nuclei by atomic electrons, as has been pointed out by Breit[7] and co-workers.

(b) Uncertainties Attributable to C_{exp}

The reliable experimental determination of ΔT_s, $\psi_s^2(0)$, and of the mass dependent isotope shifts present other problems, which Breit[9] was the first to discuss.

In Sec. 2 we sketch how C_{exp} has been derived in a typical case. The element Gd is chosen both because of its complex spectrum and its position in the interesting region of strongly deformed nuclei.

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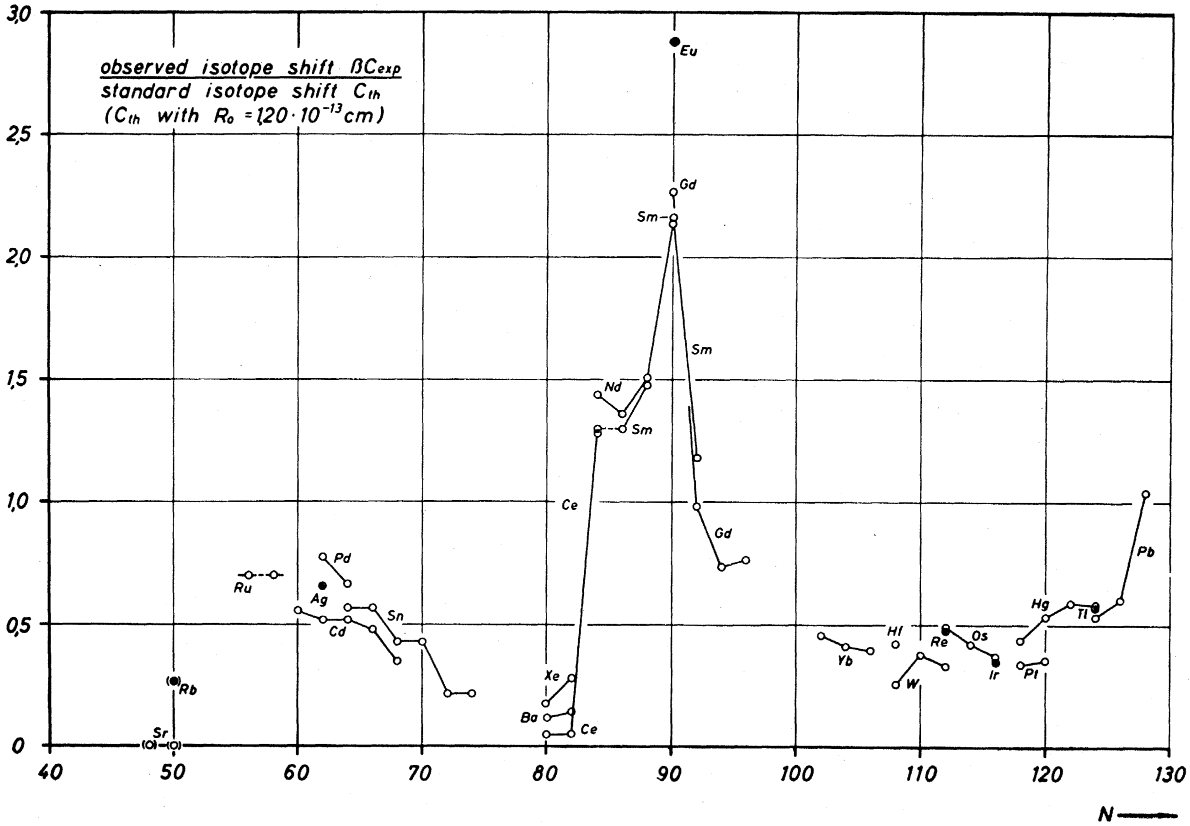

FIG. 1. Ratio of observed isotope shift to standard shift for uniform density nuclei $(r_0=R_0A^{\frac{1}{3}})$. Points are plotted *versus* neutron number N, and are placed at the higher N value for each pair of isotopes. Full circles indicate that Z is odd. The values given for Sr and Rb are rough estimates only.

2. TYPICAL EVALUATION OF $C_{\rm exp}$ FOR A COMPLEX SPECTRUM

The spectra of Gd I and Gd II have been very carefully analyzed by Russell,[10] who assigned electron configurations to many levels of the neutral and singly ionized atom. With samples of natural isotopic composition the isotope shifts are most easily measured for the pair Gd^{158}-Gd^{160}. The number of lines studied by different authors[11–14] amounts to about 120. From these the shifts of about 50 Gd I levels and 35 Gd II levels have been deduced. A discussion of perturbations finally leads to the shifts of presumably unperturbed configurations presented in Fig. 2.

All data of Fig. 2 can be described as follows: a shift of about 74×10^{-3} cm^{-1} is assigned to the $6s$ electron in the configuration $4f^7 6s$ of Gd III. The other valence electrons reduce this isotope shift by reducing the $6s$ wave function at the nucleus; these screening effects amount to $\sim20\%$ for a $5d$ or $6s$ electron and $\sim10\%$ for a $6p$ electron. A comparison with screening effects of the valence electrons in Hg,[2,15] included in Fig. 2, shows good numerical agreement. It is concluded that

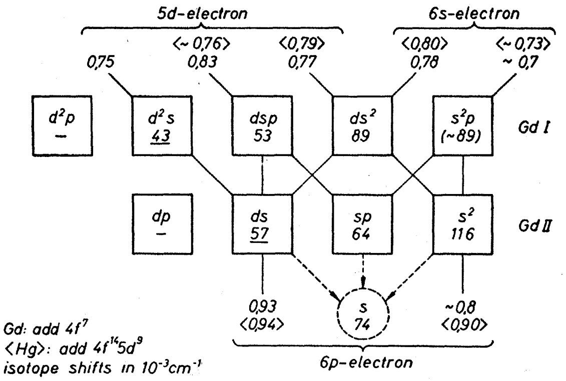

FIG. 2. Isotope shift of Gd^{158}–Gd^{160} and screening effects. The experimental shifts are given in the squares, with zero shifts assigned to levels of $5d^2 6p$ and $5d6p$, respectively, which are presumably unperturbed. Corrections for configuration interaction have been made, if necessary. From the ratios of the shifts the screening factors given at the ends of the connecting lines have been obtained. The corresponding factors for Hg are shown in brackets. The broken lines indicate an extrapolation for the "unscreened" $6s$ electron in $4f^7 6s$. The quantum numbers are always $5d$, $6s$, and $6p$.

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TABLE I. Isotope shift constants C_{th} (with $r_0=A^{\frac{1}{3}}\times 1.20\times 10^{-13}$ cm), and βC_{exp}. The factor β, explained in the text, is customarily taken as unity. The βC_{exp} values not given directly can be obtained from the relative shifts.

							Literature	
Element	Z	A_1-A_2	N	C_{th} 10^{-3} cm^{-1}	βC_{exp} 10^{-3} cm^{-1}	Relative shift	for βC_{exp}	for relat. shifts, and general
Rb	37	85– 87	50	30.1	8±12		a	b,c
Sr	38	84– 86	48	32.5	∼0		d	b,d
		86– 88	50	32.3	∼0			
Ru	44	96– 98	54	49.0	34± 9 average		e	c,f
		98–100	56	48.7				
		100–102	58	48.3				
		102–104	60	48.0				
Pd	46	106–108	62	54.0	42± 9		g	c
		108–110	64	53.7	36± 5			
Ag	47	107–109	62	58.4	38± 6		h	b,c
Cd	48	106–108	60	62.8		1.07	i	b,c,i–l
		108–110	62	62.4		1.00		
		110–112	64	62.0	32± 4	1		
		112–114	66	61.6		0.93		
		114–116	68	61.2		0.67		
Sn	50	112–114	64	70.9	40±10		m	b,c
		114–116	66	70.5	40±10			
		116–118	68	70.0	30±10			
		118–120	70	69.5	30±10			
		120–122	72	69.1	15±10			
		122–124	74	68.7	15±10			
Xe	54	132–134	80	88		∼0.7	a	b,c
		134–136	82	88	25± 6	1		
Ba	56	134–136	80	100	12± 5		n,a	b,c,n,o
		136–138	82	99	15± 5			
Ce	58	136–138	80	115		0.04	a	b,c
		138–140	82	114		0.04		
		140–142	84	113	147±30	1		
Nd	60	142–144	84	131	187±35	1.03	p	b,c,q
		144–146	86	130		0.97		
		146–148	88	129		1.08		
		148–150	90	129		1.53		
Sm	62	144–146	84	148	190±40 average	2	a	b,c
		146–148	86	148				
Sm	62	148–150	88	147		1.14		
		150–152	90	146		1.67		
		152–154	92	145		0.91		
Eu	63	151–153	90	156	450±50		r	b,c
Gd	64	152–154	90	167		3.00	s	c,t,u
		154–156	92	166		1.30		
		156–158	94	165		0.97		
		158–160	96	164	125±20	1		
Yb	70	170–172	102	242	110±10		v	b,c,w
		172–174	104	241	99± 8			
		174–176	106	240	94± 8			
Hf	72	178–180	108	272	114±13		x	b,c
W	74	180–182	108	314		0.68	a	b,c,y,z
		182–184	110	312	117±30	1		
		184–186	112	311		0.88		
Re	75	185–187	112	332	157±40		i	b
Os	76	186–188	112	357		1.3_5	i	b,c,i
		188–190	114	355		1.15		
		190–192	116	354	130±20	1		
Ir	77	191–193	116	377	130±30		aa	b,c
Pt	78	194–196	118	402	135±25	1	a	b
		196–198	120	400		1.04		
Hg	80	196–198	118	462		0.74	a	b,c,i,bb–ff
		198–200	120	460		0.90		
		200–202	122	458	270±30	1		
		202–204	124	456		0.99		
Tl	81	203–205	124	487	280±40		a	b
Pb	82	204–206	124	524		0.89	a	b,c,i,gg
		206–208	126	522	315±35	1		
		208–210	128	520		1.74		

The C_{th} values are those calculated by Humbach [Z. Physik **133**, 589 (1952)], modified for the differences in radius and, if necessary, in mass numbers. In the second last column, only the latest determination of βC_{exp} has been mentioned. All other work, including the measurements of the relative shifts, is cited in the last column mainly by reference to other compilations.

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[b] P. Brix and H. Kopfermann, in Landolt-Börnstein, *Zahlenwerte und Funktionen* (1951), sixth edition, Vol. I/5, p. 1.
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[bb] K. Murakawa and S. Suwa, J. Phys. Soc. Japan **5**, 429 (1953).
[cc] G. R. Fowles, J. Opt. Soc. Am. **44**, 85 (1954).
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all observed Gd isotope shifts can be understood on the basis of a nuclear field effect and that, therefore, Eq. (1) is applicable.

Only empirical methods[16] were available at first to evaluate $\psi_s^2(0)$ for the $5d^26s$ and the $5d6s$ configurations. With these the isotope shift constants 134×10^{-3} cm^{-1} and 120×10^{-3} cm^{-1}, respectively, were obtained from the underlined values of Fig. 2. Recently the magnetic hyperfine structure splitting constant a_s of Gd^{157} for $5d^26s$, as well as the ratio of the magnetic moments $\mu(\mathrm{Gd}^{157})/\mu(\mathrm{Eu}^{151})$ have been measured.[17,18] These data in connection with the isotope shift constant of Eu give $\sim 108\times 10^{-3}$ cm^{-1} as a third value, not dependent upon an empirical evaluation of $\psi_s^2(0)$ for Gd, and in agreement with the formerly published average

$$\beta C_{\mathrm{exp}}(\mathrm{Gd}^{158}-\mathrm{Gd}^{160})=(125\pm 20)\times 10^{-3}\ \mathrm{cm}^{-1}.$$

The factor β indicates that a further uncertainty exists due to a possible contribution to the isotope shift of

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the inner closed shells of the Gd atom: the wave functions of the $1s^2$, $2s^2$, $\cdots$ shells could possibly be affected by changes in electron configuration of the valence electrons. Crawford and Schawlow[19] and later Humbach[20] have tried to calculate β and for Hg the result seems to be $\beta \approx 1$. For Gd II the only estimate available is from the isoelectronic Eu I spectrum, where the isotope shift of $6p_{\frac{3}{2}}$ electrons has been used[13] to get the empirical estimate $\beta \approx 1$.

Mass dependent effects have been neglected. They are assumed to be unimportant for Gd for theoretical reasons, but no quantitative values can be calculated at present. The fact that the nonequidistant relative isotope shifts of Gd[17,21] and other rare earth elements are constant within experimental error is the most conclusive argument against appreciable mass effects.

We believe that the error given above for βC_{exp} is conservative and that βC_{exp} can be assumed to give a fair estimate of C_{exp} with an accuracy of about $\pm 20\%$. This uncertainty is typical for all isotope shift constants and should not be overlooked. The relative shifts, on the other hand, are usually much more accurate, with uncertainties of a few percent or less in favorable cases.

We are indebted to Dr. G. Nöldeke for his help in bringing Table I up to date.

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